



# Compilation Principle 编译原理

第6讲: 语法分析(3)

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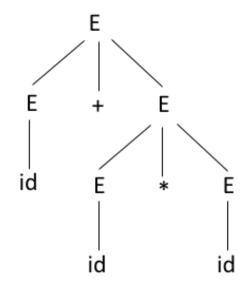


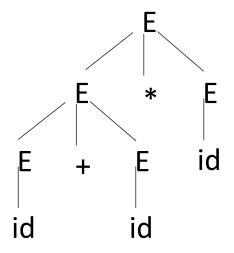


#### Review: Ambiguous Grammar

- Grammar E→E\*E | E+E | (E) | id
  - Ambiguous. Why?
  - Two distinct leftmost derivations for the sentence id + id \* id

- Are the two trees have the same meaning?
  - Above: id + (id \* id)
  - Below: (id + id) \* id
- The deepest sub-tree is traversed first, thus higher precedence







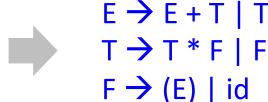


#### Review: Ambiguity Removal

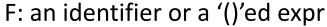
- How to remove the ambiguity?
- Specify precedence
  - The higher level of the production, the lower priority of operator
  - The lower level of the production, the higher priority of operator
- Specify associativity
  - If the operator is left associative, induce left recursion in its production
  - If the operator is right associative, induce right recursion in its production

$$E \rightarrow E + E \mid T$$
  
 $E \rightarrow E * E \mid E + E \mid (E) \mid id$   
 $T \rightarrow T * T \mid F$   
 $F \rightarrow (E) \mid id$ 

```
still possible to get
id + (id + id)
and
(id + id) + id
what if '-' (minus)?
```



Now, can only have more '+' on left E: sum of one or more terms (T) T: product of one or more factors (F)







#### Review: Top-down and Bottom-up

Consider a CFG grammar G

$$S \rightarrow AB$$

$$A \rightarrow aC$$

$$B \rightarrow bD$$

$$D \rightarrow d$$

$$C \rightarrow c$$

This language has only one sentence: L(G) = {acbd}

Top-down (Leftmost Derivation)

$$S \Rightarrow AB (1)$$

$$\Rightarrow$$
 aCB (2)

$$\Rightarrow$$
 acB (3)

$$\Rightarrow$$
 acbD (4)

$$\Rightarrow$$
 acbd (5)

Bottom-up (reverse of rightmost derivation)

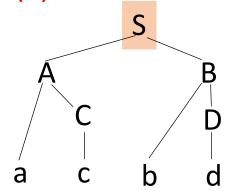
$$S \Rightarrow AB (5)$$

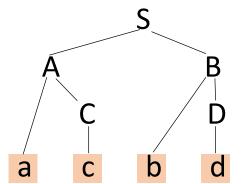
$$\Rightarrow$$
 AbD (4)

$$\Rightarrow$$
 Abd (3)

$$\Rightarrow$$
 aCbD (2)

$$\Rightarrow$$
 acbd (1)







#### Preview: Bottom-up Steps

#### Consider a CFG grammar G

S→AB

 $A \rightarrow aC$ 

 $B \rightarrow bD$ 

 $D \rightarrow d$ 

 $C \rightarrow c$ 

Stack	Input	Action
\$	acbd\$	Shift
\$a	cbd\$	Shift
\$ac	bd\$	Reduce
\$aC	bd\$	
\$A	bd\$	Reduce
\$Ab	d\$	Shift
\$Abd	\$	Shift
\$AbD	\$	Reduce
\$AB	\$	Reduce
\$S	\$	Reduce

Bottom-up (reverse of rightmost derivation)

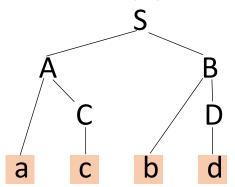
 $S \Rightarrow AB (5)$ 

 $\Rightarrow$  AbD (4)

 $\Rightarrow$  Abd (3)

 $\Rightarrow$  aCbD (2)

 $\Rightarrow$  acbd (1)

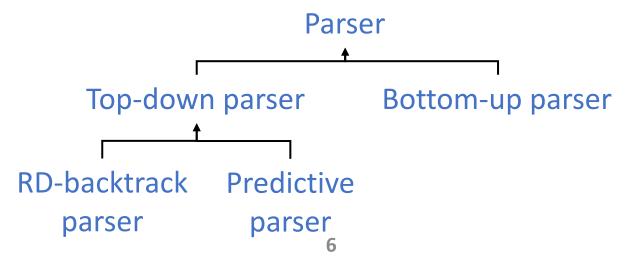






# Recursive Descent[递归下降]

- Recursive descent is a simple and general parsing strategy
  - Try and backtrack
  - Left-recursion must be eliminated first
    - Can be eliminated automatically using some algorithm
- However it is not popular because of backtracking
  - Backtracking requires re-parsing the same string
  - Which is inefficient (can take exponential time)
  - Also undoing semantic actions may be difficult
    - E.g. removing already added nodes in parse tree







