HASSLER WHITNEY
(23 March 1907–10 May 1989)

Hassler Whitney was born in New York City to a family with a tradition of contributions to world knowledge. His father was a state supreme court judge; his mother was an artist, who was also active in politics. A grandfather, William D. Whitney, was a linguist and Sanskrit scholar and another grandfather, Simon Newcomb, was an astronomer. A great-grandfather surveyed the Atlantic coastline for Thomas Jefferson and a great uncle was the first to survey Mount Whitney.

Whitney attended Yale University and received a baccalaureate degree in physics in 1928 and in music in 1929. He earned a Ph.D. in mathematics at Harvard University in 1932. From 1932 to 1952 he taught at Harvard. He moved to the Institute for Advanced Study in 1952, where he retired in 1977.

Whitney was a topologist of great originality. His contributions were broad and could be roughly divided into the following areas:

1) *Differential topology.* Nineteenth-century mathematics was mainly mathematical analysis. At the turn of the century the importance of topology began to be recognized. It proceeded in two directions: (1) point-set topology, where the spaces are very general; (2) combinatorial topology, where the spaces are locally polyhedral. Differentiation does not seem to play a role. In fact, there was, as there is now, a sentiment against the calculus. A common interesting saying was: "Whenever I see a derivative it gives me nausea."[1] In such an atmosphere Whitney created differential topology, which became a most active mathematical area in recent times.

2) *Cup products.* A fundamental problem in combinatorial topology is to define topological invariants of manifolds. The most elementary among these are the Betti numbers, based on the boundary relation. It was Emmy Noether who observed that these can be built into an algebraic structure; the homology groups of all dimensions. A much deeper concept is the introduction of a multiplication based on intersection theory.

It is remarkable that the dual of the homology groups, the cohomology groups, has most beautiful properties. Using the cup product introduced by Whitney, the direct sum of cohomology groups has a ring

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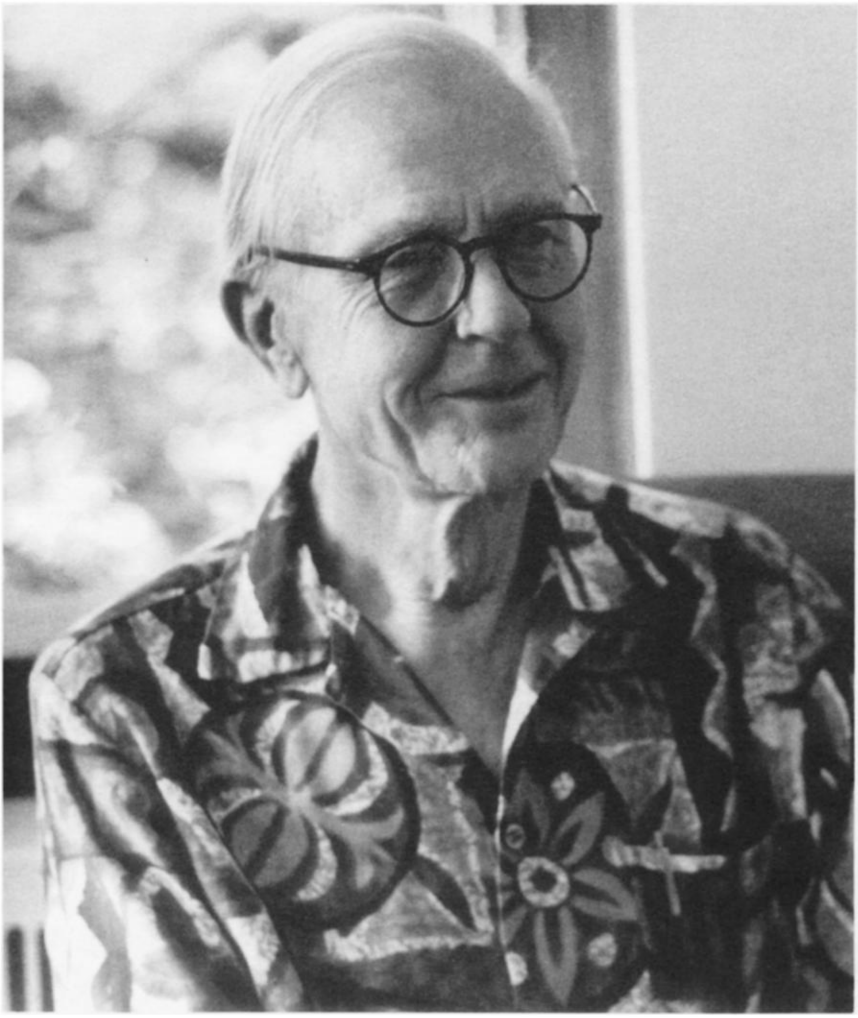
[1] Edgar R. Lorch, "Mathematics at Columbia during adolescence," *A Century of Mathematics in America*, 3: 153, American Mathematical Society, 1989. I also heard similar statements from other sources.



