



Competitive pricing decisions in a two-echelon supply chain with horizontal and vertical competition

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ABSTRACT

This study investigates the pricing decisions in a non-cooperative supply chain that consists of two retailers and one common supplier. The retailers order from the common supplier and compete in the same market. We analyze six power structures that characterize exclusively horizontal competition between the retailers and vertical competition between the supplier and the retailers, leading to different sequences of moves among the chain members. We derive the analytical forms of the equilibrium quantities under each power structure and explore the effect of retail substitutability on the equilibrium quantities among all power structures. We further investigate the performances of the game models as compared with the integrated model.

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1. Introduction

We study the competitive dynamics in a supply chain consisting of two retailers and one common supplier. Both retailers order from the common supplier and sell their products in the same market. The retailers determine the sale prices to the market, and the supplier determines the wholesale prices to the retailers. We explore the chain members' decisions under various power structures. Different power structures appear in real-world chains. With regard to vertical competition, suppliers (e.g., Microsoft and Intel) play a more dominant role than downstream members in some chains, whereas retailers (e.g., Wal-Mart and Tesco) play a more dominant role than upstream members (Ertek and Griffin, 2002) in other chains. Chain members may be engaged in vertical Bertrand competition in a small or local market, such as in the market of private brands (Cotterill and Putsis, 2001). In the light of horizontal competition, firms at the same stage of a supply chain may consider their decisions (e.g., on sale pricing policy, facility investment, R&D expenditures, etc.) as strategic moves in timing, and thus their timing of moves will lead to different power structures between them. A real-world example is that Tesco announced to follow Wal-Mart to enter India grocery market (Domain-b.com, 2008).

The analysis of supplier–buyer relationships often proceeds under a non-cooperative setting, focusing on the interaction of supply chain members and the resulting supply chain performance.

There are a number of studies concerning vertical or/and horizontal interaction in a supply chain (Almehdawe and Martin, 2010; Choi, 1991; Ertek and Griffin, 2002; Lee and Staelin, 1997; Trivedi, 1998; Tsay and Agrawal, 2004; Yao and Liu, 2005). Choi (1991), for instance, examined a two-echelon supply chain consisting of two manufacturers and one retailer, and investigated three types of power structures: manufacturer Stackelberg, retailer Stackelberg, and vertical Nash. Trivedi (1998) extended the work in Choi (1991) by considering two manufacturers selling their substitutable products through two retailers, and examined three channel structures: the decentralized channel in which the manufacturers sell their products through independent retailers, the integrated channel in which each manufacturer–retailer pair is vertically integrated, and the full distribution channel in which each manufacturer sells its product through both retailers. Trivedi also studied retailer–Stackelberg and manufacturer–Stackelberg power structures in both decentralized and full distribution channels. Lee and Staelin (1997) introduced the concept of vertical strategic interaction in a non-cooperative supply chain with two manufacturers and two retailers, and characterized the relationship between the types of vertical strategic interaction and channel price leadership. Ingene and Parry (1995) studied channel coordination under the manufacturer–Stackelberg power structure with two retailers competing on prices simultaneously. They showed that the two-part tariff has some limitations, compared to quantity discount, under the Robinson–Patman Act. Using the power structure in Ingene and Parry (1995, 1998) developed an optimal wholesale pricing strategy (a form of a two-part tariff) for the manufacturer and showed that this pricing strategy is preferable to the manufacturer, and Ingene and Parry (2000) compared two types of

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two-part tariffs and established the conditions under which one type is preferred by the manufacturer to the other. A closer look at the power structures in these aforementioned studies reveals that the chain members at the same stage are always engaged in Bertrand price competition. There could be price leadership or decisions made in chronological order at the same stage of a supply chain. Yang and Zhou (2006), for instance, considered a supply chain with one manufacturer and two retailers, in which the manufacturer is a Stackelberg leader and horizontal interaction between the retailers may take on simultaneous competition, Stackelberg competition, or collusion. Yang and Zhou (2006), however, did not examine other forms of vertical interaction such as retailer-Stackelberg and vertical Nash.

Previous studies that investigated the issues of altering the nature of competition by selecting the timing of moves include Amir and Stepanova (2006), Bárcena-Ruiz (2007), Cvsa and Gilbert (2002), Emmons and Gilbert (1998), Granot and Yin (2007), Hamilton and Slutsky (1990), Maggi (1996), Su and Rao (2011), van Damme and Hurkens (1999), van Damme and Hurkens (2004). Maggi (1996), for example, studied an investment game in which firms are able to decide their timing of moves on investment strategies in tradeoff between commitment and flexibility: they may move first to take first-mover advantage or delay moving to receive precise market information. van Damme and Hurkens (1999) investigated quantity decisions in a duopoly game in which the players have equal opportunities to choose the timing of their decisions. As a result, the Stackelberg and Cournot games between the players emerge. Bárcena-Ruiz (2007) considered endogenous order of moves in a mixed duopoly in which firms determine their pricing decisions simultaneously or sequentially. Cvsa and Gilbert (2002) analyzed the effectiveness of pricing incentives provided by a supplier to entice two competing buyers to make early purchase commitments. In their model, the supplier acts as a Stackelberg leader, and the buyers compete in order quantity and are engaged in either a Cournot game or a Stackelberg game, depending upon whether they order before the selling season (i.e., making early purchase commitments) or at the start of the selling season. Granot and Yin (2007), on the other hand, constructed a Stackelberg game to analyze sequential commitment in a decentralized newsvendor model with one manufacturer and one retailer under price-dependent demand, where the manufacturer's decisions are the wholesale price and the buyback price, and the retailer's decisions are the order quantity and the sale price. Balachander and Srinivasan (1998) studied the influence of product prices in two periods on customer expectations by considering a sequential game between a firm and its customers. By treating the product's first-period price as a signal, they found that a high first-period price credibly signals to customers that the firm has low experiential learning toward reducing the product's unit cost. It is worth noting that early wholesale price commitment is valuable for realizing downstream operational decisions before the investment is made (Gilbert and Cvsa, 2003), coordinating supply chain performance (Desai et al., 2004), or deterring the retailer from introducing a store brand (Groznik and Heese, 2010). In particular, Groznik and Heese (2010) investigated competition between the products in the presence and absence of the wholesale price commitment under a deterministic model.

We embrace the concept of sequential commitment at the same stage of a supply chain and attempt to report analytical results in a non-cooperative supply chain with one supplier and two retailers. We investigate how the supplier and the retailers determine their decisions in the presence of vertical and horizontal competition. We allow for vertical and horizontal competition to be modeled as either Stackelberg or Bertrand competition, constituting an exhaustive set of six power structures. Although

some of the power structures have been examined in the literature (e.g., the power structure with both vertical and horizontal interaction taking on Bertrand competition is, in essence, the vertical Nash structure in Choi, 1991), these six power structures as a whole have not been explored. Our power structures with the supplier being a Stackelberg leader in vertical competition are identical to those in Yang and Zhou (2006). But, we focus in this paper on competition among the chain members and do not consider collusion between the retailers, which was examined in Yang and Zhou (2006). Also, we allow for horizontal competition to be Stackelberg where the retailers' wholesale prices can be set differently and sequentially, whereas the retailers in Yang and Zhou (2006) are always engaged in simultaneous competition. Furthermore, we consider vertical Bertrand and retailer-Stackelberg competition, both of which were not reported in Yang and Zhou (2006). Our main contribution is exploring the chain members' interactive dynamics by using six game models. We derive the equilibrium decisions and profits of the chain members and investigate the effect of retail substitutability on the equilibrium decisions and profits of the chain members in these six game models. We find that the relationships of certain equilibrium values among the game models are independent of retail substitutability. We also find that the types of vertical interaction have more impact on the performances of the game models, as compared to the integrated model, than the types of horizontal interaction do when retail substitutability is low. We further assess whether or not the chain members' equilibrium decisions in Stackelberg games are enforceable.

The remainder of the paper is organized as follows. The next section details the supply chain in a non-cooperative setting and discusses decisions and profits through six game models with various combinations of vertical and horizontal competition. Section 3 derives the analytical forms of various equilibrium quantities such as prices and profits in each game model. Section 4 analyzes the results within each game model and among six game models, and provides some implications. The last section concludes with a brief summary.