# Predictive Parsers[预测分析]

- A parser with no backtracking: predict correct next production given next input terminal(s)
  - If first terminal of every alternative production is unique, then parsing requires no backtracking
  - If not unique, grammar cannot use predictive parsers

```
A \rightarrow aBD \mid bBB

B \rightarrow c \mid bce

D \rightarrow d
```

parsing input "abced" requires no backtracking





#### Predictive Parsers (cont.)

- A predictive parser chooses the production to apply solely on the basis of
  - Next input symbols
  - Current nonterminal being processed
- Patterns in grammars that prevent predictive parsing
  - Common prefix[共同前缀]:

$$A \rightarrow \alpha\beta \mid \alpha\gamma$$

Given input terminal(s)  $\alpha$ , cannot choose between two rules

- Left recursion[左递归]:

$$A \rightarrow A\beta \mid \alpha$$

Given input terminal(s)  $\alpha$ , cannot choose between two rules

What is the language of the grammar?  $\alpha\beta^*$ 





#### Rewrite Grammars for Prediction

- Left factoring[左公因子]: removes common left prefix
  - In previous example:  $A \rightarrow \alpha\beta \mid \alpha\gamma$
  - can be changed to

$$A \rightarrow \alpha A'$$
  
 $A' \rightarrow \beta \mid \gamma$ 

- Given input  $\alpha$ , A' can can choose between β or γ (Assuming β or γ do not start with  $\alpha$ )
- Left-recursion removal: same as for recursive descent
  - In previous example:  $A \rightarrow A\beta \mid \alpha$
  - can be changed to

$$A \rightarrow \alpha A'$$
  
  $A' \rightarrow \beta A' \mid \epsilon$ 

– Given input  $\alpha$ , A' can can choose between  $\beta$  or  $\varepsilon$  (Assuming  $\beta$  doesn't start with  $\alpha$  or A' isn't followed by  $\alpha$ )





#### LL(k) Parser / Grammar / Language

#### LL(k) Parser

- A predictive parser that uses k lookahead tokens
- L: scans the input from left to right
- L: produces a leftmost derivation
- k: using k input symbols of lookahead at each step to decide

#### LL(k) Grammar

- A grammar that can be parsed using an LL(k) parser
- LL(k)  $\subset$  CFG
  - Some CFGs are not LL(k): common prefix or left-recursion

#### LL(k) Language

- A language that can be expressed as an LL(k) grammar
- Many languages are LL(k) ... in fact many are LL(1)!





### LL(k) Parser Implementation

- Implemented in a recursive or non-recursive fashion
  - Recursive: recursive descent (recursive function calls)
  - Non-recursive: explicit stack to keep track of recursion
- Recursive LL(1) parser for:  $A \rightarrow B \mid C, B \rightarrow b, C \rightarrow c$ 
  - Parser consists of small functions, one for each non-terminal

```
int A() {
  int token = peekNext(); // lookahead token
  switch(token) {
    case 'b': // 'B' starts with 'b'
        return B();
    case 'c': // 'C' starts with 'c'
        return C();
    default: // Reject
        return 0;
}
```





