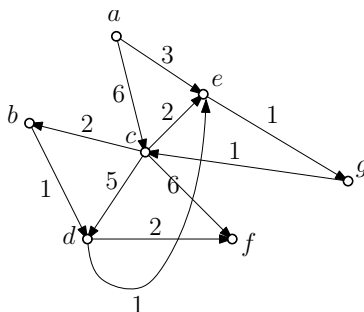


CSCI3160: Special Exercise Set 10

Prepared by Yufei Tao

Problem 1. Consider the weighted directed graph below.



Run Dijkstra's algorithm starting from vertex a . Recall that the algorithm relaxes the outgoing edges of every other vertex in turn. Give the order of vertices by which the algorithm relaxes their edges.

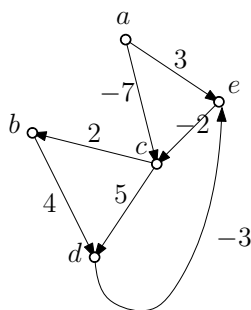
Problem 2. Consider a simple directed graph $G = (V, E)$ where each edge $(u, v) \in E$ carries a non-negative weight $w(u, v)$. Given two vertices $u, v \in V$, function $spdist(u, v)$ represents the shortest path distance from u to v . Given a vertex $v \in V$, denote by $IN(v)$ the set of in-neighbors of v . Let s and t be two distinct vertices in G . Prove:

$$spdist(s, t) = \min_{v \in IN(t)} \{spdist(s, v) + w(v, t)\}.$$

(Hint: First prove $LHS \leq RHS$, and then prove \geq .)

Problem 3. Give a counterexample to show that Dijkstra's algorithm does not work if edge weights can be negative.

Problem 4. Consider the weighted directed graph $G = (V, E)$ below.



Set the source vertex to a and run Bellman-Ford's algorithm, which performs 4 rounds of edge relaxations. Show the $dist(v)$ value of every $v \in V$ after each round.

Problem 5. The Bellman-Ford algorithm presented in the lecture computes only the shortest-path distance from the source vertex s to every vertex. Extend the algorithm to output a shortest-path tree of s . Your modified algorithm must still terminate in $O(|V||E|)$ time.

Problem 6 (SSSP with Unit Weights). Let us simplify the SSSP problem by requiring that all the edges in the input directed graph $G = (V, E)$ take the *same* positive weight, which we assume to be 1. Give an algorithm that solves the SSSP problem in $O(|V| + |E|)$ time.