以下内容参考了某大学开设的全部纯数专业的本科数学课，按照prerequisite的要求，我把课程分成了四层。只要学好每一层的基础课，就可以学这一层的全部进阶课以及下一层的全部基础课，以此类推。值得注意的是，有的进阶课不需要学完这一层全部的基础课，不过为了方便起见，以及基础课也不多，所以就不标出每门进阶课到底需要什么基础（比如学了复分析就可以学解析数论了，注意要学到复分析说明上面的基础课也学完了）。其中大部分进阶课都不是必要的课，可以按照个人兴趣学习。

第一层：

基础课：数学分析 线性代数

进阶课：无

第二层：

基础课：多元微积分 拓扑学

进阶课：常微分方程 数论 图论 数理逻辑 组合学 混沌分形动力系统 流形和微分形式

第三层：

基础课：抽象代数 实分析 复分析1

进阶课：偏微分方程 微分几何 高等概率论 复分析2

第四层：

基础课：无

进阶课：欧式几何与非欧几何 集合论 代数数论 解析数论 微分拓扑 线性算子导论 K理论和C\*代数 表示论 交换代数与代数几何 代数曲线 泛函分析 黎曼几何与微分几何 代数拓扑

北美各大高校参考书汇总（未必全，但是这些书质量很高）

第一层：

基础课1-1数学分析

1. Hass, Heil, and Weir, Thomas’ Calculus: Early Transcendentals, Single Variable, Pearson Education, 2018 (Edition: 14; ISBN: 978-0-13-443941-9).
2. Hughes-Hallett, et al., Calculus: Single and Multivariable, Wiley, 2017 (Edition: 7; ISBN: 978-1-119-26000-4)
3. Finney, Weir, Giordano, Thomas' Calculus custom edition for Princeton, Vol. 1
4. Stewart, James, Calculus (Edition: 8th; ISBN: 978-1-285-74062-1)

基础课1-2线性代数

1. Lay, Lay, and McDonald, Linear Algebra and its Applications, Pearson Education, 2016 (Edition: 5; ISBN: 978-0-321-98238-4)
2. Hubbard and Hubbard, Vector Calculus, Linear Algebra and Differential Forms: A Unified Approach, Matrix Editions, 2015 (Edition: 5; ISBN: 978-0-9715766-8-1).
3. Friedberg, Insel, and Spence, Linear Algebra, Prentice Hall PTR, 2018 (Edition: 5; ISBN: 978-0-13-486024-4).
4. Cooperstein, Bruce, Advanced Linear Algebra, CRC Press LLC, 2015 (Edition: 2; ISBN: 978-1-4822-4884-5).
5. G. Strang, Introduction to Linear Algebra

进阶课：无

第二层：

基础课2-1 多元微积分

1. Marsden and Tromba, Vector Calculus, W. H. Freeman, 2013 (Edition: 6; ISBN: 1-4292-1508-9)
2. Finney, Weir, Giordano, Thomas' Calculus customized for Princeton University, Vol. 2
3. Stewart, James, Multivariable Calculus, 8th ed.
4. Andrew Browder, Mathematical Analysis. An Introduction, Springer, 1996.
5. W. McCallum, D. Hughes-Hallett, A. Gleason, Multivariable Calculus, 3rd ed.
6. Munkres, J. Analysis on Manifolds. Cambridge, MA: Perseus Publishing, 1991. ISBN: 0201510359, ISBN: 0201315963 (paperback).
7. Spivak, M. Calculus on Manifolds: A Modern Approach to Classical Theorems of Advanced Calculus. Cambridge, MA: Perseus Publishing, 1965. ISBN: 0805390219.

基础课2-2拓扑学

1. Jänich, Klaus, Topology, Springer, 1984 (ISBN: 978-0-387-90892-2).
2. Munkres, Topology, 2nd Ed
3. Manetti, Topology, Springer, 2015 (ISBN: 9783319169576)

进阶课2-1 常微分方程

1. Brauer and Nohel, The Qualitative Theory of Ordinary Differential Equations: An Introduction, Dover Publications, 1969 (ISBN: 0-486-65846-5).
2. Boyce & DiPrima , Elementary Differential Equations and Boundary Value Problems (10th Edition) , Wiley 2012
3. W. Kohler and L. Johnson, Elementary Differential Equations

进阶课2-2 数论

1. Weissman, Martin, An Illustrated theory of Numbers, American Mathematical Society, 2017 (ISBN: 978-1-4704-3493-9).
2. Gareth A. Jones and J. Mary Jones, Elementary Number Theory
3. J. Silverman, A Friendly Introduction to Number Theory, 4th ed.
4. Ireland, Classical Introduction to Modern Number Theory (Edition: 2nd 1990; ISBN: 9780387973296)
5. J. K. Strayer, Elementary Number Theory
6. I. Niven, H. Zuckerman, H. Montegomery, An Introduction to the Theory of Numbers

进阶课2-3 图论

1. Douglas B. West, Introduction to Graph Theory, Second Edition, Prentice-Hall.
2. G. Harris, J. Hirst & M. Mossinghoff, Combinatorics and Graph Theory.
3. Balakrishnan & Kanganathan, A Textbook of Graph Theory

进阶课2-4 数理逻辑

1. Robert L. Vaught, Set Theory: An Introduction

进阶课2-5 组合学

1. Van Lint and Wilson, A Course in Combinatorics (paperback), Cambridge University Press, 2001 (Edition: 2; ISBN: 0-521-00601-5)
2. G. Harris, J. Hirst & M. Mossinghoff, Combinatorics and Graph Theory.

