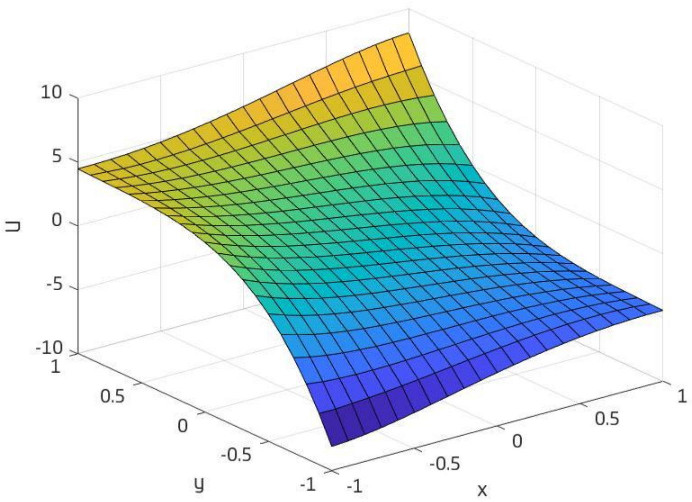
解方程$u_t = u_{xx} + u_{yy}$在时间 $0 \le t \le 1$，其中初值条件和边界条件用精确解：

$$u = \exp(1.5t)\sin(x - 0.5y)coxh(x + y)$$

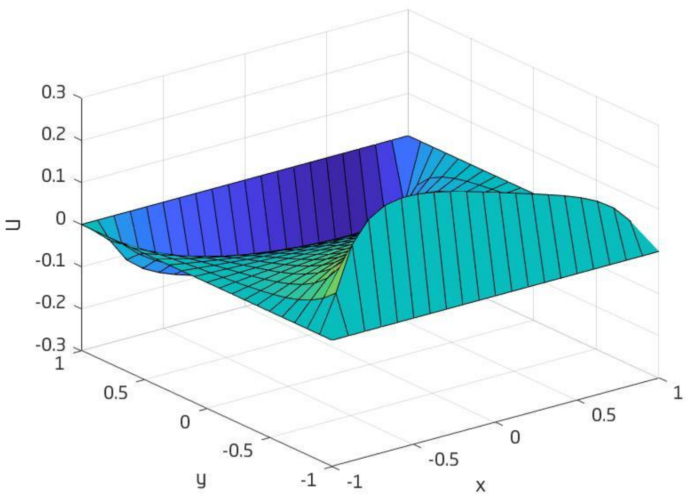
1. 用 Peaceman-Rachford 方法解
2. 用 Mitchell-Fairweather 方法解

Peaceman-Rachford 方法

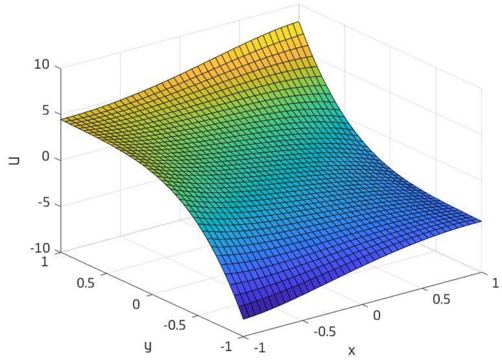
1.取$\Delta x = \Delta y = \Delta t = 1/10$



