15.094J/1.142J Robust Modeling, Optimization and Computation

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Lecture: Wednesday 9-12, E51-372.

Recitation: Friday 11-12, E51-145.

Course Content and Objectives: Today the modeling of uncertain phenomena in both theory and practice is predominantly done via probability theory, whose foundation is based on the axioms set forth by Kolmogorov in 1933. While it offers insights in understanding uncertainty, probability theory, in contrast to optimization, has not been developed with computational tractability as an objective when the dimension increases. Correspondingly, some of its major areas of application remain unsolved when the underlying systems become multidimensional: Queueing networks, auction design in multi-item, multi-bidder auctions, network information theory, pricing multi-dimensional options, optimization under uncertainty among others. At the current time, the only alternative for modeling uncertain phenomena is probability theory.

The first goal of the class is to propose a computationally efficient alternative via robust optimization (RO) for modeling uncertain phenomena. The key idea is to replace the Kolmogorov axioms and the concept of random variables as primitives of probability theory, with uncertainty sets that are derived from some of the asymptotic implications of probability theory. In this way, the performance analysis questions become highly structured optimization problems (linear, semidefinite, mixed integer) for which there exist efficient, practical algorithms that are capable of solving problems in high dimensions involving hundreds of thousands of variables and constraints.

The second goal of the class is to develop RO as a tractable methodology for solving optimization problems under uncertainty. Specifically we review under what conditions

RO problems remain tractable when modeling uncertainty via uncertainty sets. The notion of tractability used, however, is not the same as theoretical efficiency (polynomial time solvability) developed in the 1970s. The Simplex method, for instance, has proven over many decades to be practically efficient, but not theoretically efficient. It is exactly this notion of practical efficiency we use in this class: it is the ability to solve problems of realistic size relative to the application we address. For example, queueing networks with hundreds of nodes, auctions with hundreds of items and bidders with budget constraints, network information theory with hundreds of thousands of codewords, and option pricing problems with hundreds of securities.

The third goal of the class is to expose students to a large number of applications ranging from supply chains, revenue management, energy, portfolio theory, options pricing, risk management, Kalman filtering, queueing theory, information theory, statistics and engineering design. Our objective here is to teach students how to model and optimize uncertain phenomena via RO and be able to solve large scale problems involving hundreds of thousands to millions of variables and constraints.

With the availability of very large data sets arising in applications, the key principle we use in this class is that the ability to compute in high dimensions is critical. Correspondingly, **the fourth and final goal of the class is to expose students to the tools necessary for large scale computation for RO.**

Text: Research papers and class notes. All handouts can be downloaded from: https://stellar.mit.edu/S/course/15/sp17/15.094/

Recitations: The recitations will cover software for RO, computational aspects, and examples and applications that enhance the theory developed in the lectures.

Course Requirements: Problem sets, one examination, and one final team project. Grades will be determined by performance on the above requirements weighted approximately as 30% problem sets, 30% exam, and 40% final team project.

Lecture	Time	Topic	Readings
1	W, 2/08	Probability theory and its limitations	[1]
2	W, 2/08	The new primitives: Uncertainty sets	[1]
3	W, 2/15	Robust linear optimization I	[11]
4	W, 2/15	Robust linear optimization II	[29, 14]
5	W, 2/22	Robust mixed integer optimization	[30, 31]
6	W, 2/22	Robust convex optimization	[10, 32]
7	W, 3/01	Data driven robust optimization	[22]
8	W, 3/01	From data to decisions	[25]
9	W, 3/08	Adaptive optimization I	[9, 23, 8]
10	W, 3/08	Adaptive optimization II	[24, 15, 18, 17]
11	W, 3/15	Distributionally robust optimization	[33]
12	W, 3/15	Distributionally adaptive optimization	[33]
13	W, 3/22	Power of robust policies in adaptive optimization	[21, 20]
14	W, 3/22	Power of affine policies in adaptive optimization	[19, 23]
	W, 3/29	Spring break	
15	W, 4/05	RO, risk preferences and utilities	[13, 27]
16	W, 4/05	RO in energy	[26]
17	W, 4/12	Robust portfolios	[36, 28]
18	W, 4/12	Robust options pricing	[3]
19	W, 4/19	Robust steady-state queueing theory	[6]
20	W, 4/19	Robust transient queueing theory and supply chains	[7, 34]
	W, 4/26	Exam	
21	W, 5/03	Robust mechanism design	[2]
22	W, 5/03	RO in statistics	[35, 16]
23	W, 5/10	RO in control theory	[12]
24	W, 5/10	Robust information theory	[4, 5]
25	W, 5/17	Project presentations	
26	W, 5/17	Project presentations	

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