# LL(k) Parser Implementation (cont.)

• Recursive LL(1) parser for:  $A \rightarrow B \mid C, B \rightarrow b, C \rightarrow c$ 

```
int A() {
  int token = peekNext(); // lookahead token
  switch(token) {
    case 'b': // 'B' starts with 'b'
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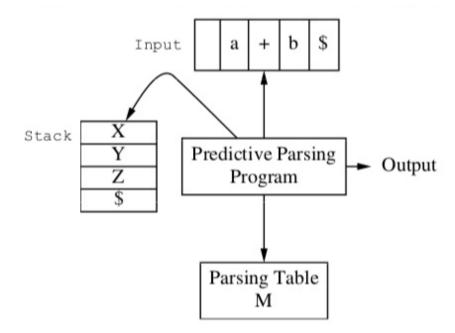
- Is there a way to express above code more concisely?
  - Non-recursive LL(k) parsers use a state transition table (Just like finite automata)
  - Easier to automatically generate a non-recursive parser





#### Non-recursive LL(1) Parser

- Table-driven parser: amenable to automatic code generation (just like lexers)
  - Input buffer: contains the string to be parsed, followed by \$
  - Stack: holds unmatched portion of derivation string
  - Parse table M(A, b): an entry containing rule " $A \rightarrow ...$ " or error
  - Parser driver (a.k.a., predictive parsing program): next action based on (stack top, current token)







#### LL(1) Parse Table: Example

Table	int	*	+	(	)	\$
E	E → TE′			E → TE′		
E'			E' → +E		E′ <del>→</del> ε	E' → ε
Т	$T \rightarrow int T'$			T → (E)		
T'		T′ → *T	T′ → ε		T′ <del>→</del> ε	T′ <del>→</del> ε

- Implementation with 2D parse table
  - First column lists all non-terminals in the grammar
  - First row lists all possible terminals in the grammar and \$
  - A table entry contains one production
    - One action for each (non-terminal, input) combination
    - It "predicts" the correct action based on one lookahead
    - No backtracking required





#### LL(1) Parsing Algorithm

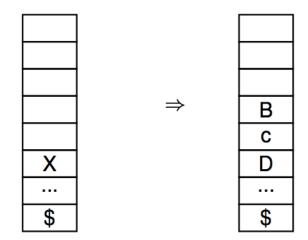
- Initial state
  - Input tape: input tokens followed by '\$'
