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概率图模型

> inference-> $P(Z|X)$->积分问题（MCMC）

GMM：样本之间是独立同分布

HMM: Dynamic Model

y: System state 隐变量

- state 离散：HMM
- state 线性: Kalman Filter
- state 非线性: Particle Filter

$$
\begin{gathered}
\lambda=(\underbrace{\pi}_{} \text{初始} prob dist, \underbrace{A}_{} \text{状态转移矩阵}, \underbrace{B}_{\text{发射矩阵} emission}) \\
\text{状态变量} i: i_1, i_2, \cdots i_t \cdots \rightarrow Q=q_1, q_2, \cdots, q_m \\
\text{观测变量} o: o_1, o_2, \cdots o_t \cdots \rightarrow V=v_1, v_2, \cdots, v_m \\
A=\left[a_{ij}\right], a_{ij}=P\left(i_{t+1}=q_i | i_t=q_i\right) \\
B=\left[b_{jk}\right], b_{jk}=P\left(Q_t=v_k | i_t=q_j\right)
\end{gathered}
$$

> transition 和 emission probability 是independent

两个假设：

- 齐次Markov 假设

$$P(i_{t+1}|i_t, t_{t-1}, \cdots, t_1, o_t, o_{t-1}, \ldots, o_1) = p(i_{t+1}|i_t)$$

- 观察独立假设

$$P(o_t|i_t, t_{t-1}, \cdots, t_1, o_t, o_{t-1}, \ldots, o_1) = p(o_t|i_t)$$

三个问题：

1. Evaluation: $P(O|\lambda) \Rightarrow$ 前向后向 Forward-backward
2. learning $\lambda = \arg\max P(O|\lambda)$ EM algorithm\baum welch
3. Decoding $\lambda = \arg\max_i P(I|O)$
    1. 预测：$P(i_{t+1}|o_1, o_2, \cdots, o_t)$
    2. 滤波：$P(i_t|o_1, o_2, \cdots, o_t)$

HMM-Evaluation

$$
\begin{aligned}
& Give \lambda, 求 P(O|\lambda) \\
& P(O|\lambda) = \sum_1 P(I, O|\lambda) = \sum_1 P(O|I, \lambda) \cdot P(I|\lambda) \\
& P(I|\lambda) = P\left(i_1, i_2, \cdots, i_T|\lambda\right) = \underbrace{P\left(i_T|i, i_2, \cdots, i_{T-1}, \lambda\right)}_{P(i_T|i_{T-1}) = a_{i_{T-1}, i_T}} P(i_1, i_2, \cdots, i_{T-1}|\lambda) = a_{i_{T-1}, i_T} \cdot a_{i_{T-2}, i_{T-1}} \cdots a_{i_1, i_2} \cdot \pi(i_1)
\end{aligned}
$$

$$
\begin{aligned}
& \pi \text{ 是初始分布} \\
& = \pi\left(a_i\right) \cdot \prod_{t=2}^{T} a_{i_{t-1}, i_t}
\end{aligned}
$$

$- - - -$

$$
p(O|I, \lambda) = \prod_{t=1}^{T} b_{i_t}\left(O_t\right)
$$

PPT