2. The model

We examine a supply chain with three independent, risk-neutral, and profit-maximizing chain members—a common supplier and two identical retailers serving a market with price-sensitive demand. Both retailers sell identical products, compete in a market based on their sale prices, and place their orders to the supplier who has sufficient capacity to satisfy all the ordered quantities. The supplier's decisions are the wholesale prices to the retailers. When the retailers move simultaneously, the supplier will decide a unique wholesale price for both retailers, as required by the Robinson-Patman Act¹; when the retailers move sequentially, the supplier will choose an advanced wholesale price to the first-mover and a normal wholesale price to the second-mover (Cachon, 2003; Cvsa and Gilbert, 2002). The retailers' sequence of moves may be attributed to their endogenous leadership (Bárcena-Ruiz, 2007; Hamilton and Slutsky, 1990; Maggi, 1996; van Damme and Hurkens, 1999).² Each retailer's decision is the sale price to the market (or equivalently the sale margin). We

¹ This guarantees that when the retailers move simultaneously, they receive the same wholesale price and there is no price discrimination between them.

² When the retailers move sequentially, they are indeed in a different level of trade, and thus are not limited by the Robinson-Patman Act. In addition, legislative equivalents of the Robinson-Patman Act hardly exist worldwide (Whelan and Marsden, 2006). Hence, our models with early commitment at large are applicable in global supply chain settings.

Table 1
Six game models with different power structures.

Vertical competition	Horizontal competition	
	Bertrand (B)	Stackelberg (S)
Supplier-Stackelberg (SS)	SSB	SSS
Retailer-Stackelberg (RS)	RSB	RSS
Bertrand (B)	BB	BS

investigate in a non-cooperative setting how the supplier and the retailers determine their decisions under various types of vertical and horizontal competition.

We allow for vertical and horizontal competition to be modeled as either a Stackelberg game or a Bertrand game. Such modeling enables us to capture the chain members' competitive dynamics under six power structures, as summarized in Table 1. The two models in the first row of Table 1 concern the power structures under which vertical interaction is supplier-Stackelberg, i.e., the supplier acts as the Stackelberg leader determining its decision prior to the retailers: the SSB model, in which the retailers are engaged in Bertrand competition (i.e., the retailers move simultaneously), and the SSS model, in which the retailers are engaged in Stackelberg competition (i.e., the retailers move sequentially). Particularly, in the SSS model, the supplier can choose the advanced and normal wholesale prices to the retailers after both retailers' reactions have been resolved, or choose these wholesale prices to the retailers sequentially once either of their reactions has been resolved. We shall show in Section 3.2 that the former alternative surpasses the latter in terms of maximizing the supplier's profit. Without loss of generality, we designate retailer 1 as the first mover and retailer 2 as the second mover when horizontal competition is Stackelberg. The RSB and RSS models, in the second row of Table 1, focus on vertical competition being retailer Stackelberg. Specifically, the RSB model considers the power structure under which the retailers are the first movers in the vertical pricing game, determining their decisions simultaneously in horizontal competition. The RSS model differs from the RSB model not only in that horizontal competition is Stackelberg but also in that the supplier's reactions to the retailers' sale margins can be resolved simultaneously or sequentially. We shall show in Section 3.4 that both alternatives of resolving the supplier's reactions to the retailers' sale margins lead to the same result. Finally, the BB model studies the power structure under which the supplier and the retailers determine their decisions simultaneously (Choi, 1991, 1996; Lee and Staelin, 1997), whereas the BS model examines the power structure under which the vertical pricing game between the supplier and each retailer is Bertrand competition and retailer 1 acts as the first mover in horizontal competition.³

In each model, we suppose that retailer i 's demand q_i , $i=1,2$, is a general linear demand function of its own sale price p_i and the rival's sale price p_j , $j \neq i$, in line with (Choi, 1991, 1996; Dong et al., 2009; Gupta and Loulou, 1998; Jeuland and Shugan, 1988; Lee and Staelin, 1997; McGuire and Staelin, 1983; Padmanabhan and Png, 1997)

$$q_i = \alpha - \beta p_i + \gamma p_j, \quad j = 3-i \text{ and } i = 1, 2, \quad (1)$$

where $\alpha > 0$ denotes the primary demand, β the store-level factor (Padmanabhan and Png, 1997), γ/β the degree of retail

substitutability between the retailers, and $\beta > \gamma > 0$.⁴ That $\beta > \gamma$ indicates that the change of p_i is more influential on retailer i 's own demand than that of the rival's sale price p_j is. Let m_i , w_A , w , and c denote retailer i 's sale margin and the supplier's advanced wholesale price, normal wholesale price, unit cost, respectively. And, let w_i denote the supplier's wholesale price received by retailer i . Thus, $p_i = m_i + w_i$, $w_i \in \{w_A, w\}$. Retailer i 's profit function Π_i , the supplier's profit function Π_s , and the supply chain profit Π_T can now be expressed as

$$\Pi_i = (p_i - w_i)q_i = m_i q_i, \quad i = 1, 2, \quad (2)$$

$$\Pi_s = \sum_{i=1}^2 (w_i - c)q_i, \quad (3)$$

$$\Pi_T = \Pi_1 + \Pi_2 + \Pi_s, \quad (4)$$

where q_i follows (1) and $w_i \in \{w_A, w\}$.

To measure the performances of the game models, we consider an integrated model in which a single decision maker has the ownership of the supplier and the retailers, and determines all the decisions that maximize the supply chain profit simultaneously. In the integrated model, we obtain the optimal sale prices, denoted by p_i^I , $i = 1, 2$, by solving $\partial \Pi_T / \partial p_1 = 0$ and $\partial \Pi_T / \partial p_2 = 0$:

$$p_1^I = p_2^I = c + \frac{\alpha - c(\beta - \gamma)}{2(\beta - \gamma)}.$$

Then, substituting p_1^I and p_2^I into Π_T in (4) gives the maximal supply chain profit Π_T^I for the integrated model:

$$\Pi_T^I = \frac{(\alpha + c(\beta - \gamma))^2}{2(\beta - \gamma)^2}.$$

3. Equilibrium

We investigate the equilibrium decisions of the chain members under six power structures, detailed in Section 2, in sequence.

3.1. The SSB model

In the SSB model, the supplier is the Stackelberg leader who first determines a profit-maximizing wholesale price w , based on which the retailers then determine their profit-maximizing sale prices in Bertrand competition. To obtain the power balance relationship among the chain members, we derive the retailers' best response functions to a given value w by setting $d\Pi_1/dm_1$ and $d\Pi_2/dm_2$ equal to zero and solving for m_1 and m_2 :

$$m_1 = m_2 = \frac{\alpha - w(\beta - \gamma)}{2\beta - \gamma}. \quad (5)$$

Substituting m_1 and m_2 in (5) into the supplier's profit Π_s in (3) and applying the first-order conditions to the resulting profit function in terms of the wholesale price w gives the supplier's wholesale price w^{SSB} at equilibrium as follows:

$$w^{SSB} = \frac{\alpha + c(\beta - \gamma)}{2(\beta - \gamma)}. \quad (6)$$

Hence, the retailers' equilibrium sale margins m_i^{SSB} in response to w^{SSB} in (6) are

$$m_1^{SSB} = m_2^{SSB} = \frac{\alpha - c(\beta - \gamma)}{2(2\beta - \gamma)}. \quad (7)$$

³ One plausible application of the BS game model is that when introducing a new product into the market, the manufacturer may encourage the downstream firms to place the orders before the selling season by offering a price commitment. As reported in Gilbert and Cvsa (2003), Intel provides forward prices of its next generation product to its partners.

⁴ Our analysis with this deterministic demand is applicable to a certain class of stochastic demand, where the randomness of the demand comes from the primary demand which has a fixed mean and a fixed variance.

Table 2

Equilibrium sale margins m_i^j , wholesale prices w_A^j and w^j , order quantities q_i^j , and profits $\Pi_i^j, \Pi_s^j, \Pi_T^j$ in model $j \in \{\text{SSB}, \text{SSS}\}$, $i=1,2$, where vertical competition is supplier Stackelberg.

