ST102 Week 6

Def. (Moments of a random variable)

For $k = 1, 2, \cdots$ define

i) the kth moment about zero as $\mu_k := E(X^k)$

ii) the kth central moment as $\mu_k' := E\{[X - E(X)]^k\}$

Remark. Clearly, $\mu_1 = E(X)$; $\mu_2' = Var(X)$

Def. (Moment generating function / mgf)

The mgf of a discrete r.v. X is defined as:

$$M_X(t) = E(e^{tX}) = \sum_{x \in \mathcal{D}} e^{tx} p(x)$$

Remark. $M_X(t)$ is a function in t, with no randomness

Prop. (Useful properties of a mgf)

i) $M_X'(0) = E(X)$ & $M_X''(0) = E(X^2)$

$\cdots$ & $M_X^{(k)}(0) = E(X^k)$, $k \geq 1$

ii) $Var(X) = E(X^2) - (EX)^2 = M_X''(0) - [M_X'(0)]^2$

iii) $M_X(t) = M_Y(t) \Rightarrow X$ & Y share the same distribution

iv) $M_{aX+b}(t) = e^{bt} M_X(at)$

v) For independent r.v.'s $X_1, \cdots, X_n$,

$$M_{\sum_{i=1}^{n} X_i}(t) = \prod_{i=1}^{n} M_{X_i}(t)$$

Def. i) Continuous r.v.

ii) probability density function $f(x)$ (p.d.f.)

R.K. $\mathbb{P}(X = x_0) = 0$, $\forall x_0 \in \mathcal{D}$

iii) $\mathbb{P}(a<x<b) = \int_a^b f(x)\, dx$

iv) Necessary conditions $\begin{cases} f \geq 0 & \forall x \\ \int_{-\infty}^{+\infty} f(x)\, dx = 1 \end{cases}$

v) cumulative distribution function (c.d.f.)

$F(x) := \mathbb{P}(X \leq x) = \int_{-\infty}^{x} f(t)\, dt$

R.K. $f(x) = \frac{d}{dx} F(x)$;

$\mathbb{P}(a<x<b) = \int_a^b f\, dx = F(b) - F(a)$

vi) $E(X) = \int_{-\infty}^{+\infty} x f(x)\, dx$

$E(g(X)) = \int_{-\infty}^{+\infty} g(x) f(x)\, dx$

$Var(X) = E\{[X - E(X)]^2\} = \int_{-\infty}^{+\infty} (X - EX)^2 f(x)\, dx$

$= E(X^2) - [E(X)]^2$

$sd(X) = \sqrt{Var(X)}$