  - Stack: start symbol followed by '\$' at bottom
- General idea: repeat one of two actions
  - Expand symbol at top of stack by applying a production
  - Match terminal symbol at top of stack with input token
- Step-by-step parsing based on (X,a)
  - X: symbol at the top of the stack
  - a: current input token
    - $\blacksquare$  If  $X \in T$ , then
      - If X == a == \$, parser halts with "success"
      - If X == a != \$, successful match, pop X from stack and advance input head
      - If X != a, parser halts and input is rejected
    - $\Box$  if  $X \in \mathbb{N}$ , then
      - if  $M[X,a] == 'X \rightarrow RHS''$ , pop X and push RHS to stack
      - if M[X,a] == empty, parser halts and input is rejected





#### Push RHS in Reverse Order

- For (X, a)
  - X: symbol at the top of the stack
  - a: current input token
- If  $M[X,a] = "X \rightarrow BcD"$



- Performs the leftmost derivation:  $\alpha \times \beta \Rightarrow \alpha \text{ BcD } \beta$ 
  - $-\alpha$ : string that has already been matched with input
  - $\beta$ : string yet to be matched, corresponding to the ... above





#### Applying LL(1) Parsing to Grammar

Consider the grammar

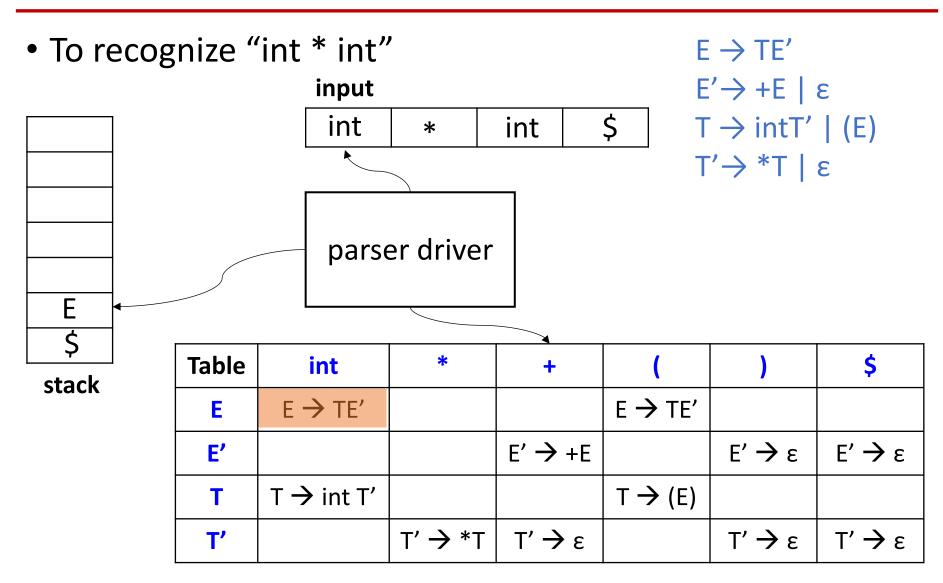
$$E \rightarrow T+E|T$$
  
 $T \rightarrow int*T | int | (E)$ 

- No left recursion
- But require left factoring
- After rewriting grammar, we have

$$E \rightarrow TE'$$
 $E' \rightarrow +E \mid \epsilon$ 
 $T \rightarrow intT' \mid (E)$ 
 $T' \rightarrow *T \mid \epsilon$ 

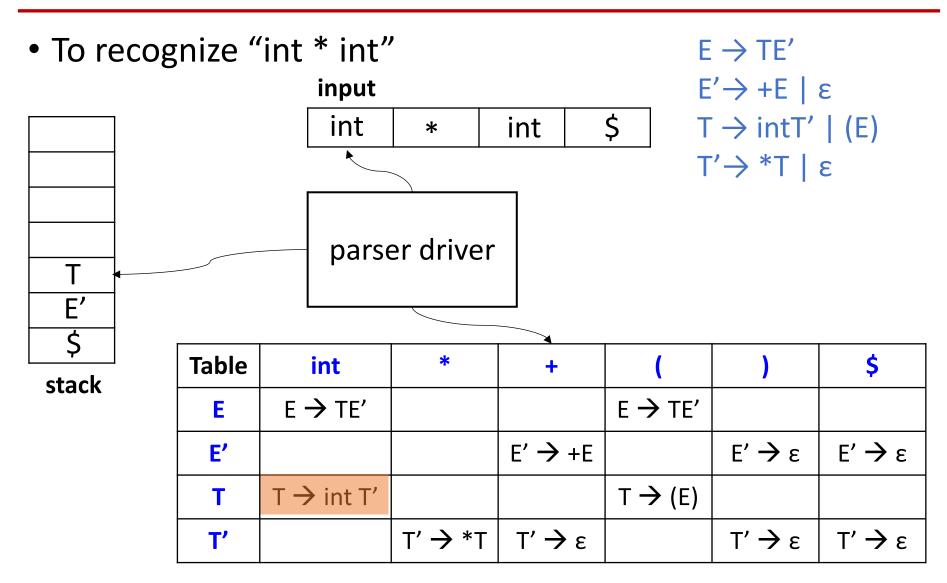






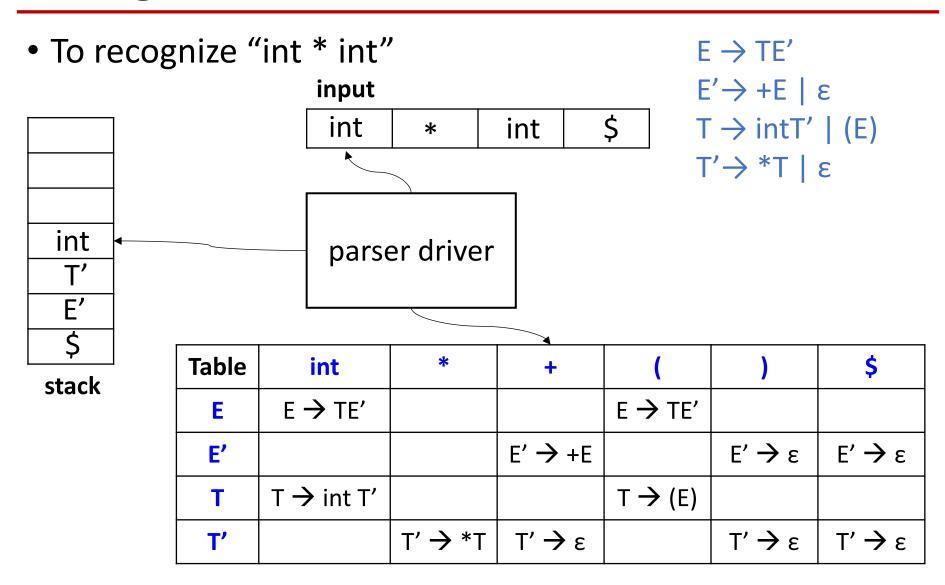






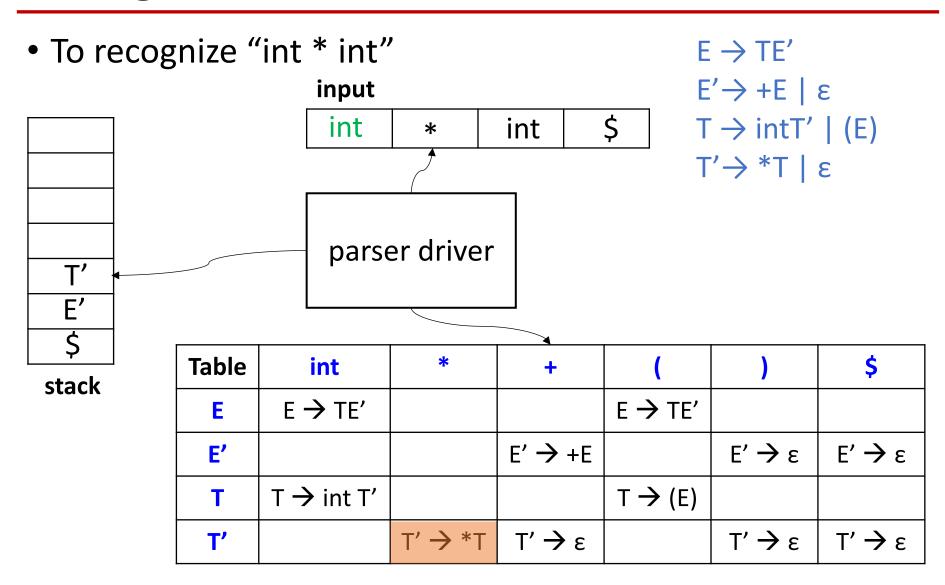






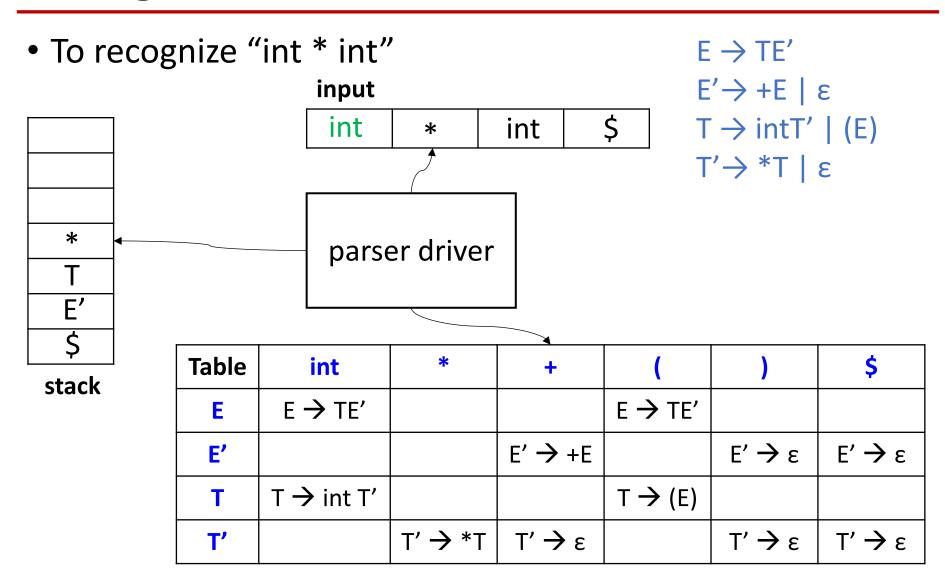