Equilibrium values in model j	SSB	SSS
m_1^j	$\frac{\alpha - c(\beta - \gamma)}{2(2\beta - \gamma)}$	$\frac{(2\beta + \gamma)(\alpha - c(\beta - \gamma))}{8\beta^2 - 4\gamma^2}$
m_2^j	$\frac{\alpha - c(\beta - \gamma)}{2(2\beta - \gamma)}$	$\frac{(4\beta^2 + 2\beta\gamma - \gamma^2)(\alpha - c(\beta - \gamma))}{16\beta^3 - 8\beta\gamma^2}$
w_A^j	–	$\frac{\alpha + c(\beta - \gamma)}{2(\beta - \gamma)}$
w^j	$\frac{\alpha + c(\beta - \gamma)}{2(\beta - \gamma)}$	$\frac{\alpha + c(\beta - \gamma)}{2(\beta - \gamma)}$
q_1^j	$\frac{\beta(\alpha - c(\beta - \gamma))}{2(2\beta - \gamma)}$	$\frac{(2\beta + \gamma)(\alpha - c(\beta - \gamma))}{8\beta}$
q_2^j	$\frac{\beta(\alpha - c(\beta - \gamma))}{2(2\beta - \gamma)}$	$\frac{(4\beta^2 + 2\beta\gamma - \gamma^2)(\alpha - c(\beta - \gamma))}{8(2\beta^2 - \gamma^2)}$
Π_1^j	$\frac{\beta(\alpha - c(\beta - \gamma))^2}{4(2\beta - \gamma)^2}$	$\frac{(2\beta + \gamma)^2(\alpha - c(\beta - \gamma))^2}{64\beta^3 - 32\beta\gamma^2}$
Π_2^j	$\frac{\beta(\alpha - c(\beta - \gamma))^2}{4(2\beta - \gamma)^2}$	$\frac{(-4\beta^2 - 2\beta\gamma + \gamma^2)^2(\alpha - c(\beta - \gamma))^2}{64\beta(2\beta^2 - \gamma^2)^2}$
Π_s^j	$\frac{\beta(\alpha - c(\beta - \gamma))^2}{2(2\beta^2 - 3\beta\gamma + \gamma^2)}$	$\frac{(8\beta^3 + 4\beta^2\gamma - 3\beta\gamma^2 - \gamma^3)(\alpha - c(\beta - \gamma))^2}{16\beta(\beta - \gamma)(2\beta^2 - \gamma^2)}$
Π_T^j	$\frac{\beta(3\beta - 2\gamma)(\alpha - c(\beta - \gamma))^2}{2(\beta - \gamma)(2\beta - \gamma)^2}$	$\frac{(96\beta^5 + 32\beta^4\gamma - 96\beta^3\gamma^2 - 28\beta^2\gamma^3 + 23\beta\gamma^4 + 5\gamma^5)(\alpha - c(\beta - \gamma))^2}{64\beta(\beta - \gamma)(2\beta^2 - \gamma^2)^2}$

With these equilibrium values in (6) and (7), we can obtain the supplier's equilibrium profit Π_s^{SSB} and the retailers' equilibrium profits Π_i^{SSB} , $i=1,2$, as

$$\Pi_s^{\text{SSB}} = \frac{\beta(\alpha - c(\beta - \gamma))^2}{2(2\beta^2 - 3\beta\gamma + \gamma^2)}, \quad \Pi_1^{\text{SSB}} = \Pi_2^{\text{SSB}} = \frac{\beta(\alpha - c(\beta - \gamma))^2}{4(2\beta - \gamma)^2},$$

and the equilibrium supply chain profit Π_T^{SSB} as

$$\Pi_T^{\text{SSB}} = \frac{\beta(3\beta - 2\gamma)(\alpha - c(\beta - \gamma))^2}{2(\beta - \gamma)(2\beta - \gamma)^2}.$$

We summarize the equilibrium pricing decisions, demand, and profits of the SSB model in the second column of Table 2, which conform with the corresponding results in Yang and Zhou (2006).

3.2. The SSS model

In the SSS model with the retailers moving sequentially, the supplier as the Stackelberg leader in vertical competition can choose the wholesale prices after both retailers' reactions have been resolved, or it can choose the wholesale prices to the retailers once either of their reactions has been resolved. The supplier's first alternative leads to the following sequence of events: (i) the supplier announces the advanced and normal wholesale prices w_A and w , respectively⁵; (ii) retailer 1 chooses the sale margin m_1 in response to w_A ; and (iii) retailer 2 decides the sale margin m_2 according to the above price information. We obtain the members' best response functions in reverse chronological order to the aforementioned events. We solve the first-order condition of Π_2 in (2) with respect to m_2 for retailer 2's best response function:

$$m_2 = \frac{\alpha - \beta w + \gamma(w_A + m_1)}{2\beta}. \quad (8)$$

Substituting m_2 in (8) into Π_1 in (2) and solving $d\Pi_1/dm_1 = 0$ for m_1 gives retailer 1's best response function:

$$m_1 = \frac{2\beta(\alpha - w_A\beta) + (\alpha + w\beta)\gamma + w_A\gamma^2}{4\beta^2 - 2\gamma^2}. \quad (9)$$

Further substituting m_2 in (8) and m_1 in (9) into the supplier's profit function and maximizing the resulting profit in terms of w_A and w yields the supplier's equilibrium wholesale prices w_A^{SSS} and w^{SSS} :

$$w_A^{\text{SSS}} = w^{\text{SSS}} = \frac{\alpha + c(\beta - \gamma)}{2(\beta - \gamma)}, \quad (10)$$

with which we obtain the equilibrium values of m_1 and m_2 by, in turn, using (9) and (8)

$$m_1^{\text{SSS}} = \frac{(2\beta + \gamma)(\alpha - c(\beta - \gamma))}{8\beta^2 - 4\gamma^2}, \quad (11)$$

$$m_2^{\text{SSS}} = \frac{(4\beta^2 + 2\beta\gamma - \gamma^2)(\alpha - c(\beta - \gamma))}{16\beta^3 - 8\beta\gamma^2}. \quad (12)$$

The last column of Table 2 summarizes the equilibrium values of the chain members' decisions and profits and the supply chain profit.

The supplier's second alternative of choosing the wholesale prices sequentially leads to the following sequence of moves: (i) the supplier announces the advanced wholesale price w_A to retailer 1; (ii) retailer 1 chooses the sale margin m_1 in response to w_A ; (iii) the supplier determines the wholesale price w ; and (iv) retailer 2 decides the sale margin m_2 according to the above price information. Solving the first-order condition of Π_2 in (2) yields retailer 2's best response function \tilde{m}_2 :

$$\tilde{m}_2 = \frac{\alpha - \beta w + \gamma(w_A + m_1)}{2\beta}. \quad (13)$$

Substituting \tilde{m}_2 in (13) into Π_s in (3) and solving $\partial\Pi_s/\partial w = 0$ for w gives the supplier's best response function \tilde{w} :

$$\tilde{w} = \frac{\alpha + c(\beta - \gamma) + \gamma(2w_A + m_1)}{2\beta}. \quad (14)$$

⁵ Such a setting differs from Yang and Zhou (2006) which assumes the retailers with distinct timing of moves receive the same wholesale price.

Based on \tilde{m}_2 in (13) and \tilde{w} in (14), we can further derive retailer 1's best response function as

$$\tilde{m}_1 = \frac{\alpha(4\beta+3\gamma) - (\beta-\gamma)(-c\gamma+4w_A(\beta+\gamma))}{8\beta^2-6\gamma^2}. \quad (15)$$

Further substituting the best response functions $\tilde{m}_2, \tilde{w}, \tilde{m}_1$ in (13)–(15), respectively, into the supplier's profit function in (3), and solving the first-order condition of the supplier's profit function for w_A gives the supplier's equilibrium price w'_A to retailer 1:

$$w'_A = \frac{13c}{28} + \frac{\alpha}{2(\beta-\gamma)} + \frac{16c\beta^4 - 7\beta(2\alpha+c\beta)\gamma^2 - 7(\alpha+c\beta)\gamma^3}{28(16\beta^4 - 21\beta^2\gamma^2 + 7\gamma^4)},$$

with which we obtain the equilibrium values of m_1, w, m_2 by, in turn, using (15), (14), and (13)

$$m'_1 = \frac{(2\beta+\gamma)(2\beta^2-\gamma^2)(\alpha-c(\beta-\gamma))}{16\beta^4 - 21\beta^2\gamma^2 + 7\gamma^4}, \quad (16)$$

$$w' = \frac{c(\beta-\gamma)(2\beta^2-\gamma^2)(16\beta^3 - 4\beta^2\gamma - 11\beta\gamma^2 + 3\gamma^3)}{4\beta(\beta-\gamma)(16\beta^4 - 21\beta^2\gamma^2 + 7\gamma^4)} + \frac{\alpha(32\beta^5 + 8\beta^4\gamma - 46\beta^3\gamma^2 - 10\beta^2\gamma^3 + 17\beta\gamma^4 + 3\gamma^5)}{4\beta(\beta-\gamma)(16\beta^4 - 21\beta^2\gamma^2 + 7\gamma^4)}, \quad (17)$$