进阶课2-6 混沌 分形 动力系统

1. Hirsch, Smale, and Devaney, Differential Equations, Dynamical Systems, and an Introduction to Chaos, Elsevier Science & Technology Books, 2012 (Edition: 3; ISBN: 978-0-12-382010-5)
2. Devaney, Robert, An Introduction to Chaotic Dynamical Systems, CRC Press, 2003 (Edition: 2; ISBN: 978-0-8133-4085-2).
3. Kathleen T. Alligood, Tim D. Sauer, and James A. Yorke, Chaos: An Introduction to Dynamical Systems
4. Strogatz, S. Nonlinear Dynamics And Chaos: With Applications To Physics, Biology, Chemistry, And Engineering. Westview Press, 2001. ISBN: 9780738204536.
5. Berge, P., Y. Pomeau, and C. Vidal. Order within Chaos. Wiley-VCH, 1987. ISBN: 9780471849674.

进阶课2-7 流形和微分形式

1. do Carmo, Manfredo, Differential Forms and Applications, Springer, 1998 (ISBN: 978-3-540-57618-1)

第三层：

基础课3-1 抽象代数

1. Beachy and Blair, Abstract Algebra, Waveland Press, Inc., 2019 (Edition: 4; ISBN: 978-1-4786-3869-8).
2. Dummit and Foote, Abstract Algebra, John Wiley & Sons, 2004 (Edition: 3; ISBN: 0-471-43334-9).
3. Michael Artin, Algebra, 2nd Edition, 2011
4. Serge Lang, Algebra (ISBN: 0-387-95385-X) [Main Resource for Field Theory]
5. Thomas Judson, Abstract Algebra [Group and Ring Theory]
6. Topics in Algebra, 2nd Ed.
7. Saracino, Abstract Algebra: First Course, (Edition: 2nd 08; ISBN: 9781577665366)
8. Serge Lang, Undergraduate Algebra.
9. I. Stewart, Galois Theory
10. D. Cox, Galois Theory
11. Herstein, I. N. Abstract Algebra. Macmillan, 1986. ISBN: 9780023538209.

基础课3-2 实分析

1. Strichartz, Robert, The Way of Analysis (revised edition), Jones & Bartlett Publishers, 2000 (ISBN: 0-7637-1497-6).
2. Rudin, Walter, Principles of Mathematical Analysis, McGraw-Hill, 1976 (Edition: 3; ISBN: 0-07-054235-X).
3. Stein and Shakarchi, Real Analysis: Measure Theory, Integration, and Hilbert Spaces, Princeton University Press, 2005 (ISBN: 0-691-11386-6).
4. Bartle, Robert, The Elements of Integration and Lebesgue Measure, John Wiley & Sons, 1966 (ISBN: 0-471-04222-6)
5. Royden & Fitzpatrick, Real Analysis, 4th Ed
6. Russel A. Gordon, “Real Analysis, A First Course”, second edition, published by Addison Wesley. Available at Wheelock Books.

基础课3-3 复分析1

1. Levinson and Redheffer, Complex Variables, Holden-Day, Inc., 1970 (ISBN: 0-8162-5104-5).
2. Stein and Shakarchi, Complex Analysis, Princeton University Press, 2003 (ISBN: 0-691-11385-8).
3. Andrew Pressley, Elementary Differential Geometry
4. Serge Lang, Complex Analysis
5. T. Gamelin, Complex Analysis
6. Brown, James Ward and Ruel V. Churchill. Complex Variables and Applications. 9th ed. McGraw-Hill Education, 2013. ISBN: 9780073383170.
7. Ahlfors, Lars V. Complex Analysis: An Introduction to the Theory of Analytic Functions of One Complex Variable. 3rd ed. New York, NY: McGraw-Hill, 1979. ISBN: 9780070006577.

进阶课3-1 偏微分方程

1. Zachmanoglou and Thoe, Introduction to Partial Differential Equations with Applications, Dover Publications, 1986 (ISBN: 0-486-65251-3).
2. Taylor, Michael, Partial Differential Equations I: Basic Theory, Springer, 2011 (Edition: 2; ISBN: 978-1-4419-7054-1)
3. Taylor, Michael, Partial Differential Equations II: Qualitative studies of linear equations, Springer, 2011 (Edition: 2; ISBN: 978-1-4419-7051-0)
4. Evans, Lawrence, Partial Differential Equations, American Mathematical Society, 2010 (Edition: 2; ISBN: 978-0-8218-4974-3)
5. Pressley, Andrew, Elementary Differential Geometry.
6. Strauss, Partial Differential Equations
7. Birkhoff, Garret, and Gian-Carlo Rota. Ordinary Differential Equations. 4th ed. New York, NY: Wiley, 1989. ISBN: 9780471860037.