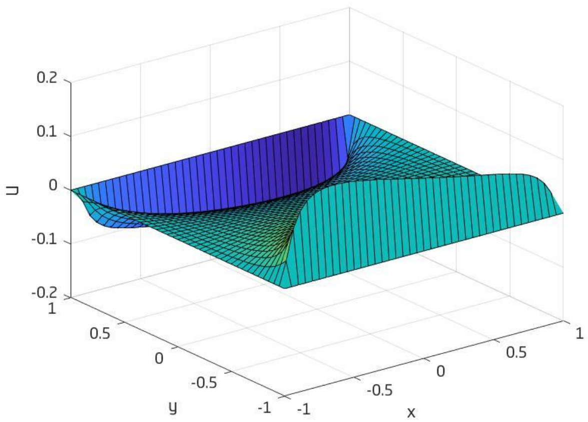
其误差为



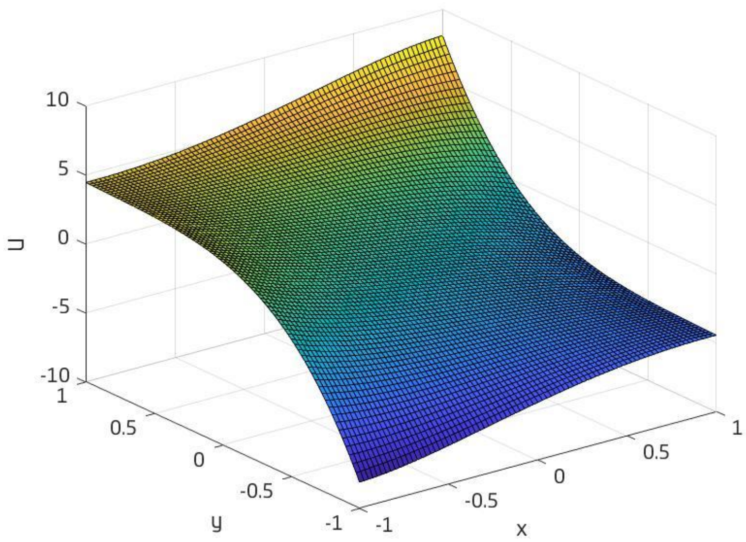
2.取$\Delta x = \Delta y = \Delta t = 1/10$



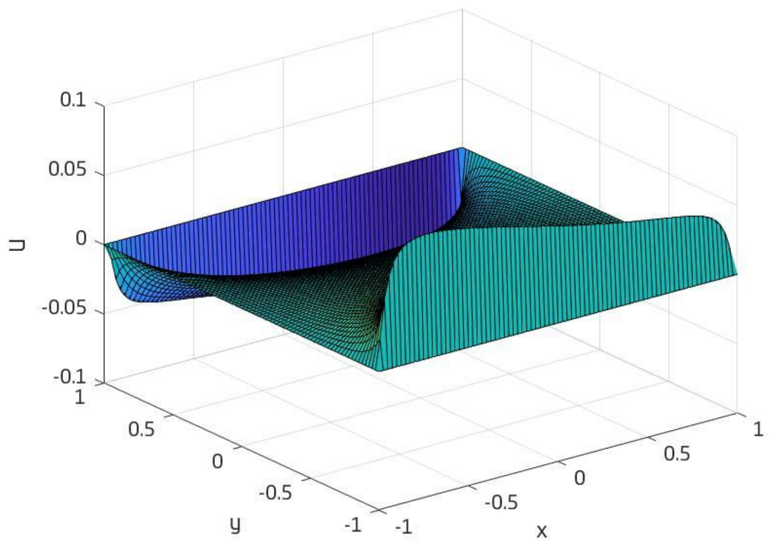
其误差为



3.取$\Delta x = \Delta y = \Delta t = 1/10$

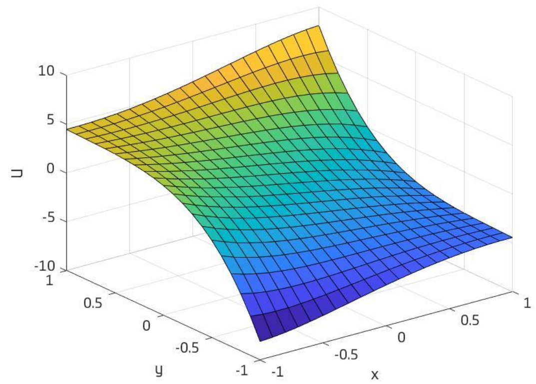


其误差为：

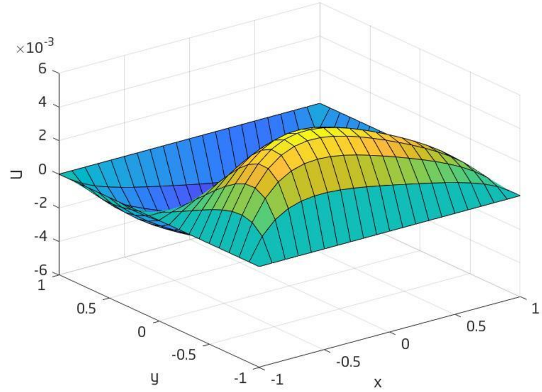


Mitchell-Fairweather 方法

取$\Delta x = \Delta y = 1/10$，$\Delta t = 1/30$



其误差为：



通过误差的图像可以看出，随着网格的加细，解的误差在减小，Peaceman-Rachford 方法的误差要比 Mitchell-Fairweather 方法的误差要小。