$$m'_2 = \frac{(16\beta^4 + 4\beta^3\gamma - 19\beta^2\gamma^2 - 2\beta\gamma^3 + 6\gamma^4)(\alpha-c(\beta-\gamma))}{64\beta^5 - 84\beta^3\gamma^2 + 28\beta\gamma^4}, \quad (18)$$

and the equilibrium profit Π'_s

$$\Pi'_s = \frac{(64\beta^5 + 32\beta^4\gamma - 82\beta^3\gamma^2 - 34\beta^2\gamma^3 + 27\beta\gamma^4 + 9\gamma^5)\Omega^2}{16\beta(\beta-\gamma)(16\beta^4 - 21\beta^2\gamma^2 + 7\gamma^4)}, \quad (19)$$

where $\Omega = \alpha - c(\beta - \gamma)$. To determine which alternative the supplier will adopt, we compare the supplier's equilibrium profit Π_s^{SSS} (in Table 2) with the first alternative and its equilibrium profit Π'_s in (19) with the second alternative. The difference of Π_s^{SSS} and Π'_s is

$$\Pi_s^{SSS} - \Pi'_s = \frac{(\alpha-c(\beta-\gamma))^2}{16\beta(\beta-\gamma)} \cdot \frac{(\beta-\gamma)\gamma^2(2\beta+\gamma)^2(3\beta^2-2\gamma^2)}{32\beta^6 - 58\beta^4\gamma^2 + 35\beta^2\gamma^4 - 7\gamma^6} > 0,$$

because $32\beta^6 - 58\beta^4\gamma^2 + 35\beta^2\gamma^4 - 7\gamma^6 > 0$ for all $\beta > \gamma$. We therefore conclude that the supplier will employ the first alternative. The above analysis shows that the supplier's choice of identical equilibrium wholesale prices w_A^{SSS} and w^{SSS} is an endogenous result rather than an exogenous assumption which is made in Yang and Zhou (2006) and that setting equal equilibrium wholesale prices indeed benefit the supplier most.

3.3. The RSB model

The retailers in the RSB model act as Stackelberg leaders in vertical competition, and determine their sale margins m_1 and m_2 simultaneously. The supplier's best response function of w to m_1 and m_2 are obtained by solving the first-order conditions $d\Pi_s/dw=0$:

$$w = \frac{\alpha - (m_1 - c)(\beta - \gamma)}{2(\beta - \gamma)}. \quad (20)$$

We substitute the supplier's best response functions in (20) into the retailers' profit functions in (2) and solve the first-order conditions of the retailers' profit functions for the equilibrium values of m_1 and m_2 :

$$m_1^{RSB} = m_2^{RSB} = \frac{\alpha - c(\beta - \gamma)}{2\beta - \gamma}. \quad (21)$$

With m_1^{RSB} and m_2^{RSB} in (21), we obtain from (20) that the suppliers' equilibrium wholesale prices are

$$w^{RSB} = \frac{\alpha\beta + 3c\beta^2 - 5c\beta\gamma + 2c\gamma^2}{4\beta^2 - 6\beta\gamma + 2\gamma^2}. \quad (22)$$

Table 3 summarizes the equilibrium values of the chain members' decisions and profits and the supply chain profit in the RSB model.

3.4. The RSS model

Similar to the SSS model, the Stackelberg leaders in vertical competition in this model have two alternatives. The first alternative is to choose the sale margins after both of the supplier's reactions to the retailers' sale margins have been resolved, whereas the second alternative is to choose the sale margins once either of the supplier's reactions has been resolved. The sequence of moves in consideration of the first alternative is as follows: (i) retailer 1 chooses the sale margin m_1 ; (ii) retailer 2 decides the sale margin m_2 ; and (iii) the supplier determines the advanced and normal wholesale prices w_A and w , respectively. We can obtain the supplier's best response functions w_A and w as

$$w_A = \frac{\alpha - (m_1 - c)(\beta - \gamma)}{2(\beta - \gamma)}, \quad w = \frac{\alpha - (m_2 - c)(\beta - \gamma)}{2(\beta - \gamma)}. \quad (23)$$

Substituting w in (23) into retailer 2's profit and solving the first-order condition of the resulting profit for m_2 gives retailer 2's best response function m_2 :

$$m_2 = \frac{\alpha - c\beta + \gamma(m_1 + c)}{2\beta}. \quad (24)$$

Further solving the first-order condition of retailer 1's profit function for m_1 gives retailer 1's equilibrium sale margin:

$$m_1^{RSS} = \frac{(2\beta + \gamma)(\alpha - c(\beta - \gamma))}{2(2\beta^2 - \gamma^2)}, \quad (25)$$

with which we can determine the equilibrium values of m_2, w_A, w by, in turn, using (24) and (23), and the equilibrium profits of the supplier and the retailers, as summarized in the third column of Table 3.

The sequence of events under the retailers' second alternative is as follows: (i) retailer 1 chooses the sale margin m_1 ; (ii) the supplier determines the advanced wholesale price w_A ; (iii) retailer 2 decides the sale margin m_2 ; and (iv) the supplier determines the normal wholesale price w . Solving $\partial\Pi_s/\partial w=0$ yields the supplier's best response function \hat{w} :

$$\hat{w} = \frac{\alpha - \beta(m_2 - c) + \gamma(m_1 + 2w_A - c)}{2\beta}. \quad (26)$$

Substituting \hat{w} in (26) into retailer 2's profit and solving the first-order condition of the resulting profit for m_2 gives retailer 2's best response function \hat{m}_2 :

$$\hat{m}_2 = \frac{\alpha - c\beta + \gamma(m_1 + c)}{2\beta}. \quad (27)$$

With \hat{w} in (26) and \hat{m}_2 in (27), we can further derive the supplier's best response function \hat{w}_A as

$$\hat{w}_A = \frac{\alpha - (\beta - \gamma)(m_1 - c)}{2(\beta - \gamma)}. \quad (28)$$

Finally, substituting these best response functions $\hat{w}, \hat{m}_2, \hat{w}_A$ in (26)–(28), respectively, into retailer 1's profit function, and solving the first-order condition of retailer 1's profit function for m_1 gives retailer 1's equilibrium sale margin:

$$m_1' = \frac{(2\beta + \gamma)(\alpha - c(\beta - \gamma))}{2(2\beta^2 - \gamma^2)}, \quad (29)$$

Table 3

Equilibrium sale margins m_i^j , wholesale prices w_A^j and w^j , order quantities q_i^j , and profits $\Pi_i^j, \Pi_s^j, \Pi_T^j$ in model $j \in \{\text{RSB}, \text{RSS}\}$, $i=1,2$, where vertical competition is retailer Stackelberg.

Equilibrium values in model j	RSB	RSS
m_1^j	$\frac{\alpha - c(\beta - \gamma)}{2\beta - \gamma}$	$\frac{(2\beta + \gamma)(\alpha - c(\beta - \gamma))}{2(2\beta^2 - \gamma^2)}$
m_2^j	$\frac{\alpha - c(\beta - \gamma)}{2\beta - \gamma}$	$\frac{(4\beta^2 + 2\beta\gamma - \gamma^2)(\alpha - c(\beta - \gamma))}{4\beta(2\beta^2 - \gamma^2)}$
w_A^j	–	$\frac{1}{2} \left[c + \frac{\alpha}{\beta - \gamma} - \frac{(2\beta + \gamma)(\alpha - c(\beta - \gamma))}{2(2\beta^2 - \gamma^2)} \right]$
w^j	$\frac{\alpha\beta + 3c\beta^2 - 5c\beta\gamma + 2c\gamma^2}{4\beta^2 - 6\beta\gamma + 2\gamma^2}$	$\frac{1}{8} \left[7c - \frac{\alpha}{\beta} + \frac{4\alpha}{\beta - \gamma} - \frac{c\gamma}{\beta} - \frac{2(c\beta^2 + \alpha(\beta + \gamma))}{2\beta^2 - \gamma^2} \right]$
q_1^j	$\frac{\beta(\alpha - c(\beta - \gamma))}{2(2\beta - \gamma)}$	$\frac{(2\beta + \gamma)(\alpha - c(\beta - \gamma))}{8\beta}$
q_2^j	$\frac{\beta(\alpha - c(\beta - \gamma))}{2(2\beta - \gamma)}$	$\frac{(4\beta^2 + 2\beta\gamma - \gamma^2)(\alpha - c(\beta - \gamma))}{8(2\beta^2 - \gamma^2)}$
Π_1^j	$\frac{\beta(\alpha - c(\beta - \gamma))^2}{2(2\beta - \gamma)^2}$	$\frac{(2\beta + \gamma)^2(\alpha - c(\beta - \gamma))^2}{16\beta(2\beta^2 - \gamma^2)}$
Π_2^j	$\frac{\beta(\alpha - c(\beta - \gamma))^2}{2(2\beta - \gamma)^2}$	$\frac{(-4\beta^2 - 2\beta\gamma + \gamma^2)^2(\alpha - c(\beta - \gamma))^2}{32\beta(2\beta^2 - \gamma^2)^2}$
Π_s^j	$\frac{\beta^2(\alpha - c(\beta - \gamma))^2}{2(\beta - \gamma)(2\beta - \gamma)^2}$	$\frac{(32\beta^5 + 32\beta^4\gamma - 16\beta^3\gamma^2 - 20\beta^2\gamma^3 + \beta\gamma^4 + 3\gamma^5)(\alpha - c(\beta - \gamma))^2}{64\beta(\beta - \gamma)(2\beta^2 - \gamma^2)^2}$
Π_T^j	$\frac{\beta(3\beta - 2\gamma)(\alpha - c(\beta - \gamma))^2}{2(\beta - \gamma)(2\beta - \gamma)^2}$	$\frac{(96\beta^5 + 32\beta^4\gamma - 96\beta^3\gamma^2 - 28\beta^2\gamma^3 + 23\beta\gamma^4 + 5\gamma^5)(\alpha - c(\beta - \gamma))^2}{64\beta(\beta - \gamma)(2\beta^2 - \gamma^2)^2}$

