Physics	logic
Quantum Mechanics	Encoding: qubits manipulating: "4 postulates"

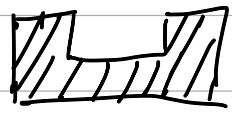
The 4 Postulates for Single-Qubit System

Classical:

0 or 1 (classical bit)

register position

Quantum:



a quantum register.

A qubit: $|0\rangle$, $|1\rangle$.

$$\alpha \cdot |0\rangle + \beta \cdot |1\rangle$$

where $\alpha, \beta \in \mathbb{R}$
and $\alpha^2 + \beta^2 = 1$

(I'm cheating)

The special symbol : $| \rangle$ <u>ket</u>.

by mathematician / physicist Paul Dirac

[The other half $\langle |$ is <u>bra</u>

so, $\langle | \rangle$ <u>bracket</u>]

Linear Algebra?

$$|\psi\rangle = \alpha \cdot |0\rangle + \beta \cdot |1\rangle$$

Is it reminiscent of linear algebra?

- $\{|0\rangle, |1\rangle\}$ are basis "vectors".
  - they are "orthogonal".
  - they are "unit" vectors.
  - natural choice of notation:

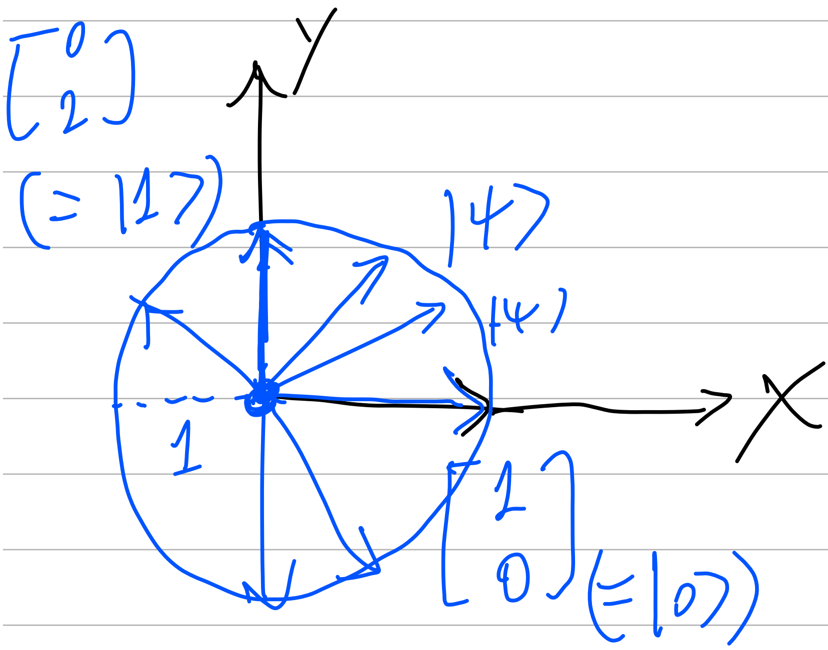
$|0\rangle$ $\xrightarrow{\text{rename}}$ $\underset{\vec{e}_x}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}$, $|1\rangle$ $\xrightarrow{\text{rename}}$ $\underset{\vec{e}_y}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}$

- $|\psi\rangle$ is a linear combination of basis vectors.

$$|\psi\rangle = \alpha \cdot |0\rangle + \beta \cdot |1\rangle$$

$$= \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

- Caveat: α, β have constraints.



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\alpha^2 + \beta^2 = 1$$

$$|0\rangle = 1 \cdot |0\rangle + 0 \cdot |1\rangle$$

Postulate 1: A quantum register encode a "unit vector" $|\psi\rangle = \alpha \cdot |0\rangle + \beta \cdot |1\rangle$.

Measurement:

- a basic operation to qubits.
- no analog in classic computation.

Postulate 2: When we "measure" a qubit $|\psi\rangle = \alpha \cdot |0\rangle + \beta \cdot |1\rangle$, it "collapses" to $|0\rangle$ with probability α^2, and "collapses" to $|1\rangle$ with probability β^2.

Evolution (manipulating)

$$|\psi\rangle \xrightarrow{\text{physical procedure}} |\phi\rangle$$

$\boxed{\text{}}$Postulate 3: Evolution of a single qubit must be a 2X2 orthogonal matrix multipled on the left side of the qubit

I.e. $|\phi\rangle = M \cdot |\psi\rangle$, where M is a 2X2 orthogonal matrix

Orthogonal matrix: (with real numbers)

$$M \cdot M^T = 1 \qquad \left(\begin{array}{l} \text{or } M^T \cdot M = 1 \\ \text{or } \underline{M^T = M^{-1}} \end{array} \right)$$

$\pitchfork$

$\mathbb{R}$

Some rationale behind this choice.

- Orthogonal transforms preserve length.

(So, being consistent with Postulate 1) and Postulate 2.

Proof:

$$M \cdot M^T = 1$$

$$\sqrt{(M\vec{u})^T (M \cdot \vec{u})} = \sqrt{\vec{u}^T \cdot \underbrace{M^T \cdot M}_{1} \vec{u}}$$

$$= \underline{\sqrt{\vec{u}^T \cdot \vec{u}}}$$

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\sqrt{\vec{u}^T \cdot \vec{u}}$$

$$\sqrt{\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}$$

$$= \sqrt{1^2 + 2^2 + 3^2}$$