

ST102 Week 9

Def. (Cont uniform distribution) $X \sim \text{Uniform}[a, b]$
($b > a$)

$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

$$F(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & x > b \end{cases}$$

$$E(X) = \frac{a+b}{2} = \text{median}$$

$$\text{Var}(X) = \frac{(b-a)^2}{12}$$

Def. (Exponential) $X \sim \text{Exp}(\lambda)$ ($\lambda > 0$)

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-\lambda x} & x \geq 0 \end{cases}$$

$$E(X) = \frac{1}{\lambda}, \quad \text{Var}(X) = \frac{1}{\lambda^2}$$

$$M_X(t) = \frac{\lambda}{\lambda - t} \quad \text{for } t < \lambda$$

RK. If "#events/unit time" follows Poisson(λ),
then the "waiting time between 2 successive
events" follows Exp(λ).

Def. (Normal) $\underline{X} \sim N(\mu, \sigma^2)$ ($\sigma^2 > 0$)

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \quad x \in \mathbb{R}$$

$$E(\underline{X}) = \mu, \quad \text{Var}(\underline{X}) = \sigma^2$$

$$M_{\underline{X}}(t) = \exp\left\{\mu t + \frac{\sigma^2 t^2}{2}\right\} \quad t \in \mathbb{R}$$

RK. (Standard Normal)

$$\underline{Z} \sim N(0, 1),$$

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{z^2}{2}\right\} \quad z \in \mathbb{R}$$

$$\Phi(x) := F(\underline{Z} \leq x) = \int_{-\infty}^x f(z) dz$$

↑
Statistical table