

# The Supplementary Material of the Paper “Improving Decomposition-based MOEAs for Combinatorial Optimisation by Intensifying Corner Weights”

Table 1: MS results (mean and SD) of MOEA/D-CenWI and MOEA/D-CWI.

Problem	MOEA/D-CenWI	MOEA/D-CWI
KP (2, $D = 250$ )	1.015E+03(4.6E+01) <sup>+</sup>	<b>1.283E+03(1.7E+02)</b>
KP (2, $D = 500$ )	1.875E+03(9.5E+01) <sup>+</sup>	<b>2.116E+03(2.2E+02)</b>
KP (3, $D = 250$ )	3.002E+03(1.3E+02) <sup>≈</sup>	<b>3.113E+03(5.3E+01)</b>
KP (3, $D = 500$ )	4.426E+03(2.1E+02) <sup>+</sup>	<b>5.200E+03(3.7E+02)</b>
NK (2, $K = 2$ )	2.336E-01(2.7E-02) <sup>+</sup>	<b>2.567E-01(2.5E-02)</b>
NK (2, $K = 3$ )	2.007E-01(2.2E-02) <sup>+</sup>	<b>2.402E-01(2.2E-02)</b>
NK (2, $K = 5$ )	2.121E-01(3.7E-02) <sup>+</sup>	<b>2.667E-01(3.2E-02)</b>
NK (3, $K = 2$ )	4.097E-01(2.1E-02) <sup>≈</sup>	<b>4.181E-01(1.5E-02)</b>
NK (3, $K = 3$ )	4.331E-01(1.5E-02) <sup>≈</sup>	<b>4.437E-01(2.1E-02)</b>
NK (3, $K = 5$ )	4.196E-01(4.3E-02) <sup>≈</sup>	<b>4.248E-01(2.6E-02)</b>
TSP (2, $\rho = 0.2$ )	6.457E+00(5.1E-01) <sup>+</sup>	<b>7.990E+00(1.0E+00)</b>
TSP (2, $\rho = 0$ )	8.908E+00(1.3E+00) <sup>+</sup>	<b>1.189E+01(1.1E+00)</b>
TSP (2, $\rho = -0.2$ )	1.498E+01(1.0E+00) <sup>+</sup>	<b>1.779E+01(1.8E+00)</b>
TSP (3, $\rho = 0.2$ )	1.648E+01(1.4E+00) <sup>≈</sup>	<b>1.698E+01(8.9E-01)</b>
TSP (3, $\rho = 0$ )	2.192E+01(9.6E-01) <sup>≈</sup>	<b>2.218E+01(7.9E-01)</b>
TSP (3, $\rho = -0.2$ )	2.897E+01(1.7E+00) <sup>≈</sup>	<b>2.959E+01(8.0E-01)</b>
+ / $\approx$ / -	9/7/0	-

The symbols +,  $\approx$  and - indicate that MOEA/D-CWI is statistically better than, equivalent to and worse than MOEA/D-CenWI at a 0.05 level by Wilcoxon’s rank sum test. The better mean for each case is highlighted in boldface.

Table 2: MS results (mean and SD) of MOEA/D and MOEA/D-CWI.

