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# Fuzzy Logic Model

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***Abstract*—Traditional classification problem is based on the precision of classical crisp logic, such as Neural network, Support vector machine, and the solution of problem is just like Black-Box operation. However, many variables in reality are not expressed quantitatively, for example, temperature can be expressed as cold, warm or hot. Fuzzy logic intend to solve such problems. Therefore, it may be viewed as a bridge over the excessively wide gap between the precision of classical crisp logic and the imprecision of both the real world and its human interpretation. As to mechanism, actually, it is a tool for embedding human structured knowledge into workable algorithms. In this paper, in order to master the basic knowledge, we have done related experiments using fuzzy logic model..**

***Index Terms*—Fuzzy logic, fuzzy set, fuzzy relation, fuzzy inference, membership function**

## I. Introduction

In reality, there are many applications, which can not be solved by traditional classification tool, such as Neural network, support vector machine. Because these classification tools can just deal with precise expression about the problem. For example, classifying different digits into ten classes from zero to nine. However, many variables are not expressed quantitatively. For example, temperature can be expressed as cold, warm, or hot; ***x*** is much smaller than 10. Fuzzy logic intend to solve such problems. It is a tool for embedding human structured knowledge into workable algorithms.

In fuzzy logic model, it includes two process: fuzzification and defuzzification. In fuzzification, using fuzzy sets or classes, crisp input values are transformed into a vector of their membership degrees. The membership degrees show how much they belong to different fuzzy sets for one variable. In defuzzification, the acquired fuzzy values are transformed into a crisp value using different defuzzification methods. The detail will be explained in the following parts.

The rest of paper is organized as follows: In section II, fuzzy logic model is given. The result is presented in section III. The following part is conclusion.

## II. FUZZY logic model

### A. Fuzzification

Fuzzification refers to transform crisp input values into a vector of their degree for further operation. Related basic concepts are illustrated as following.

1) Fuzzy sets

Sets could be variously defined, by a list elements, by some property, or by a membership function. However, no matter which method to be used. The essence of fuzzy sets is to expressed an object as a matter of degree. The following examples give a intuitionistic impression.

$S_1 =$ { "***x*** is much smaller than 10 "};

$S_2 =$ {x | x < 3};

In fuzzy logic model, each variable, in fact, is a combination of several fuzzy sets. For example, temperature={cold, warm, hot}. It shows that variable "temperature" has three classes, which are expressed by fuzzy sets.

2) Membership function

Membership function is used to denote the trend of changing for each fuzzy set. The membership function may have different shapes. The choice of a shape for each particular linguistic variable (attribute or fuzzy set) is both subjective and problem dependent. The most common membership function are shown in the following figure.

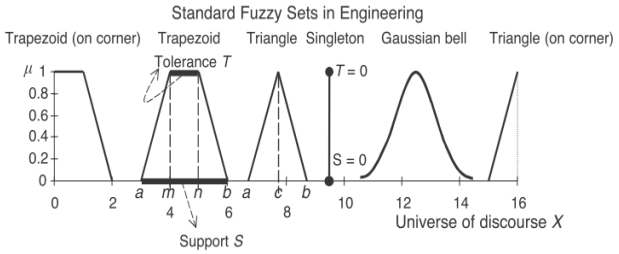


Fig 1. membership function

3) Fuzzy relation

A relation is a set of ordered pairs. Relations express connections between different sets. A crisp relation represents the presence or absence of association, interaction, or interconnetedness between the elements of two or more sets. Fuzzy relation allow various degrees or strengths of relations between elements.

In fuzzy logic model, Fuzzy relation act as a reference, in other words, it is the structured knowledge. Each element in fuzzy relation is a "IF-THEN" rule.

4) Fuzzy inference

Fuzzy inference is the process to fuzzified crisp input value and fuzzy relation. The basic implication rules states that the true value of the conclusion must not be larger that that of premise. The operator of inference is MAX-MIN, with alternatives MAX-PROD or MAX-AVE.

### B. Defuzzification

The procedure for obtaining a crisp output value from the resulting fuzzy set is called defuzzification. Three popular methods are First-of-maxima, Middle-of-maxima, and Center-of-area for singletons.