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structure and becomes the cohomology ring. This can be defined for spaces more general than manifolds. If there is a continuous mapping $f : X \to Y$, it induces a ring homomorphism $f^* : H^*(Y) \to H^*(X)$, where $H^*(X)$ and $H^*(Y)$ are the cohomology rings of the spaces X and Y respectively.

Cohomology groups are useful in the applications of topology. They are closely related to multiple integration. Whitney wrote a book on integration which is full of original geometrical ideas but unappreciated by the analysts.

From Betti numbers to the cohomology ring was a great development in topology. Whitney's cup products made a crucial contribution.

3) *Sphere bundles.* This is perhaps the contribution for which Whitney is best known, even the name originating from him. It is a remarkable fact that a family of spheres which is locally a topological product may not be a global product. The first invariants of such a phenomenon are the so-called Stiefel-Whitney characteristic classes. They were discovered almost simultaneously by Whitney and E. Stiefel, the latter in his thesis written under the supervision of Heinz Hopf. Stiefel restricted himself to the tangent bundle of a manifold and drew beautiful conclusions, including the theorem that a closed orientable 3-dimensional manifold is parallelizable. Whitney saw the merit of the general notion of a sphere bundle over any space. In particular, this leads to algebraic operations on bundles. The description of the characteristic classes of the sum of two sphere bundles is the important Whitney duality theorem. Whitney's original proof covers the general case of local coefficients. It was very long and was never published. (I can still remember when he showed me the proof on a snowy Sunday when I visited his home near Watertown; it was like a book. The first proof, for an important special case, was given by Wu Wen-Tsun; cf. *Annals of Mathematics* 49 [1948]: 641–653.)

Fiber bundles have since become a fundamental notion in topology.

4) *Stratified manifolds and singularities.* Whitney realized that the notion of a smooth manifold is not broad enough. For example, a cube is not a smooth manifold and algebraic varieties generally allow singularities. He introduced the notion of a stratification of a manifold. Personally it is my feeling that stratified manifolds will be the main object in differential geometry. They already play an important role in McPherson's theory of preverse sheaves.

5) *Miscellaneous.* Whitney liked mathematical problems and was able to come up with ingenious solutions. An example was his characterization of the closed 2-cell. I think it is not unfair to say that he has worked on the four-color problem. He realized its difficulty. He was happy with the computer proof by W. Haken and K. Appel.

Another result is the Graustein-Whitney theorem on the regular homotopy of closed curves in the plane, an elementary result leading to much development.

In his last years he devoted much time to the mathematical education of children, arousing their interest by asking challenging questions.

As mentioned in the beginning he had a degree in music. He was an accomplished violin and viola player.

Whitney received the National Medal of Science in 1976, the Wolf Prize in 1982, and the Steele Prize of the American Mathematical Society in 1985.

In an article[2] on the occasion of the centennial of the American Mathematical Society Whitney gave a report on the Topology Conference in 1935 in Moscow. The article is both personal and mathematical and is full of interesting facts and anecdotes. Being an avid mountaineer himself, he recounted how he met J. Alexander and G. de Rham in the Swiss mountains. He was a member of the Geneva section of the Swiss Alpine Club. On 20 August 1989, his ashes were placed at the summit of the Swiss mountain Dent Blanche by Oscar Burlet, a mathematician and member of the Swiss Alpine Club.

ELECTED 1947

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[2] Hassler Whitney, Moscow 1935, "Topology moving toward America," *A Century of Mathematics in America*, 1: 97–117. There is an error on page 117: the simple proof of the duality theorem was given by Wu Wen-Tsun, cf. text.