which is identical to m_1^{RSS} in (25). We can show that the equilibrium values of w_A , m_2 , w at $m_1 = m_1^{\text{RSS}}$ in (29) are identical to w_A^{RSS} , m_2^{RSS} , w^{RSS} , respectively, in Table 3. We hence conclude that both alternatives yield the same profits for the retailers.

3.5. The BB model

When both vertical and horizontal interaction is Bertrand competition, the chain members will determine their decisions simultaneously. Thus, solving the first-order conditions $d\Pi_1/dm_1 = 0$, $d\Pi_2/dm_2 = 0$, and $d\Pi_1/dw = 0$ for m_1 , m_2 , and w gives the chain members' equilibrium strategies:

$$m_1^{\text{BB}} = m_2^{\text{BB}} = \frac{\alpha - c(\beta - \gamma)}{3\beta - \gamma},$$

$$w^{\text{BB}} = \frac{\alpha\beta + c(2\beta^2 - 3\beta\gamma + \gamma^2)}{3\beta^2 - 4\beta\gamma + \gamma^2}.$$

With these equilibrium strategies, we can obtain the equilibrium profits of the chain members, as tabularized in the second column of Table 4.

3.6. The BS model

In the BS model, the supplier chooses the wholesale price w_A and retailer 1 chooses the sale margin m_1 in Bertrand competition. After the advanced wholesale price w_A and the sale margin m_1 have been determined, the supplier chooses the wholesale price w and retailer 2 chooses the sale margin m_2 simultaneously. To obtain the chain members' equilibrium strategies, we solve $d\Pi_2/dm_2 = 0$ and $\partial\Pi_s/\partial w = 0$ for m_2 and w to yield the supplier's and retailer 2's best response functions:

$$m_2 = \frac{\alpha - c\beta + \gamma(m_1 + c)}{3\beta}, \quad (30)$$

$$w = \frac{\alpha + 2c\beta + \gamma(-2c + m_1 + 3w_A)}{3\beta}. \quad (31)$$

Incorporating m_2 in (30) and w in (31) into retailer 1's and the supplier's profit functions and solving the first-order conditions of these two profit functions in terms of m_1 and w_A gives retailer 1's sale margin m_1^{BS} and the supplier's wholesale price w_A^{BS} at equilibrium as follows:

$$m_1^{\text{BS}} = \frac{(3\beta + \gamma)(\alpha - c(\beta - \gamma))}{9\beta^2 - 5\gamma^2}, \quad (32)$$

$$w_A^{\text{BS}} = \frac{\alpha(3\beta - 2\gamma)(\beta + \gamma) + c(\beta - \gamma)(6\beta^2 - \beta\gamma - 3\gamma^2)}{(\beta - \gamma)(9\beta^2 - 5\gamma^2)}. \quad (33)$$

Substituting m_1^{BS} and w_A^{BS} back into (30) and (31) gives retailer 2's sale margin m_2^{BS} and the supplier's wholesale price w^{BS} at equilibrium:

$$m_2^{\text{BS}} = \frac{(9\beta^2 + 3\beta\gamma - 4\gamma^2)(\alpha - c(\beta - \gamma))}{3(9\beta^3 - 5\beta\gamma^2)},$$

$$w^{\text{BS}} = \frac{\alpha\Gamma + c(18\beta^4 - 21\beta^3\gamma - 8\beta^2\gamma^2 + 13\beta\gamma^3 - 2\gamma^4)}{3\beta(\beta - \gamma)(9\beta^2 - 5\gamma^2)},$$

where $\Gamma = (9\beta^3 + \beta^2\gamma - 4\beta\gamma^2 - 2\gamma^3)$. The last column of Table 4 summarizes the equilibrium values of the chain members' pricing decisions, demand, and profits in this model.

4. Analysis

We proceed to analyze the equilibrium results derived in the previous section. We first compare the equilibrium values within each game model. Next, we explore the effect of retail substitutability on the ordinal relationships of the equilibrium values among the six game models. We then assess the performances of the game models in comparison with the integrated model. Finally, we investigate whether or not the chain members' equilibrium decisions in Stackelberg games are enforceable. For brevity, we include the proofs of the propositions in the Appendix.

Table 4

Equilibrium sale margins m_i^j , wholesale prices w_A^j and w^j , order quantities q_i^j , and profits $\Pi_i^j, \Pi_s^j, \Pi_T^j$ in model $j \in \{BB, BS\}$, $i=1,2$, where vertical interaction is Bertrand competition.

Equilibrium values in model j	BB	BS
m_1^j	$\frac{\alpha - c(\beta - \gamma)}{3\beta - \gamma}$	$\frac{(3\beta + \gamma)(\alpha - c(\beta - \gamma))}{9\beta^2 - 5\gamma^2}$
m_2^j	$\frac{\alpha - c(\beta - \gamma)}{3\beta - \gamma}$	$\frac{(9\beta^2 + 3\beta\gamma - 4\gamma^2)(\alpha - c(\beta - \gamma))}{3(9\beta^3 - 5\beta\gamma^2)}$
w_A^j	–	$\frac{\alpha(3\beta - 2\gamma)(\beta + \gamma) + c(\beta - \gamma)(6\beta^2 - \beta\gamma - 3\gamma^2)}{(\beta - \gamma)(9\beta^2 - 5\gamma^2)}$
w^j	$\frac{\alpha\beta + c(2\beta^2 - 3\beta\gamma + \gamma^2)}{3\beta^2 - 4\beta\gamma + \gamma^2}$	$\frac{\alpha\Gamma + c(18\beta^4 - 21\beta^3\gamma - 8\beta^2\gamma^2 + 13\beta\gamma^3 - 2\gamma^4)}{3\beta(\beta - \gamma)(9\beta^2 - 5\gamma^2)}$
q_1^j	$\frac{\beta(\alpha - c(\beta - \gamma))}{3\beta - \gamma}$	$\frac{(3\beta + \gamma)(3\beta^2 - 2\gamma^2)(\alpha - c(\beta - \gamma))}{3(9\beta^3 - 5\beta\gamma^2)}$
q_2^j	$\frac{\beta(\alpha - c(\beta - \gamma))}{3\beta - \gamma}$	$\frac{(9\beta^2 + 3\beta\gamma - 4\gamma^2)(\alpha - c(\beta - \gamma))}{3(9\beta^2 - 5\gamma^2)}$
Π_1^j	$\frac{\beta(\alpha - c(\beta - \gamma))^2}{(-3\beta + \gamma)^2}$	$\frac{(3\beta + \gamma)^2(3\beta^2 - 2\gamma^2)(\alpha - c(\beta - \gamma))^2}{3\beta(9\beta^2 - 5\gamma^2)^2}$
Π_2^j	$\frac{\beta(\alpha - c(\beta - \gamma))^2}{(-3\beta + \gamma)^2}$	$\frac{(9\beta^2 + 3\beta\gamma - 4\gamma^2)^2(\alpha - c(\beta - \gamma))^2}{9\beta(9\beta^2 - 5\gamma^2)^2}$
Π_s^j	$\frac{2\beta^2(\alpha - c(\beta - \gamma))^2}{(\beta - \gamma)(-3\beta + \gamma)^2}$	$\frac{2(9\beta^3 + 6\beta^2\gamma - 4\beta\gamma^2 - 2\gamma^3)(\alpha - c(\beta - \gamma))^2}{9\beta(\beta - \gamma)(9\beta^2 - 5\gamma^2)}$
Π_T^j	$\frac{2\beta(2\beta - \gamma)(\alpha - c(\beta - \gamma))^2}{(\beta - \gamma)(3\beta - \gamma)^2}$	$\frac{2(18\beta^3 + 3\beta^2\gamma - 11\beta\gamma^2 - \gamma^3)(\alpha - c(\beta - \gamma))^2}{9\beta(\beta - \gamma)(9\beta^2 - 5\gamma^2)}$