Problem	MOEA/D	MOEA/D-CWI
KP (2, $D = 250$ )	1.152E+03(2.2E+01) <sup>+</sup>	<b>1.283E+03(1.7E+02)</b>
KP (2, $D = 500$ )	2.037E+03(8.3E+01) <sup>+</sup>	<b>2.116E+03(2.2E+02)</b>
KP (3, $D = 250$ )	3.099E+03(2.5E+02) <sup>≈</sup>	<b>3.113E+03(5.3E+01)</b>
KP (3, $D = 500$ )	4.624E+03(1.1E+02) <sup>+</sup>	<b>5.200E+03(3.7E+02)</b>
NK (2, $K = 2$ )	2.457E-01(5.2E-02) <sup>+</sup>	<b>2.567E-01(2.5E-02)</b>
NK (2, $K = 3$ )	2.119E-01(8.4E-02) <sup>+</sup>	<b>2.402E-01(2.2E-02)</b>
NK (2, $K = 5$ )	2.137E-01(1.2E-02) <sup>+</sup>	<b>2.667E-01(3.2E-02)</b>
NK (3, $K = 2$ )	4.092E-01(4.8E-02) <sup>+</sup>	<b>4.181E-01(1.5E-02)</b>
NK (3, $K = 3$ )	4.384E-01(9.1E-02) <sup>≈</sup>	<b>4.437E-01(2.1E-02)</b>
NK (3, $K = 5$ )	4.186E-01(8.6E-02) <sup>≈</sup>	<b>4.248E-01(2.6E-02)</b>
TSP (2, $\rho = 0.2$ )	6.747E+00(4.9E-01) <sup>+</sup>	<b>7.990E+00(1.0E+00)</b>
TSP (2, $\rho = 0$ )	8.935E+00(4.2E+00) <sup>+</sup>	<b>1.189E+01(1.1E+00)</b>
TSP (2, $\rho = -0.2$ )	1.611E+01(4.9E+00) <sup>+</sup>	<b>1.779E+01(1.8E+00)</b>
TSP (3, $\rho = 0.2$ )	1.670E+01(1.2E+00) <sup>≈</sup>	<b>1.698E+01(8.9E-01)</b>
TSP (3, $\rho = 0$ )	2.184E+01(1.0E-01) <sup>+</sup>	<b>2.218E+01(7.9E-01)</b>
TSP (3, $\rho = -0.2$ )	2.883E+01(1.3E+00) <sup>≈</sup>	<b>2.959E+01(8.0E-01)</b>
+ / $\approx$ / -	11/5/0	-

The symbols +,  $\approx$  and - indicate that MOEA/D-CWI is statistically better than, equivalent to and worse than MOEA/D at a 0.05 level by Wilcoxon’s rank sum test. The better mean for each case is highlighted in boldface.

Table 3: MS results (mean and SD) of the four algorithms.