进阶课3-2 微分几何

1. Pressley, Andrew, Elementary Differential Geometry, Springer-Verlag, 2010 (Edition: 2; ISBN: 978-1-84882-890-2)
2. M. DoCarmo, Differential Geometry of Curves and Surfaces
3. J. Thorpe. Elementary Topics in Differential Geometry
4. Kuhnel, Wolfgang. Differential Geometry: Curves – Surfaces – Manifolds. Student mathematical library, vol. 16. Providence, RI: American Mathematical Society, 2002. ISBN: 9780821826560.
5. Spivak, Michael. A Comprehensive Introduction to Differential Geometry. Vol. 2. Boston, MA: Publish or Perish, 1999. ISBN: 9780914098713.
6. Gray, Alfred, Simon Salamon, and Elsa Abbena. Modern Differential Geometry of Curves and Surfaces with Mathematica. Boca Raton, FL: Chapman & Hall/CRC, 2006. ISBN: 9781584884484.

进阶课3-3 高等概率论

进阶课3-4 复分析2（同复分析1）

第四层：

基础课：无

进阶课4-1 欧式几何与非欧几何

进阶课4-2 集合论

1. Elements of Set Theory, Herbert B. Enderton, Academic Press, 1977. ISBN: 978-0122384400

进阶课4-3 代数数论

1. Samuel, Pierre. Algebraic Theory of Numbers. Translated by Allan J. Silberger. Mineola, NY: Dover, 2008. ISBN: 9780486466668.
2. Milne, J. S. Algebraic Number Theory, 2009.

进阶课4-4 解析数论

1. Davenport, Multiplicative Number Theory (Edition: 3; ISBN: 978-1-4757-5929-7)
2. Serre, Course in Arithmetic (Edition: 1973; ISBN: 978-1-4684-9884-4)

进阶课4-5 微分拓扑

进阶课4-6 线性算子导论

进阶课4-7 K理论和C\*代数

进阶课4-8 表示论

进阶课4-9 交换代数与代数几何

1. Eisenbud, David, Commutative Algebra with a View Toward Algebraic Geometry, Springer-Verlag, 1995 (ISBN: 0-387-94269-6)
2. M.F. Atiyah & I.G. Macdonald, Introduction to Commutative Algebra
3. Miles Reid, Undergraduate Commutative Algebra
4. Hartshorne, Algebraic Geometry
5. Kollar, Janos, Lectures on Resolution of Singularities, Princeton University Press, 2007
6. C. Weibel, An Introduction to Homological Algebra
7. J. Harris, Algebraic Geometry, A First Course
8. Kempf, G. Algebraic Varieties. Cambridge University Press, 1993. ISBN: 9780521426138
9. Shafarevich, Igor R. Basic Algebraic Geometry 1: Varieties in Projective Space. Springer, 2013. ISBN: 9783642379550

进阶课4-10 代数曲线

1. Liu, Qing, Algebraic Geometry and Arithmetic Curves (Oxford Graduate Texts in Mathematics)

进阶课4-11 泛函分析

1. Stein and Shakarchi, Functional Analysis: Introduction to Further Topics in Analysis, Princeton University Press, 2011 (ISBN: 978-0-691-11387-6).
2. Real Analysis (fourth edition) by H. L. Royden and P. M. Fitzpatric
3. S. Sternberg, Theory of functions of a Real Variables
4. W. Rudin, Functional analysis
5. P. Lax, Functional analysis
6. G. Folland, Real analysis
7. Muscat, Functional Analysis (Edition: 2014; ISBN: 9783319067278)
8. J. B. Conway, A Course in Functional Analysis
9. S. Lang, Real and Functional Analysis
10. R. Zimmer, Essential Results of Functional Analysis
11. W. Milman, Functional Analysis, an Introduction

进阶课4-12 黎曼几何与微分几何

1. Manfredo P. do Carmo, Riemannian Geometry
2. Tobias Colding & William Minicozzi, Notes on a course in minimal surfaces
3. Peter Topping, Lectures on the Ricci Flow
4. Richard Schoen & Shing-Tung Yau, Lectures on Differential Geometry

进阶课4-13 代数拓扑

1. Hatcher, Allen, Algebraic Topology (paperback), Cambridge University Press, 2001 (0-521-79540-0).
2. P. Shanahan, Atiyah-Singer Index Theorem, Springer-Verlag Berlin Heidelberg.

参考资料：

1. <https://math.cornell.edu/textbooks>
2. <https://registrar.princeton.edu/course-offerings?term=1212&subject=MAT>
3. <https://columbia.bncollege.com/shop/columbia/page/find-textbooks>
4. <https://insite.browntextbook.com/CourseMaterials>
5. <https://math.yale.edu/undergrad-programs/recent-textbooks>
6. <https://ocw.mit.edu/courses/mathematics/>