附录

```
%%追赶法 Thomas Algorithm 解三对角矩阵
function [ w] = TA( A,v,d )
n=length(d);%n存储将要求解未知量的个数m-1个
p=zeros(1,n+1);
q=zeros(1,n+1);
w=zeros(1,n+2);%w0放在第一个位置，wi放在第i+1个位置
w(1)=v(1);
w(n+2)=v(2);
q(1)=v(1);%为p1, q1赋初值
for i=1:1:n %先求出pq的值
    p(i+1)=-1/(A(1)*p(i)+A(2))*A(3);
    q(i+1)=1/(A(1)*p(i)+A(2))*(d(i)-A(1)*q(i));
end
for j=n:-1:1
    w(j+1)=p(j+1)*w(j+2)+q(j+1);
end
end
```

Peaceman-Rachford方法的代码为

```
clear all;
clc;
delt=1/10;%时间间隔
tal=1/10;%x和y的空间间隔
t=0:delt:1;
x=-1:tal:1;
y=-1:tal:1;

U=zeros(length(x),length(y));%初始条件
for i=1:length(x)
    for j=1:length(y)
        U(i,j)=sin(x(i)-0.5*y(j))*cosh(x(i)+y(j));
    end
end

for it=2:length(t)  %%%%开始循环一层一层的计算
```

```
    U2=zeros(length(x),length(y));%%%存储n+1/2层的数值
    U3=zeros(length(x),length(y));%%%存储n+1层的数
    for j=1:length(y)%%%边界条件

U3(1,j)=exp(1.5*(t(1)+(it-1)*delt))*sin(x(1)-0.5*y(j))*cosh(x(1)+y(j));

U3(length(x),j)=exp(1.5*(t(1)+(it-1)*delt))*sin(x(length(x))-0.5*y(j))*cosh
(x(length(x))+y(j));
    end
    for i=2:length(x)-1

U3(i,1)=exp(1.5*(t(1)+(it-1)*delt))*sin(x(i)-0.5*y(1))*cosh(x(i)+y(1));

U3(i,length(y))=exp(1.5*(t(1)+(it-1)*delt))*sin(x(i)-0.5*y(length(y)))*cosh
(x(i)+y(length(y)));
    end

     for j=2:length(y)-1%%%第n+1/2层的边界条件
         for i=1:length(x)-1:length(x)

U2(i,j)=1/2*(U(i,j)+U3(i,j))+1/2*delt*(U(i,j+1)-2*U(i,j)+U(i,j-1)-U2(i,j+1)
+2*U2(i,j)-U2(i,j-1));
         end
     end

        A=ones(1,3);
        A(1)= -1/2*delt;        %存储a
        A(2)= delt+tal*tal;   %存储b
        A(3)=-1/2*delt;       %存储c
        d=zeros(1,length(x)-2);
        for j=2:1:(length(y)-1)
             v=[U2(1,j),U2(length(x),j)]';
             for i=1:length(d)

d(i)=tal*tal*U(i+1,j)+delt*(U(i+1,j+1)-2*U(i+1,j)+U(i+1,j-1));
             end
              [U2(:,j)]=TA(A,v,d); %%v，d在不同步骤中不同，需要分别求
        end

        A(1)= -delt;        %存储a
        A(2)= 2*delt+tal*tal;  %存储b
        A(3)=-delt;       %存储c
        d1=zeros(1,length(y)-2);
        for i=2:1:(length(x)-1)
```

```
            v=[U3(i,1),U3(i,length(y))]';
            for j=1:length(d)

d1(j)=tal*tal*U2(i,j+1)+1/2*delt*(U2(i+1,j+1)-2*U2(i,j+1)+U2(i-1,j+1));
            end
             [U3(i,:)]=TA(A,v,d1); %%v，d在不同步骤中不同，需要分别求
        end

    U=U3;
end
Ud=zeros(length(x),length(y));%初始条件
for i=1:length(x)
    for j=1:length(y)
         Ud(i,j)=exp(1.5)*sin(x(i)-0.5*y(j))*cosh(x(i)+y(j));
    end
end
Ue=U-Ud;
surf(x,y,Ue);
xlabel('x');
ylabel('y');
zlabel('U');
```