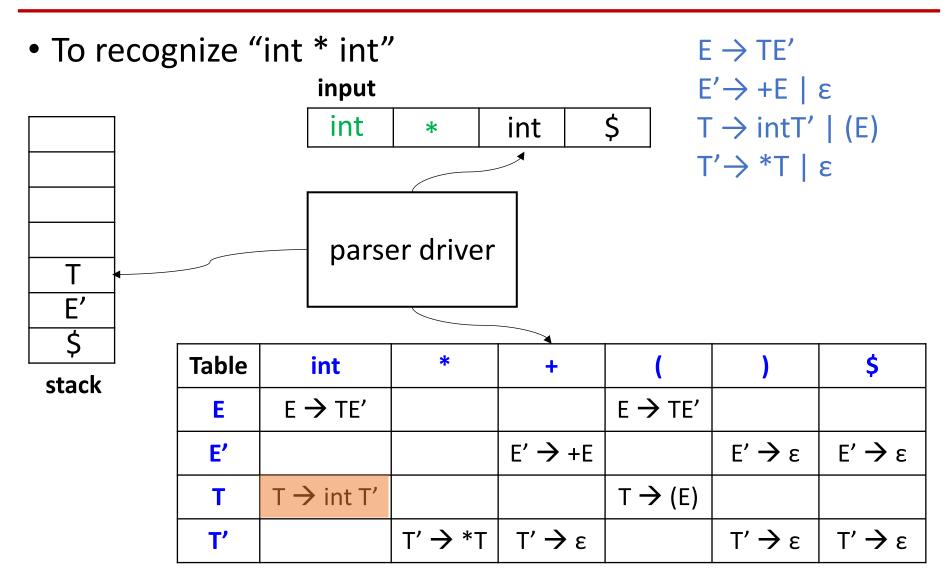






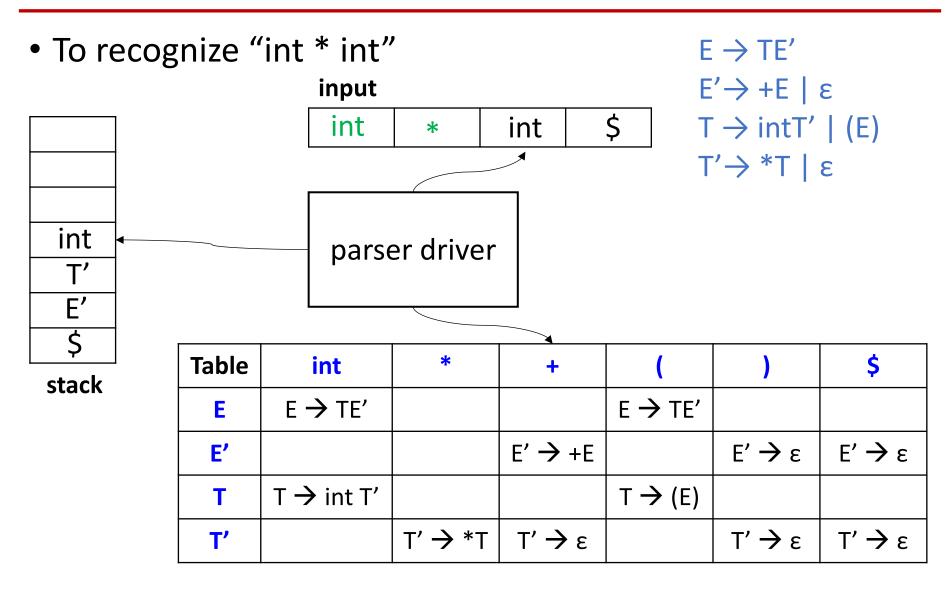






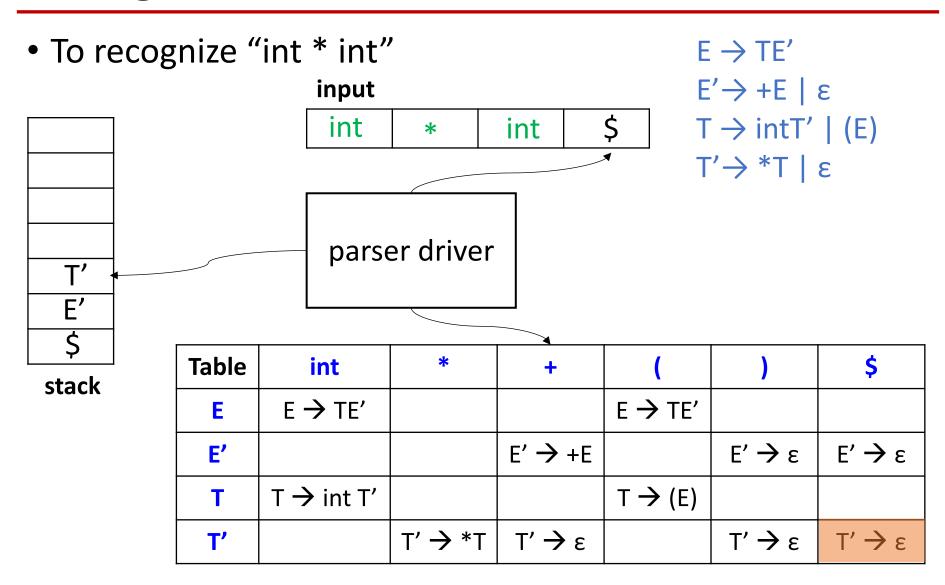






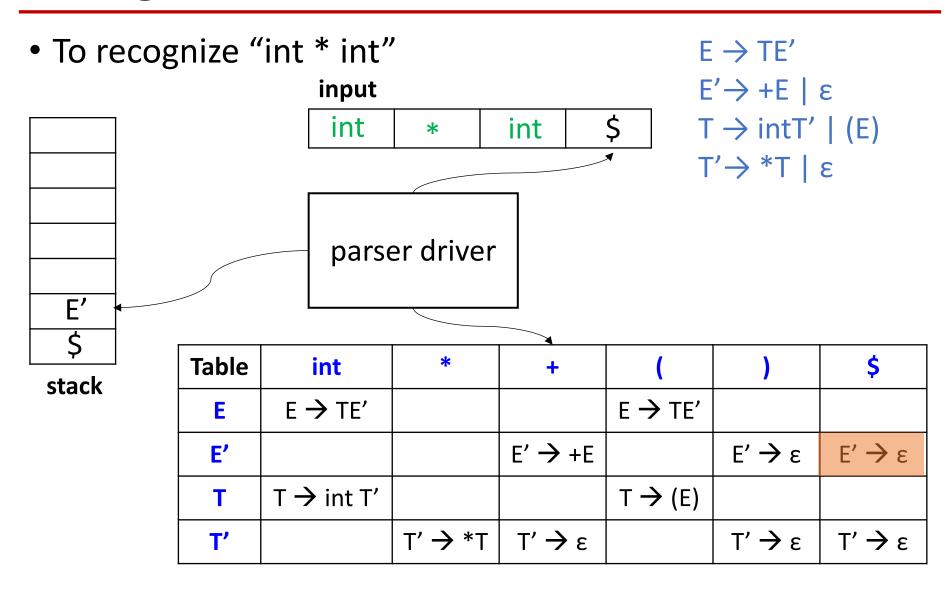






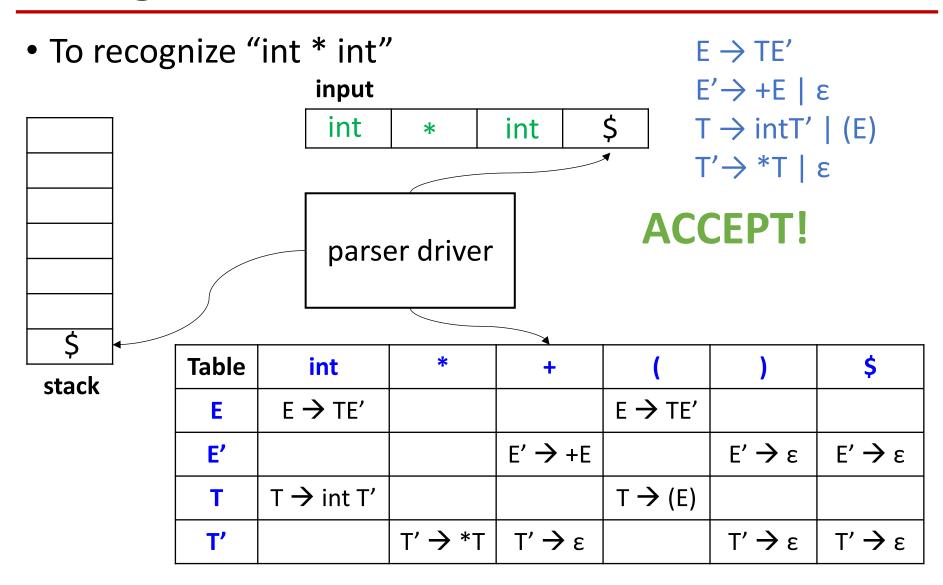
















#### Recognizing Sequence

Stack	Input	Action
E\$	int * int \$	E → TE'
T E' \$	int * int \$	$T \rightarrow int T'$
int T' E' \$	int * int \$	match
T' E' \$	* int \$	T′ → *T
* T E' \$	* int \$	match
T E' \$	int \$	T → int T'
int T' E' \$	int \$	match
T' E' \$	\$	T′ → ε
E' \$	\$	E' → ε
\$	\$	Halt and accept

$$E \rightarrow TE'$$
  
 $E' \rightarrow +E \mid \epsilon$   
 $T \rightarrow intT' \mid (E)$   
 $T' \rightarrow *T \mid \epsilon$ 

- Contents of stack correspond to remaining input
- Actions correspond to productions in leftmost derivation





#### Review Questions (1)

What is Recursive Descent?
 Parsing by trying and backtracking to produce the leftmost derivation

- Why do we prefer to use Predictive Parser?
   Requires no backtracking, more efficient
- How to predict the next production to use?
   Next input symbol, current nonterminal being processed
- What are the grammar requirements of predictive parse?
   No common prefix, no left recursion [唯一性]
- What does LL(k) mean?
  - L: scans the input from left to right
  - L: produces a leftmost derivation
  - K: using k input symbols of lookahead





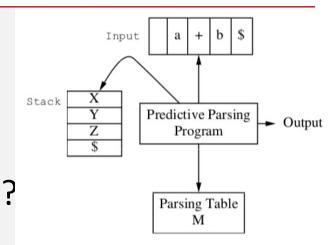
#### Review Questions (2)

What is the initial state of the parser?

Input: input tokens followed by \$

Stack: start symbol followed by \$

General idea of the table-driven parse?
 Expand on non-terminal, match on terminal



- How do we expand?
   If M[X, a] = "X → RHS", pop X and push RHS to stack