$$\Gamma = (9\beta^3 + \beta^2\gamma - 4\beta\gamma^2 - 2\gamma^3).$$

4.1. Comparison of the equilibrium values within each game model

Based on Tables 2–4 we establish the following relationships for the supplier's equilibrium wholesale prices to the retailers:

$$w_A^j = w^j, \quad j \in \{SSS\},$$

$$w_A^j < w^j, \quad j \in \{RSS, BS\}, \quad (34)$$

in which we observe that the supplier's wholesale prices to the retailers are different in the RSS and BS models, and the supplier will never set the advanced wholesale price higher than the normal wholesale price. This result is usually assumed in the previous studies, e.g. Choi and Chow (2008). But, it is attainable through the endogenous decision-making process in this study.

We proceed to characterize the retailers' equilibrium sale margins and order quantities as follows:

$$m_1^j = m_2^j, \quad q_1^j = q_2^j, \quad j \in \{SSB, RSB, BB\}, \quad (35)$$

$$m_1^j > m_2^j, \quad q_1^j < q_2^j, \quad j \in \{SSS, RSS, BS\}. \quad (36)$$

The relationships in (35) and (36) reveal that when the retailers move simultaneously, both retailers' sale margins and order quantities are the same. On the other hand, when the retailers move sequentially, retailer 1's sale margin is higher than retailer 2's sale margin yet retailer 1's order quantity is lower than retailer 2's order quantity. How would the sale margins and the order quantities jointly influence the retailers' profits across the game models? The following ordinal relationships answer this question:

$$\Pi_s^j > \Pi_1^j = \Pi_2^j, \quad j \in \{SSB, RSB, BB\}, \quad (37)$$

$$\Pi_s^j > \Pi_2^j > \Pi_1^j, \quad j \in \{SSS, RSS\}, \quad (38)$$

and for the BS model,

$$\begin{cases} \Pi_s^{\text{BS}} > \Pi_1^{\text{BS}} > \Pi_2^{\text{BS}}, & \text{if } 0 < \gamma/\beta < \frac{3(-1+2\sqrt{3})}{11} \\ \Pi_s^{\text{BS}} > \Pi_2^{\text{BS}} \geq \Pi_1^{\text{BS}}, & \text{if } \frac{3(-1+2\sqrt{3})}{11} \leq \gamma/\beta < 1. \end{cases} \quad (39)$$

We reason from (37) to (39) that the retailers have the same profits when they are engaged in Bertrand competition, as expected. However, when the retailers are engaged in a Stackelberg game, the second-mover advantage prevails for the retailers whether in the supplier- or retailer-Stackelberg competition. Such a result is consistent with the previous studies, e.g., Amir and Stepanova (2006), Gal-Or (1985), van Damme and Hurkens (2004), where the second-mover advantage usually exists in the pricing competition with symmetric players under the general condition on demand. A closer look at the equilibrium quantities reveals that at high retail substitutability, the difference between q_2^{BS} and q_1^{BS} increases significantly with retail substitutability, leading to a lower profit for the first-mover, even though the first-mover's sales margin is greater than the second-mover's sales margin. However, when retail substitutability is not high, the difference between q_2^{BS} and q_1^{BS} is less significant. In this case, a higher sales margin accounts for a higher profit, and thus the first-mover can secure a higher profit than the second mover can. The implication of this result is that if the supplier wants to receive the retailers' orders commitment in advance, negotiating or coordinating with the retailers is essential for the supplier to reverse the second-mover advantage (Cachon, 2003; Kostamis and Duenyas, 2009).

4.2. Comparison of the equilibrium quantities among the game models

The analysis of comparing the equilibrium quantities among the game models is likely to be affected by demand parameters. In this study, we focus on the effect of retail substitutability γ/β on the equilibrium quantities. We begin by analyzing the supplier's equilibrium wholesale prices in six game models.

Proposition 1. $w^{\text{SSB}} = w^{\text{SSS}} = w_A^{\text{SSS}} > w^{\text{BB}} > w^{\text{BS}} > w_A^{\text{BS}} > w^{\text{RSB}} > w^{\text{RSS}} > w_A^{\text{RSS}}$ for all $0 < \gamma/\beta < 1$.

Proposition 1 clearly shows that the ordinal relationship of the supplier's wholesale prices in all game models is independent of retail substitutability, even though the values of the supplier's

wholesale prices are affected by retail substitutability. The supplier will employ high wholesale prices when the supplier is a Stackelberg leader, and low wholesale prices when the retailers are the Stackelberg leaders. Further, the supplier will not charge lower wholesale prices when the retailers move simultaneously than when they move sequentially.

Proposition 2. *The ordinal relationship of the retailers' equilibrium sales margins in all game models as a function of retail substitutability is given below:*

- (i) If $\rho_1 < \gamma/\beta < 1$, then $m_1^{RSS} > m_2^{RSS} > m_1^{RSB} = m_2^{RSB} > m_1^{BS} > m_2^{BS} > m_1^{SSS} > m_2^{SSS} > m_1^{BB} = m_2^{BB} > m_1^{SSB} = m_2^{SSB}$;
- (ii) if $\rho_2 < \gamma/\beta \leq \rho_1$, then $m_1^{RSS} > m_2^{RSS} > m_1^{RSB} = m_2^{RSB} > m_1^{BS} > m_2^{BS} > m_1^{SSS} > m_2^{SSS} > m_1^{BB} = m_2^{BB} \geq m_2^{SSS} > m_1^{SSB} = m_2^{SSB}$;
- (iii) if $\rho_3 < \gamma/\beta \leq \rho_2$, then $m_1^{RSS} > m_2^{RSS} > m_1^{RSB} = m_2^{RSB} > m_1^{BS} > m_2^{BS} \geq m_1^{SSS} > m_1^{BB} = m_2^{BB} > m_2^{SSS} > m_1^{SSB} = m_2^{SSB}$;
- (iv) if $0 < \gamma/\beta \leq \rho_3$, then $m_1^{RSS} > m_2^{RSS} > m_1^{RSB} = m_2^{RSB} > m_1^{BS} > m_2^{BS} > m_1^{BB} = m_2^{BB} \geq m_1^{SSS} > m_2^{SSS} > m_1^{SSB} = m_2^{SSB}$; and

the numerical values of ρ_1, ρ_2, ρ_3 are 0.796, 0.775, 0.667, respectively.

It immediately follows from Proposition 2 that for all $0 < \gamma/\beta < 1$, $m_1^{RSS} > m_2^{RSS} > m_1^{RSB} = m_2^{RSB} > m_1^{BS} > m_2^{BS} > m_1^{BB} = m_2^{BB} > m_1^{SSB} = m_2^{SSB}$. Among the six game models, retailer 1's sale margin in the RSS model is the largest and the retailers' sale margins in the SSB model are the lowest. We find that the retailers' sale margins in the games with vertical competition being retailer Stackelberg are higher than those in the games with vertical competition being Bertrand or supplier Stackelberg. With the same vertical interaction, the retailers' sale margins when the retailers move sequentially are greater than their sale margins when they move simultaneously. We also find from Proposition 2 that the ranking of the retailers' sale margins in the SSS model varies with retail substitutability. It may be due to the fact that the retailers' sale margins in the SSS model are revealed at the last stage of the decision-making process, and thus a change in retail substitutability causes the retailers' equilibrium sale margins to vary considerably.

