一. For each blank in the following statement, choose the best answer from the choices given below. (15 points)

1. This system of linear equations $\begin{cases} x_1 + 2x_2 + 4x_3 = 5, \\ 2x_1 + 4x_2 + 5x_3 = 4 \\ 4x_1 + 5x_2 + 4x_3 = 2 \end{cases}$ has (　　) (A)no solution (B) exactly one solution (C) exactly two solutions (D) infinitely many solutions

2. Choose the false statement in the following four statements (　　).

   (A) The columns of a matrix A are linearly independent if the equation $\mathrm{Ax} = 0$ has only the trivial solution.

   (B) If S is a linearly dependent set, then each vector is a linear combination of the other vectors in S.

   (C) The columns of any 4×5 matrix are linearly dependent.

   (D) If x and y are linearly independent, and if $\{\mathrm{x}, \mathrm{y}, \mathrm{z}\}$ is linearly dependent, then z is in $\mathrm{Span}(\mathrm{x}, \mathrm{y})$.

3. Let $\mathrm{T}: \mathrm{R}^3 \to \mathrm{R}^2$ be a linear transformation such that $\mathrm{T}(\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3) = (x_1 + x_2 + x_3, x_2 + x_3)$, Then the standard matrix A of T is (　　).

   (A) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. (B) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$

4. Choose the right statement in the following four statements on matrices (　　).

   (A) If $\mathrm{BC} = \mathrm{BD}$, then $\mathrm{C} = \mathrm{D}$. (B) If $\mathrm{AC} = 0$, then $\mathrm{A} = 0$ or $\mathrm{C} = 0$,.

   (C) If A and B are $\mathrm{n} \times \mathrm{n}$, then $(\mathrm{A} + \mathrm{B})(\mathrm{A} - \mathrm{B}) = \mathrm{A}^2 - \mathrm{B}^2$.

   (D) An elementary $\mathrm{n} \times \mathrm{n}$ matrix has either n or $\mathrm{n} + 1$ nonzero entries.

5. The rank of the matrix $\begin{bmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix}$ is 2. Then k is (　　). (A) 0; (B) -1; (c) -2; (D) -3.

二、 Fill the correct answer in the blanks (15 points)

6. Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, then the reduced echelon form of A is _______. 7. Let $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 4 & 3 \\ 0 & 1 & 2 \end{bmatrix}$, then $A^{-1} =$ __. 8. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 3 \\ 4 & 4 & 1 \end{bmatrix}$=___

9. The rank of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 3 \\ 4 & 4 & 1 \end{bmatrix}$ is ______. 10. Let $u = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ and $\mathrm{v} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$,

then $(\mathrm{vu})^3$ =______.

三. Determine if the system of linear equations $\begin{cases} x_1+x_2+x_3=1, \\ x_1+2x_2+x_3=3 \\ 5x_1+8x_2-x_3=11 \end{cases}$ is consistent. If it is consistent, please describe all the solutions of this system in parameteric vector form. (10 points)

四. Let $\mathrm{A}=\begin{bmatrix} 1 & -2 & 3 & 2 \\ -2 & 4 & -6 & 2 \\ 0 & 1 & -1 & 3 \end{bmatrix}$ and $\mathbf{u}=\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. (1) Determine if the columns of the matrix A form a linearly independent set. (2) Determine if $\mathbf{u}$ is a linear combination of the columns of the matrix A. (10 points)

五. Let $\mathrm{A}=\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 1 & -2 & 2 \end{bmatrix}$ and $\mathrm{B}=\begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 3 \\ 2 & 2 & 1 \end{bmatrix}$. (1) Compute AB, BA and $\mathrm{AB}-\mathrm{BA}$; (2) Compute $3\mathrm{A}^{\mathrm{T}}\mathrm{B}-\mathrm{B}$ (10 points)

六. Let $\mathrm{A}=\begin{bmatrix} 1 & 2 & -1 \\ -4 & -7 & 3 \\ 2 & 3 & -2 \end{bmatrix}$. (1) Compute the inverse of A; (2) Solve the matrix equation $\mathrm{AX}=\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. (10 points)

七. Solve the following matrix equations

(1) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}X=\begin{bmatrix} 0 & 2 & 1 \\ 2 & -1 & 3 \end{bmatrix}$; (2) $X\begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}=\begin{bmatrix} 5 & 4 \\ -2 & 3 \end{bmatrix}$; (3) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}X\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}=\begin{bmatrix} 3 & 2 & 1 \\ 9 & 8 & 7 \\ 6 & 5 & 4 \end{bmatrix}$ (10 points)

八. Let $\mathrm{A}=\begin{bmatrix} 1 & 3 & 2 & -6 \\ 3 & 9 & 1 & 5 \\ 2 & 6 & -1 & 9 \\ 5 & 15 & 0 & 14 \end{bmatrix}$ and $\mathbf{u}=\begin{bmatrix} 3 \\ 4 \\ 1 \\ 5 \end{bmatrix}$. (1) Find a basis for the vector space of Col(A); (2) Determine if $\mathbf{u}$ is in Col(A), and if it is, find the coordinate vector of $\mathbf{u}$ (relative to the basis for Col(A)). (8 points)

九. Let $\mathrm{A}=\begin{bmatrix} 1 & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix}$ and $\mathbf{u}=\begin{bmatrix} 1 \\ k \\ k^2 \end{bmatrix}$. (1) Is the equation $\mathrm{AX}=\mathbf{u}$ consistent for all possible k? ; (2) Describe all the solutions of $\mathrm{AX}=\mathbf{u}$ if this linear system is consistent. (7 points)

十. Let A be an $\mathrm{n}\times\mathrm{n}$ matrix. (1) If $\mathrm{A}^3=0$, compute $(\mathrm{I}-\mathrm{A})(\mathrm{I}+\mathrm{A}+\mathrm{A}^2)$ and prove that $\mathrm{I}-\mathrm{A}$ is invertible; (2) If $\mathrm{A}^2-3\mathrm{A}+\mathrm{I}=0$, prove that $2\mathrm{I}-\mathrm{A}$ is invertible. (5 points)