The resulting crisp output is calculated as

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$$y' = \frac{\sum_{i=1}^{N} y_i H_i}{\sum_{i=1}^{N} H_i}$$

where N is the number of the output membership functions.

*C. Mathematical similarities between neural networks and fuzzy logic models*

Neural networks and fuzzy logic models are based on very similar, sometimes equivalent, underlying mathematics.

$$y = F(x) = \frac{\sum_{i=1}^{N} G(x, c_i) r_i}{\sum_{i=1}^{N} G(x, c_i)}$$

In neural network, N stands for the number of neurons. The training step is to find weights. In the above form, it can get $r_i$. The simulation decision boundary is the combination of the result of different neurons.

In fuzzy logic, N stands for the number of rules. If there are two inputs X and Y, each one has three elements. Therefore, totally, there are nine rules. It means N is equal to nine. $G(x, c_i)$ is the result from each rule. $r_i$ is the assigned output degree corresponding to each rule. The simulation decision boundary is the combination of the result of different rules.

Therefore, we could see the similarities of neural network and fuzzy logic models. In our experiment part, what we use is the method, which is similar with neural network.

## III. Result

1) Experiment

- task 1

Design a fuzzy logic model for the function, requiring that membership function is triangles and Gaussian, apply singletons for the output variable.

- task 2

Design a fuzzy logic model to keep distance between two cars. Assuming three input variable (Distance, Velocity and the state of the road ). Require that each variable has three states, use MIN and PROD operators for logical AND, and center of gravity for the defuzzification.

2) Result

- task 1

In this task, for input variable, we use six membership function, centered at [1 2 3 4 5 6], and we use three membership function, centered at [0.6 -0.3 -0.1]. Fig.2 and Fig.3 are the result using triangle membership function and Guassian membership function separately.

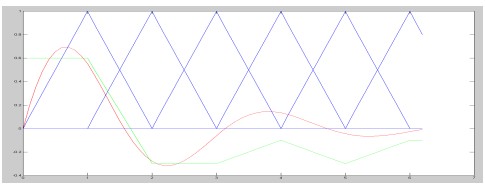


Fig.2 Simulation using triangle membership function

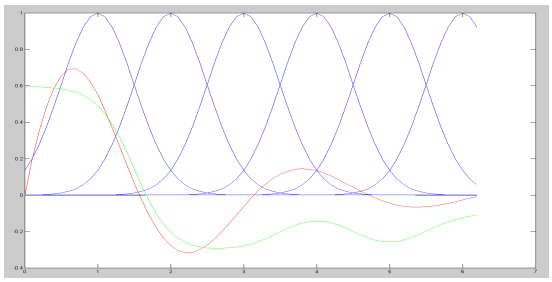


Fig 3. Simulation using Guassian membership function

- task 2

In this task, for distance, we vary distance from 10 to 100, centered at [10,50,80], velocity from 10 to 100, centered at[20,60,80], and state of road from 10 to 100, centered at [20 50 80]. We assign output as [0 10 20 30 40 50 60 70 80 90 100], which stands for percentage of braking.

Fig 4,5,6 shows the result under MIN operator. Fig 7,8,9 shows the result under PROD operator.