- What are stored in the parsing table?
   Actions the parser should take based on input token and stack top

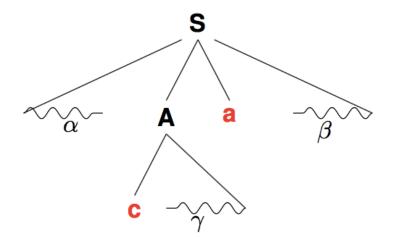
Table	int	*	+	(	)	\$
E	E → TE′			E → TE′		
E'			E′ → +E		E′ <b>→</b> ε	E′ <del>→</del> ε
Т	$T \rightarrow int T'$			T → (E)		
T'		T′ → *T	T′ <del>→</del> ε		T′ <del>→</del> ε	T′ <b>→</b> ε





#### To Construct Parsing Table

- The parsing table stores the actions the parser should take based on the <u>input token</u> and the <u>stack top</u>
- The parsing table can be constructed using two sets
  - FIRST(A): set of terminals that begin strings derived from A
    - □ E.g., c ∈ FIRST(A)
    - □ If A ⇒\* ε, then ε is also in FIRST(A)
  - FOLLOW(A): set of terminals that can appear following A
    - □ E.g., a ∈ FOLLOW(A)
    - If A is rightmost of a sentential form, then \$ is also in FOLLOW(A)







#### Use FIRST and FOLLOW

- Why do we need FIRST and FOLLOW in parsing?
- FIRST[开始集]
  - FIRST( $\alpha$ ): set of terminals that start strings derived from  $\alpha$
  - Consider A  $\rightarrow \alpha \mid \beta$ , where FIRST( $\alpha$ ) and FIRST( $\beta$ ) are disjoint sets
  - We can then choose by looking at the next input symbol a
    - $\square$  since  $\alpha$  can be in at most FIRST( $\alpha$ ) or FIRST( $\beta$ ), not both
- FOLLOW[后继集]
  - FOLLOW(A): set of terminals that can appear right after A
  - If there's a derivation of A that results in ε
    - □ In this case, A could be replace by nothing and the next token would be the first token of the symbol following A in the sentence being parsed
    - □ Thus, parser needs to consider to choose the path A ⇒\* ε





#### Example