Proposition 3. *The ordinal relationship of the retailers' order quantities at equilibrium in all game models as a function of retail substitutability is given below:*

- (i) If $\rho_4 < \gamma/\beta < 1$, then $q_2^{BS} > q_2^{SSS} = q_2^{RSS} > q_1^{BB} = q_2^{BB} > q_1^{RSB} = q_2^{RSB} = q_1^{SSB} = q_2^{SSB} > q_1^{SSS} = q_1^{RSS} > q_1^{BS}$;
- (ii) if $\rho_5 < \gamma/\beta \leq \rho_4$, then $q_2^{BS} > q_2^{SSS} = q_2^{RSS} > q_1^{BB} = q_2^{BB} > q_1^{RSB} = q_2^{RSB} = q_1^{SSB} = q_2^{SSB} > q_1^{BS} \geq q_1^{SSS} = q_1^{RSS}$;
- (iii) if $\rho_6 < \gamma/\beta \leq \rho_5$, then $q_2^{BS} > q_1^{BB} = q_2^{BB} \geq q_2^{SSS} = q_2^{RSS} > q_1^{RSB} = q_2^{RSB} = q_1^{SSB} = q_2^{SSB} > q_1^{BS} > q_1^{SSS} = q_1^{RSS}$;
- (iv) if $\rho_7 < \gamma/\beta \leq \rho_6$, then $q_2^{BS} > q_1^{BB} = q_2^{BB} > q_2^{SSS} = q_2^{RSS} > q_1^{RSB} \geq q_1^{SSB} = q_2^{SSB} > q_1^{BS} > q_1^{SSS} = q_1^{RSS}$;
- (v) if $0 < \gamma/\beta \leq \rho_7$, then $q_2^{BS} > q_1^{BB} = q_2^{BB} > q_1^{BS} \geq q_2^{SSS} = q_2^{RSS} > q_1^{RSB} = q_2^{RSB} = q_1^{SSB} = q_2^{SSB} > q_1^{SSS} = q_1^{RSS}$; and

the numerical values of $\rho_4, \rho_5, \rho_6, \rho_7$ are 0.899, 0.796, 0.686, 0.633, respectively.

We can obtain from Proposition 3 that for all $0 < \gamma/\beta < 1$, $q_2^{BS} > q_2^{SSS} = q_2^{RSS} > q_1^{RSB} = q_2^{RSB} = q_1^{SSB} = q_2^{SSB} > q_1^{SSS} = q_1^{RSS}$ and $q_1^{BB} = q_2^{BB} > q_1^{BS}$. We find that retailer 2's order quantity in the BS model is the largest among all the game models and that the ranking of q_1^{BB} (q_2^{BB}) and q_1^{BS} is nonincreasing in retail substitutability. Further, the equalities $q_1^{RSB} = q_1^{SSB}$ and $q_1^{SSS} = q_1^{RSS}$, $i = 1, 2$, imply that the types of

vertical Stackelberg competition have no influence on each retailer's order quantity. And, together with Proposition 2, we conclude that the influence of the types of vertical Stackelberg competition is confined to the chain members' price decisions. We now turn our attention to the supply chain profits in all game models.

Proposition 4. *The ordinal relationship of the supply chain profits at equilibrium in all game models as a function of retail substitutability is given below:*

- (i) If $\rho_8 < \gamma/\beta < 1$, then $\Pi_T^{BB} > \Pi_T^{RSB} = \Pi_T^{SSB} > \Pi_T^{SSS} = \Pi_T^{RSS} > \Pi_T^{BS}$;
- (ii) if $\rho_9 < \gamma/\beta \leq \rho_8$, then $\Pi_T^{BB} > \Pi_T^{RSB} = \Pi_T^{SSB} > \Pi_T^{BS} \geq \Pi_T^{SSS} = \Pi_T^{RSS}$;
- (iii) if $0 < \gamma/\beta \leq \rho_9$, then $\Pi_T^{BB} > \Pi_T^{BS} \geq \Pi_T^{RSB} = \Pi_T^{SSB} > \Pi_T^{SSS} = \Pi_T^{RSS}$;

and the numerical values of ρ_8 and ρ_9 are 0.981 and 0.923, respectively.

It follows from Proposition 4 that $\Pi_T^{BB} > \Pi_T^{RSB} = \Pi_T^{SSB} > \Pi_T^{SSS} = \Pi_T^{RSS}$ independent of retail substitutability. The ranking of the supply chain profit of the BS model, however, is subject to the change of retail substitutability, especially when retail substitutability is high. We find that as long as vertical competition is Stackelberg, the supply chain profit is the same for both types of horizontal competition. This result, together with that in Proposition 3, implies that vertical leadership reallocates the supply chain profit among the chain members through the chain members' price decisions. Furthermore, the game models with simultaneous moves will outperform the other models with sequential moves in both vertical and horizontal competition. This is because the decision-making process with sequential moves is more complex and thus the chain members' equilibrium decisions are likely to deviate from the system optimum.

4.3. Comparing the game models with the integrated model

In this section, we illustrate the influence of retail substitutability on the performance of the game models with respect to the integrated model. For comparative purposes, we measure the performance of the game models by using the percentage difference Δ^j which is defined as

$$\Delta^j = \frac{\Pi_T^I - \Pi_T^j}{\Pi_T^I} \times 100\%, \quad j \in \{SSS, SSB, RSS, RSB, BS, BB\},$$

where smaller Δ^j means a higher performance of model j . As each measure Δ^j involves only the parameters β and γ , we use the graphic approach to represent the trend of Δ^j over the range of retail substitutability, as shown in Fig. 1.

We observe from Fig. 1 that the relationships among the performances of the game models echo Proposition 4, and that when retail substitutability increases, all Δ^j decrease and converge to zero, implying that when the retailers are more substitutable, all game models perform similarly and the types of vertical and horizontal competition have less impact on the supply chain profits. This further implies that at high retail substitutability, the impact of coordination, if undertaken, is insignificant under any power structure. On the other hand, when retail substitutability is low, we observe that $\Delta^{SSS}(\Delta^{RSS})$ and $\Delta^{SSB}(\Delta^{RSB})$ converge to nearly 25% and Δ^{BB} and Δ^{BS} converge to nearly 11% as retail substitutability decreases. It is clear that the difference in performance between game models $j = BB, BS$ and game models $j = SSB, RSB, SSS, RSS$, is significant at low retail substitutability. Further, the difference in performance between these game models widens as retail substitutability increases. Nevertheless, there is room for chain performance improvement under every game model by deploying coordinating mechanisms. A closer look

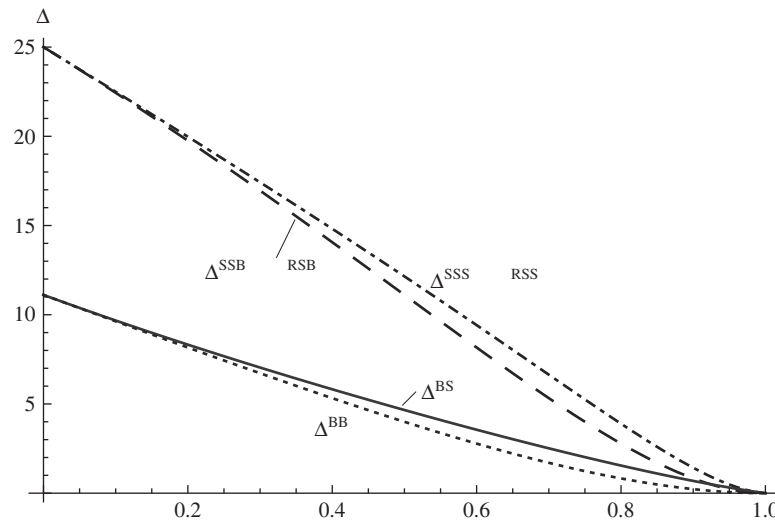


Fig. 1. The performances, Δ^i , of the game models in terms of retail substitutability γ/β .

at the performances of the game models at low retail substitutability reveals that the sequence of moves in horizontal competition has less impact on the supply chain profit. This is because the retailers have increasingly independent demand at lower retail substitutability. However, the types of vertical interaction at low retail substitutability play a determinant role in the performances of the game models. At extremely low retail substitutability, the types of vertical interaction account for nearly 14% in performance difference. The above analysis helps enrich our understanding about how horizontal and vertical interaction influence the performances of the game models. Finally, we obtain from Fig. 1 that the performance difference between $\Delta^{SSS}(\Delta^{RSS})$ and $\Delta^{SSB}(\Delta^{RSB})$ is noticeable when retail substitutability is not very high, with the maximal difference less than 2%. Similarly, the performance difference between Δ^{BB} and Δ^{BS} is noticeable at medium values of retail substitutability, with the maximal difference less than 1%.