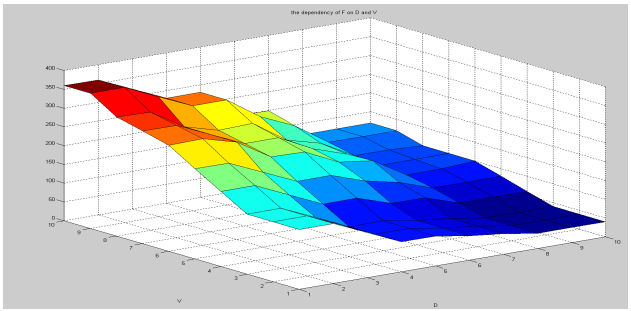


Fig. 4 the dependency of F on D and V under MIN

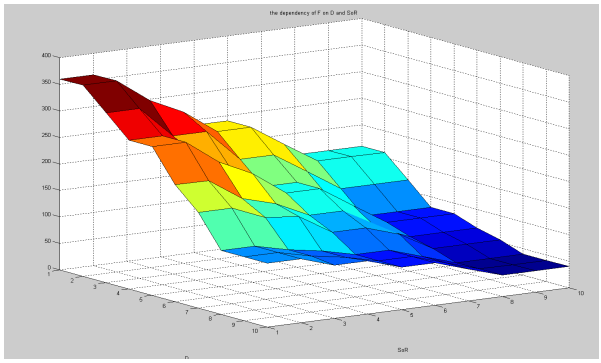


Fig. 5 the dependency of F on D and SoR under MIN

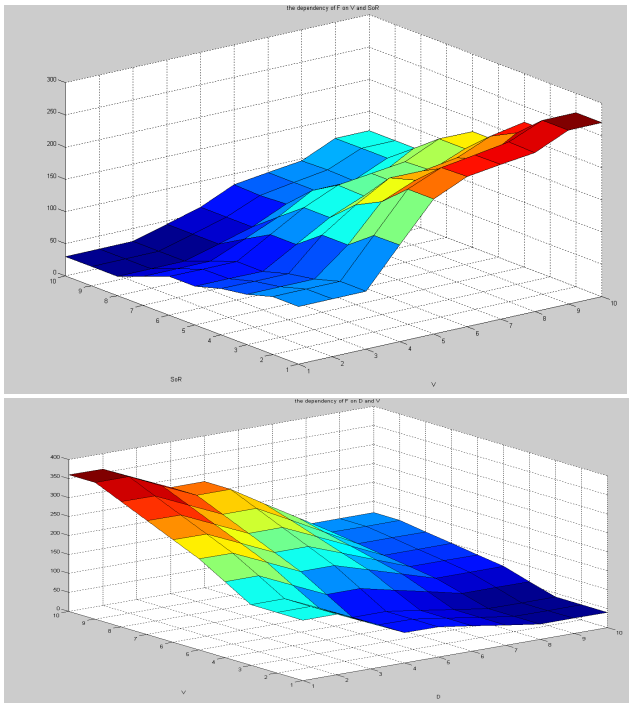


Fig. 7 the dependency of F on D and V under PROD

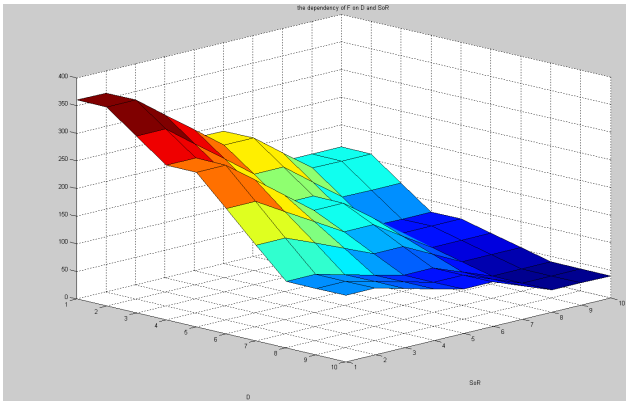
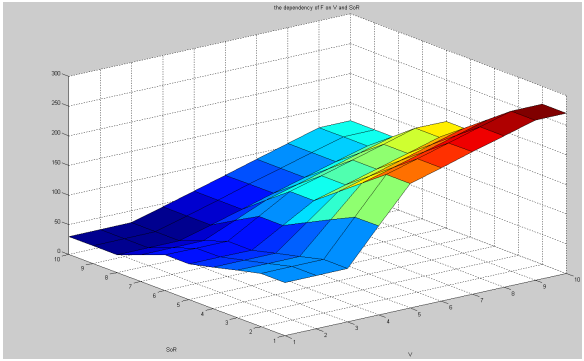


Fig. 9 the dependency of F on V and SoR under PROD



## IV. CONCLUSION

Fuzzy logic model attempts to model the way of reasoning that goes in the human brain in the form of the IF-THEN rules. In reality, when we solve problem using fuzzy logic model, what we need to pay attention include the number of membership function used in each variable, the operator, ect. Because the number of membership function will cause conflict between time consumption and precision, and the operator exhibit the smooth trend of simulation. Therefore, how to design fuzzy logic model depends on exact requirement of the tasks.

## REFERENCES

[1] Vojislav Kecman. "Learning and Soft Computing: support vector machine, neural networks, and fuzzy logic models". The MIT Press, Cambridge, MA, pp.365-410, 2001