2.2

xuyang1014@pku.edu.cn

2.2.1

$$\mathrm{Var}(S_n/n - \nu_n) = \frac{1}{n^2}\sum \mathrm{var}(X_i)$$
$$\le \frac{1}{n}\sum \mathrm{var}(X_i)/i$$
$$\to 0$$

故 L_2 收敛，依概率收敛

2.2.2

只需证 $\frac{1}{n^2}E(X_1+\cdots+X_n)^2 \to 0$

$$\frac{1}{n^2}E(X_1+\cdots+X_n)^2 = \frac{1}{n^2}\sum EX_i^2 + \frac{2}{n^2}\sum_{i<j} EX_iX_j$$
$$\le \frac{1}{n}r(0) + \frac{2}{n}(r(1)+\cdots+r(n-1))$$
$$\longrightarrow 0$$

进而 $\frac{1}{n}(X_1+\cdots+X_n) \xrightarrow{L^2} 0$，故 $\frac{1}{n}(X_1+\cdots+X_n) \xrightarrow{P} 0$

2.2.3

(i) $Ef(U_1) = \int_0^1 f(x)dx$

故由 WLLN 得证

(ii) $P(|I_n - I| > an^{-\frac{1}{2}}) \le n/a^2 \, \mathrm{Var}\, I_n$

$$= \frac{1}{a^2}\mathrm{Var}\, I_1$$

2.2.4

$E|X_i| = C\cdot\sum \frac{1}{k\log k} = +\infty$

但

$$nP(|X_i| \ge n) = C\cdot n\cdot\sum_{j=n}^{+\infty}\frac{1}{j^2\log j}$$
$$\le C\cdot n\cdot\frac{1}{\log n}\sum_{j=n}^{+\infty} j^{-2}$$
$$\lesssim C_0\frac{1}{\log n} \to 0$$

由 WLLN 得证

2.2.5

$E|X_i| \ge \int_1^{+\infty}\frac{e}{x\log x}dx = +\infty$

但 $xP(|X_i|>x) = e\frac{1}{\log x} \to 0$

由 WLLN 得证

xuyang1014@pku.edu.cn

2.2.6

(i) 显然

(ii) $EX^2 = \sum_{n=1}^{+\infty} (2n-1) P(X \geqslant n)$

2.2.7

$$
\begin{aligned}
E H(X) &= \int_{-\infty}^{+\infty} H(x)\, dP \\
&= \int_{-\infty}^{+\infty} \left(\int_{-\infty}^{x} h(y)\, dy \right) dP \\
&= \int_{-\infty}^{+\infty} \left(h(y) \int_{y}^{+\infty} dP \right) dy \\
&= \int_{-\infty}^{+\infty} h(y)\, P(X \geqslant y)\, dy
\end{aligned}
$$

228 229 not finished yet