```
Input: ada
                                                                                         Input: ade
Grammar:
S \rightarrow aBC
                                                   \Rightarrow aBC
B \rightarrow bC b \in FIRST(B)
                                                                                         \Rightarrow aBC
                                                  \Rightarrow adBC
B \rightarrow dB d \in FIRST(B)
                                                                                         \Rightarrow adB\bigcirc/
                                                                                        \Rightarrow adC/
                                                   \Rightarrow ad^{\triangleright}
B \rightarrow \epsilon
                                                   \Rightarrow ada
C \rightarrow c \quad c \in FOLLOW(B)
C \rightarrow a \quad a \in FOLLOW(B)
D \rightarrow e
```

Both FIRST and FOLLOW should be used to construct the parsing table





#### **FIRST**

- Compute FIRST(X) for all grammar symbols X, apply the following rules until no terminal or ε can be added to any FIRST set
  - If X ∈ T, then FIRST(X)={X} [终结符]
  - If X ∈ N and X → ε exists, then add ε to FIRST(X) [非终结符,空式]
  - If  $X \in \mathbb{N}$  and  $X \rightarrow Y_1Y_2Y_3...Y_k$ , then
    - □ Add  $\alpha$  to FIRST(X), if for some i,  $\alpha$  is in FIRST(Y<sub>i</sub>), and  $\epsilon$  is in all of FIRST(Y<sub>1</sub>), ..., FIRST(Y<sub>i-1</sub>), i.e., Y<sub>1</sub>...Y<sub>i-1</sub> ⇒\*  $\epsilon$ . E.g.,
      - Everything in FIRST(Y<sub>1</sub>) is surely in FIRST(X)
      - If  $Y_1$  doesn't derive  $\varepsilon$ , then we add nothing more
      - But if Y1  $\Rightarrow$ \*  $\epsilon$ , then we add FIRST(Y2), and so on
    - □ Add  $\varepsilon$  to FIRST(X), if  $\varepsilon$  is in FIRST(Y<sub>i</sub>) for all j=1,2,...k





#### FIRST(cont.)

• Compute FIRST(X) for all grammar symbols X [符号]

- Next, we can compute FIRST for any string  $\alpha = X_1X_2...X_n$ [符号串]
  - Add FIRST( $X_1$ ) all non- $\varepsilon$  symbols to FIRST( $\alpha$ )
  - Add FIRST(X<sub>i</sub>) ε), 2≤i≤k, to FIRST(α), if FIRST(X<sub>1</sub>), ..., FIRST(X<sub>k-1</sub>) all contain ε
    - $\square$  Add non-ε symbols of FIRST(X<sub>2</sub>), if ε is in FIRST(X<sub>1</sub>)
    - $\square$  Add non-ε symbols of FIRST(X<sub>3</sub>), if ε is in FIRST(X<sub>1</sub>) and FIRST(X<sub>2</sub>)
    - **-** ...
  - Add  $\varepsilon$  to FIRST( $\alpha$ ), if FIRST( $X_1$ ), ..., FIRST( $X_k$ ) all contain  $\varepsilon$





#### **FOLLOW**

- To compute FOLLOW(A) to all non-terminals A, apply following rules until no terminal or ε can be added to any FOLLOW set
  - Place \$ in FOLLOW(S), where S is the start symbol
  - If there is a production A  $\rightarrow \alpha B \beta$ , then everything in FIRST(β) except ε is in FOLLOW(B)
  - If there is a production A  $\rightarrow \alpha B$ , or a production A  $\rightarrow \alpha B\beta$ , where FIRST(β) contains ε, then everything in FOLLOW(A) is in FOLLOW(B)





#### Example: FIRST and FOLLOW

- FIRST(T) = FIRST(E) = {int, (}
  - E has only one production, and its body starts with T
  - T doesn't derive ε, E is same with T
- FIRST(E') =  $\{+, \varepsilon\}$
- FIRST(T') =  $\{*, \epsilon\}$

- $E \rightarrow TE'$
- $E' \rightarrow +E \mid \varepsilon$
- $T \rightarrow intT' \mid (E)$
- $T' \rightarrow *T \mid \epsilon$

- FOLLOW(E) = FOLLOW(E') = {), \$}
  - E is start symbol, thus \$ must be contained; production body (E)
  - E' appears at the ends of E-productions, same as FOLLOW(E)
- FOLLOW(T) = FOLLOW(T') = {+, ), \$}
  - +: T appears in bodies only followed by E', thus FIRST(E')- ε
  - ), \$: FIRST(E') contains ε, and E' is the entire str following T, so
     FOLLOW(E') is in FOLLOW(T)
  - T' is only at ends of T-productions, FOLLOW(T')=FOLLOW(T)



# Example: FIRST and FOLLOW (cont.)

Symbol	FIRST	FOLLOW
E	int, (	), \$
E'	+, ε	), \$
Т	int, (	+, ), \$
T'	*, ε	+, ), \$

$E \rightarrow TE'$	
$E' \rightarrow + E \mid \epsilon$	
$T \rightarrow intT'$	(E)
$T'\rightarrow *T \epsilon$	

A  ightarrow lpha (RHS)	FIRST
E  o TE'	int, (
E'  o + E	+
extstyle  o int $ extstyle  o$	int
T → (+E)	(
$T' \rightarrow *T$	*





#### Construct LL(1) Parse Table

- To construct, rule  $A \rightarrow \alpha$  is added to M[A, a] if either:
  - For each terminal a in FIRST( $\alpha$ )
  - If  $\varepsilon$  is in FIRST( $\alpha$ ), or  $\alpha=\varepsilon$ , a is in FOLLOW(A) (Epsilon production)

• If  $\epsilon$  is in FIRST( $\alpha$ ) and  $\varphi$  is in FOLLOW(A), add A  $\xrightarrow{}$   $\alpha$  to M[A,  $\varphi$ ] as well

- If after performing the above, there is no production at all in M[A, a], then set M[A, a] to error
  - Which is normally represented by an empty entry in the table





#### Construct LL(1) Parse Table (cont.)

$A  ightarrow \alpha$ (RHS)	FIRST
E  o TE'	int, (
$E' \rightarrow +E$	+
extstyle  o int $ extstyle  o$	int
$T \rightarrow (E)$	(
$T' \rightarrow *T$	*
$E'  ightarrow \epsilon$	FOLLOW
$T'  o \epsilon$	FOLLOW

Symbol	FIRST	FOLLOW
E	int, (	),\$
E'	+, ε	),\$
Т	int, (	+, ), \$
T'	*, ε	+, ), \$

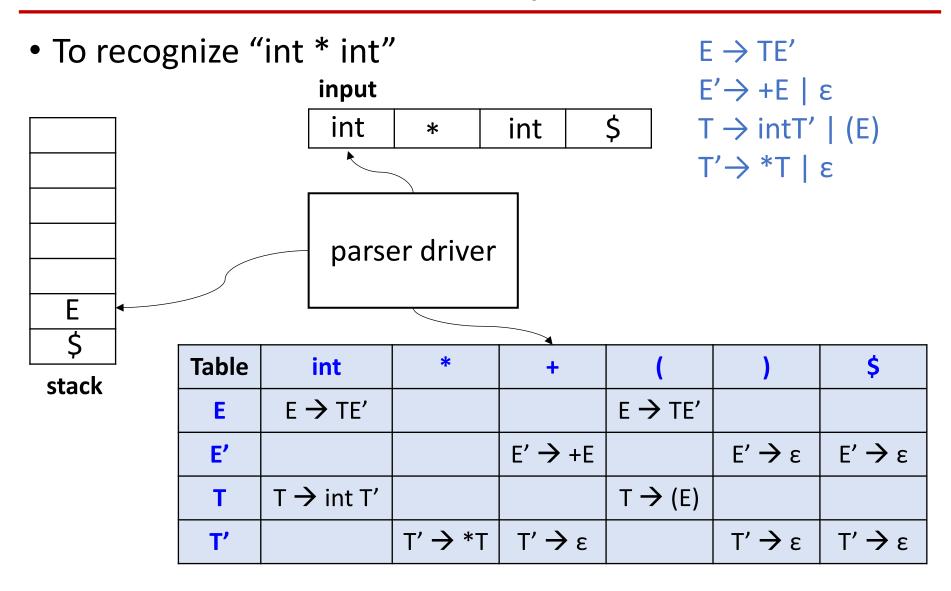
$E \rightarrow TE'$	
$E' \rightarrow + E \mid \epsilon$	
$T \to intT' \mid$	<b>(E)</b>
$T'\rightarrow *T \epsilon$	

Table	int	*	+	(	)	\$
E	E → TE′			E → TE′		
E'			E' → +E		E' <b>→</b> ε	E' <b>→</b> ε
Т	$T \rightarrow int T'$			T → (E)		
T'		T′ → *T	T′ <del>→</del> ε		T′ → ε	T′ <del>→</del> ε





#### Use the Table [already examined]







#### Determine if Grammar is LL(1)

- Observation
  - If a grammar is LL(1), then each of its LL(1) table entry contains at most one rule
  - Otherwise, it is not LL(1).
- Two methods to determine if a grammar is LL(1) or not
  - Construct LL(1) table, and check if there is a multi-rule entry
  - Checking each rule as if the table is getting constructed.

G is LL(1) iff for a rule A  $\rightarrow \alpha \mid \beta$ 

- □ FIRST(α)  $\cap$  FIRST(β) = Φ
- $_{\mbox{\scriptsize $\square$}}$  At most one of  $\alpha$  and  $\beta$  can derive  $\epsilon$
- □ If β derives ε, then FIRST(α)  $\cap$  FOLLOW(A) =  $\varphi$

保证预测的唯一性





#### Non-LL(1) Grammars

- Suppose a grammar is not LL(1). What then?
- Case-1: the language may still be LL(1).
  - Try to rewrite grammar to LL(1) grammar:
    - Apply left-factoring
    - Apply left-recursion removal
  - Try to remove ambiguity in grammar:
    - Encode precedence into rules
    - Encode associativity into rules
- Case-2: If Case-1 fails, language may not be LL(1)
  - Impossible to resolve conflict at the grammar level
  - Programmer chooses which rule to use for conflicting entry (if choosing that rule is always semantically correct)
  - Otherwise, use a more powerful parser (e.g. LL(k), LR(1))





# LL(1) Time and Space Complexity

- Linear time and space relative to length of input
- Time: each input symbol is consumed within a constant number of steps
  - If symbol at top of stack is a terminal:
    - Matched immediately in one step
  - If symbol at top of stack is a non-terminal:
    - $\blacksquare$  Matched in at most N steps, where N = number of rules
    - Since no left-recursion, cannot apply same rule twice without consuming input
- Space: smaller than input (after removing  $X \rightarrow \varepsilon$ )
  - RHS is always longer or equal to LHS
    - Derivation string expands monotonically
    - Derivation string is always shorter than final input string
  - Stack is a subset of derivation string (unmatched portion)





#### Some Thoughts ...

- LL(1) table-driven parser is basically DFA + Stack
  - Capable to count ⇒ CFG is more powerful than RE
- We have studied LL(1), what about LL(0), LL(2) or LL(k)?
- Is LL(0) useful at all?
  - Grammar where rules can be predicted with no lookahead
  - $-\Rightarrow$  That means, there can only be one rule per non-terminal
  - $\Rightarrow$  That means, this language can have only one string
- What would prevent LL(2) ... LL(k) from wide usage?
  - Size of parse table =  $O(|N|*|T|^k)$ 
    - $\blacksquare$  where N = set of non-terminals, T = set of terminals





#### Summary: Predictive Parser

 FIRST and FOLLOW sets are used to construct predictive parsing tables

- Intuitively, FIRST and FOLLOW sets guide the choice of rules
  - For non-terminal A and lookahead t, use the production rule A
     → α where t ∈ FIRST(α)
     OR
  - For non-terminal A and lookahead t, use the production rule A  $\rightarrow \alpha$  where  $\epsilon \in FIRST(\alpha)$  and  $t \in FOLLOW(A)$
  - There can only be ONE such rule
    - Otherwise, the grammar is not LL(1)