Mitchell-Fairweather 方法的代码为：

```
clear all;
clc;
delt=1/10;%时间间隔
tal=1/10;%x和y的空间间隔
t=0:delt:1;
x=-1:tal:1;
y=-1:tal:1;
mu=tal/delt/delt;

U=zeros(length(x),length(y));%初始条件
for i=1:length(x)
    for j=1:length(y)
         U(i,j)=sin(x(i)-0.5*y(j))*cosh(x(i)+y(j));
    end
end

for it=2:length(t)  %%%%开始循环一层一层的计算

    U2=zeros(length(x),length(y));%%%存储n+1/2层的数值
```

```
    U3=zeros(length(x),length(y));%%%存储n+1层的数
    for j=1:length(y)%%%边界条件

U3(1,j)=exp(1.5*(t(1)+(it-1)*delt))*sin(x(1)-0.5*y(j))*cosh(x(1)+y(j))
;

U3(length(x),j)=exp(1.5*(t(1)+(it-1)*delt))*sin(x(length(x))-0.5*y(j))
*cosh(x(length(x))+y(j));
    end
    for i=2:length(x)-1

U3(i,1)=exp(1.5*(t(1)+(it-1)*delt))*sin(x(i)-0.5*y(1))*cosh(x(i)+y(1))
;

U3(i,length(y))=exp(1.5*(t(1)+(it-1)*delt))*sin(x(i)-0.5*y(length(y)))
*cosh(x(i)+y(length(y)));
    end

     for j=2:length(y)-1%%%第n+1/2层的边界条件
         for i=1:length(x)-1:length(x)

a=(mu+1/6)*(  U(i,j)+1/2*(2*mu+1/6)*(U(i,j+1)-2*U(i,j)+U(i,j-1))      )
;

b=(mu-1/6)*(  U3(i,j)-1/2*(2*mu-1/6)*(U3(i,j+1)-2*U3(i,j)+U3(i,j-1))
    );
            U2(i,j)=1/(2*mu)*(a+b);
         end
     end

        A=ones(1,3);
        A(1)= -1/2*(mu-1/6);        %存储a
        A(2)= 1+(mu-1/6);   %存储b
        A(3)=-1/2*(mu-1/6);        %存储c
        d=zeros(1,length(x)-2);
        for j=2:1:(length(y)-1)
             v=[U2(1,j),U2(length(x),j)]';
             for i=1:length(d)

d(i)=U(i+1,j)+1/2*(2*mu+1/6)*( U(i+1,j-1)-2*U(i+1,j)+U(i+1,j+1) );
                  end
              [U2(:,j)]=TA(A,v,d); %%v, d在不同步骤中不同，需要分别求
        end
```

```
         A(1)= -1/2*(2*mu-1/6);       %存储a
        A(2)= 1+(2*mu-1/6);  %存储b
        A(3)=-1/2*(2*mu-1/6);       %存储c
        d1=zeros(1,length(y)-2);
        for i=2:1:(length(x)-1)

             v=[U3(i,1),U3(i,length(y))]';
             for j=1:length(d)

d1(j)=U2(i,j+1)+1/2*(mu+1/6)*( U2(i+1,j+1)-2*U2(i,j+1)+U2(i-1,j+1) );
             end
              [U3(i,:)]=TA(A,v,d1);%%v，d在不同步骤中不同，需要分别求
        end
    U=U3;
end
Ud=zeros(length(x),length(y));%初始条件
for i=1:length(x)
    for j=1:length(y)
         Ud(i,j)=exp(1.5)*sin(x(i)-0.5*y(j))*cosh(x(i)+y(j));
    end
end
Ue=U-Ud;
surf(x,y,Ue);
xlabel('x');
ylabel('y');
zlabel('U');
```