4.4. Enforceability in games with vertical Stackelberg interaction

The chain members' equilibrium decisions in each game model are derived by deploying the rollback approach. It is, however, not guaranteed that the chain members will stick to their equilibrium strategies in actual play, especially in games involving sequential moves in horizontal interaction. This section thus hopes to resolve this doubt by examining these games. We first consider the SSB model. The supplier will certainly use its equilibrium wholesale price w^{SSB} which is expected to give the maximal profit. Once w^{SSB} is announced, the retailers' engagement in Bertrand competition will not be self-enforceable, because $w_i^{SSB} = w_i^{SSS}$ and $\Pi_i^{SSS} > \Pi_i^{SSB} = \Pi_i^{SSB}$, $i = 1, 2$. That is, the retailers have an incentive to deviate from simultaneous moves and engage in sequential moves instead.⁶ This suggests that in order to sustain the SSB model, the supplier shall exercise its power and design certain mechanisms or contracts under which the retailers will be obliged to move simultaneously.

Consider next the RSS and RSB game models. When the retailers are Stackelberg leaders in vertical interaction, the supplier could hardly defect because the wholesale prices are set for

the downstream retailers. From Section 4.2, we find that if both retailers have equal power, they will be engaged in the RSB model, and if one retailer, say retailer 2, plays a more dominant role than retailer 1, retailer 2 will choose to be a follower in horizontal competition, because of $\Pi_2^{RSS} < \Pi_1^{RSS}$. In either game, the supplier will not deviate from its equilibrium wholesale prices because they give the supplier the maximal profit once the retailers' sales margins are determined. Namely, the supplier's choice of the equilibrium wholesale prices is enforceable.

5. Summary

We have characterized the equilibrium decisions in a non-cooperative supply chain that consists of one common supplier and two retailers in the presence of horizontal and vertical competition by using six game models. Our analysis reveals that the ordinal relationships of certain equilibrium values among the game models are a function of retail substitutability. Yet, the game models perform similarly well when the retailers are highly substitutable. Our analysis also reveals that vertical interaction has more impact on the performances of the game models than horizontal interaction does when retail substitutability is low. Finally, we have found that because the retailers' engagement in Bertrand competition in the SSB model is not self-enforceable, the supplier should take preventive actions to resolve this problem.

Several extensions to this study are possible. First, in this study we have assumed symmetric retailers, which is widely adopted in the marketing and economics literature (Choi, 1991, 1996; McGuire and Staelin, 1983; Maggi, 1996; Padmanabhan and Png, 1997; Trivedi, 1998). A plausible research direction is to consider asymmetric retailers and investigate how retailer asymmetry and substitutability collectively influence the performances of the game models. Other generalization directions include, for example, consideration of multiple suppliers or multiple products, randomness and nonlinearity in demand, risk attitudes, etc. These directions, however, require restructuring of the current models, which inevitably raises analytical complexities. Finally, we have found in this study that when retail substitutability is low, the game models perform poorly as compared with the integrated model. Therefore, it is worthwhile to further investigate performance-improving mechanisms for the existing game models.

⁶ In a non-cooperative setting, when the retailers intend to be engaged in Stackelberg competition, it is not clear who shall act first. Hence, the retailers will then be engaged in a game to determine their sequence of moves. It is not difficult to see that there are multiple equilibria in this game and any retailer would prefer the equilibria where it is the follower rather than the equilibria wherein both retailers act simultaneously.

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Appendix

Proof of Proposition 1. The ordinal relationship of the supplier's wholesale prices w_i^j , $i = \{1, 2\}$, $j \in \{SSB, SSS, BB, BS, RSB, RSS\}$, can be established by pairwise comparison for all $0 < \gamma/\beta < 1$. \square

Proof of Proposition 2. Let $\rho = \gamma/\beta$. It is clear that $m_1^j = m_2^j$, $j \in \{SSB, BB, RSB\}$. From Tables 2–4 we obtain the following inequalities by pairwise comparison: $m_1^{RSS} > m_2^{RSS} > m_1^{RSB} = m_2^{RSB} > m_1^{BS} > m_2^{BS} > m_1^{BB} = m_2^{BB} > m_1^{SSB} = m_2^{SSB}$ and $m_1^{SSS} > m_2^{SSS}$. We see that the value of ρ that satisfies $m_2^{SSS} = m_1^{BB}$ is $\rho = \rho_1$ such that $4 - 2\rho_1 - 3\rho_1^2 - \rho_1^3 = 0$; the value of ρ that satisfies $m_1^{SSS} = m_2^{BS}$ is $\rho = \rho_2$ such that $18 - 3\rho_2 - 38\rho_2^2 + 3\rho_2^3 + 16\rho_2^4 = 0$; and the value of ρ that satisfies $m_1^{SSS} = m_1^{BB}$ is $\rho = \rho_3 = 2/3$. Both ρ_1 and ρ_2 can be uniquely determined over the interval (0,1), and the numerical values of ρ_1 and ρ_2 are 0.976 and 0.775, respectively. We can now show that if $\rho_1 < \rho < 1$, the ordinal relationship (i) holds; if $\rho_2 < \rho \leq \rho_1$, the ordinal relationship (ii) holds; if $\rho_3 < \rho \leq \rho_2$, the ordinal relationship (iii) holds; and if $0 < \rho \leq \rho_3$, the ordinal relationship (iv) holds. \square

Proof of Proposition 3. Let $\rho = \gamma/\beta$. By pairwise comparison, we obtain that $q_2^{BS} > q_2^{SSS} = q_2^{RSS} > q_1^{RSB} = q_2^{RSB} = q_1^{SSB} = q_2^{SSB} > q_1^{SSS} = q_1^{RSS}$ and $q_1^{BB} = q_2^{BB} > q_1^{BS}$ for $0 < \rho < 1$. We see that the value of ρ that satisfies $q_1^{BS} = q_1^{SSS}$ is $\rho = \rho_4$ such that $-18 + 3\rho_4 + 18\rho_4^2 + \rho_4^3 = 0$; the value of ρ that satisfies $q_1^{BS} = q_1^{RSB}$ is $\rho = \rho_6 = (-3 + \sqrt{33})/4 \approx 0.686$; and the value of ρ that satisfies $q_1^{BS} = q_2^{SSS}$ is $\rho = \rho_7$ such that $36 - 6\rho_7 - 81\rho_7^2 - 26\rho_7^3 + 33\rho_7^4 + 16\rho_7^5 = 0$. The value of ρ that satisfies $q_1^{BB} = q_2^{SSS}$ is $\rho = \rho_5$ such that $-4 + 2\rho_5 + 3\rho_5^2 + \rho_5^3 = 0$. The constants ρ_4, ρ_5 and ρ_7 can be uniquely determined over the interval (0,1), with the numerical values $\rho_4 = 0.899, \rho_5 = 0.796$ and $\rho_7 = 0.633$. We can then show that if $\rho_4 < \rho < 1$, the ordinal relationship (i) holds; if $\rho_5 < \rho \leq \rho_4$, the ordinal relationship (ii) holds; if $\rho_6 < \rho \leq \rho_5$, the ordinal relationship (iii) holds; if $\rho_7 < \rho \leq \rho_6$, the ordinal relationship (iv) holds; and if $0 < \rho \leq \rho_7$, the ordinal relationship (v) holds. \square

Proof of Proposition 4. Let $\rho = \gamma/\beta$. By pairwise comparison, we obtain that $\Pi_T^{BB} > \Pi_T^{RSB} = \Pi_T^{SSB} > \Pi_T^{SSS} = \Pi_T^{RSS}$ for $0 < \rho < 1$. We see that the value of ρ that satisfies $\Pi_T^{BS} = \Pi_T^{SSS}$ is $\rho = \rho_8$ such that $1440 + 384\rho_8 - 2368\rho_8^2 - 708\rho_8^3 + 1045\rho_8^4 + 276\rho_8^5 - 97\rho_8^6 = 0$ and the value of ρ that satisfies $\Pi_T^{BS} = \Pi_T^{RSB}$ is $\rho = \rho_9$ such that $45 - 33\rho_9 - 50\rho_9^2 + 32\rho_9^3 + 4\rho_9^4 = 0$. The values of ρ_8 and ρ_9 can be uniquely determined over the interval (0,1) as 0.981 and 0.923, respectively. We can then show that if $\rho_8 < \rho < 1$, the ordinal relationship (i) holds; if $\rho_9 < \rho \leq \rho_8$, the ordinal relationship (ii) holds; and if $0 < \rho \leq \rho_9$, the ordinal relationship (iii) holds. \square

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