Problem	NSGA-II	IBEA	NSGA-III	MOEA/D-CWI
KP ( $m = 2, D = 250$ )	<b>1.641E+03(4.5E+01)</b> <sup>-</sup>	1.637E+03(2.5E+02) <sup>-</sup>	1.581E+03(2.4E+02) <sup>-</sup>	1.283E+03(1.7E+02)
KP ( $m = 2, D = 500$ )	1.975E+03(3.0E+02) <sup>+</sup>	1.846E+03(2.8E+02) <sup>+</sup>	1.786E+03(1.1E+02) <sup>+</sup>	<b>2.116E+03(2.2E+02)</b>
KP ( $m = 3, D = 250$ )	2.975E+03(2.1E+02) <sup>+</sup>	2.877E+03(1.8E+02) <sup>+</sup>	2.866E+03(1.4E+02) <sup>+</sup>	<b>3.113E+03(5.3E+01)</b>
KP ( $m = 3, D = 500$ )	4.901E+03(1.2E+02) <sup>+</sup>	4.848E+03(3.3E+02) <sup>+</sup>	4.804E+03(3.6E+01) <sup>+</sup>	<b>5.200E+03(3.7E+02)</b>
NK ( $m = 2, K = 2$ )	<b>2.761E-01(5.1E-03)</b> <sup>≈</sup>	2.416E-01(1.5E-02) <sup>≈</sup>	2.446E-01(6.0E-03) <sup>≈</sup>	2.567E-01(2.5E-02)
NK ( $m = 2, K = 3$ )	2.231E-01(2.6E-02) <sup>+</sup>	1.910E-01(4.4E-02) <sup>+</sup>	2.082E-01(4.7E-02) <sup>+</sup>	<b>2.402E-01(2.2E-02)</b>
NK ( $m = 2, K = 5$ )	2.605E-01(2.1E-02) <sup>≈</sup>	2.549E-01(4.8E-02) <sup>≈</sup>	2.603E-01(4.2E-02) <sup>≈</sup>	<b>2.667E-01(3.2E-02)</b>
NK ( $m = 3, K = 2$ )	3.927E-01(7.7E-03) <sup>+</sup>	<b>4.367E-01(2.8E-02)</b> <sup>≈</sup>	3.972E-01(1.3E-02) <sup>+</sup>	4.181E-01(1.5E-02)
NK ( $m = 3, K = 3$ )	4.292E-01(3.2E-02) <sup>+</sup>	4.055E-01(2.7E-02) <sup>+</sup>	4.260E-01(1.4E-02) <sup>+</sup>	<b>4.437E-01(2.1E-02)</b>
NK ( $m = 3, K = 5$ )	4.319E-01(3.3E-02) <sup>≈</sup>	4.404E-01(2.8E-02) <sup>≈</sup>	<b>4.533E-01(4.5E-02)</b> <sup>-</sup>	4.248E-01(2.6E-02)
TSP ( $m = 2, \rho = 0.2$ )	7.986E+00(4.9E-01) <sup>≈</sup>	7.568E+00(3.1E-01) <sup>+</sup>	7.182E+00(8.9E-01) <sup>+</sup>	<b>7.990E+00(1.0E+00)</b>
TSP ( $m = 2, \rho = 0$ )	<b>1.227E+01(1.0E+00)</b> <sup>≈</sup>	1.086E+01(9.7E-01) <sup>≈</sup>	1.143E+01(1.8E+00) <sup>≈</sup>	1.189E+01(1.1E+00)
TSP ( $m = 2, \rho = -0.2$ )	1.773E+01(1.4E+00) <sup>≈</sup>	1.504E+01(1.2E+00) <sup>+</sup>	1.433E+01(9.9E-01) <sup>+</sup>	<b>1.779E+01(1.8E+00)</b>
TSP ( $m = 3, \rho = 0.2$ )	1.587E+04(1.1E+00) <sup>≈</sup>	1.536E+04(6.2E-01) <sup>≈</sup>	1.539E+04(1.1E+00) <sup>≈</sup>	<b>1.698E+04(8.9E-01)</b>
TSP ( $m = 3, \rho = 0$ )	<b>2.228E+01(1.9E+00)</b> <sup>≈</sup>	1.993E+01(1.7E+00) <sup>≈</sup>	1.859E+01(1.5E+00) <sup>≈</sup>	2.218E+01(7.9E-01)
TSP ( $m = 3, \rho = -0.2$ )	2.726E+01(1.7E+00) <sup>≈</sup>	2.885E+01(1.6E+00) <sup>≈</sup>	2.808E+01(8.2E-01) <sup>≈</sup>	<b>2.959E+01(8.0E-01)</b>
+ / ≈ / -	6/9/1	7/8/1	8/6/2	-

The symbols +, ≈ and - indicate that MOEA/D-CWI is statistically better than, equivalent to and worse than the peer algorithms at a 0.05 level by Wilcoxon's rank sum test. The best mean for each case is highlighted in boldface.

Table 4: MS results (mean and SD) of compared algorithms

Problem	MOEA/D-DRA-UT	MOEA/D-DRA-UT-CWI	MOEA/D-LdEA	MOEA/D-LdEA-CWI
KP (2, $D = 250$ )	1.984E+03(2.0E+02) <sup>+</sup>	<b>2.203E+03(9.6E+01)</b>	1.751E+03(8.5E+01) <sup>+</sup>	<b>1.987E+03(3.1E+02)</b>
KP (2, $D = 500$ )	<b>2.237E+03(1.7E+02)</b> <sup>≈</sup>	2.155E+03(2.2E+02)	2.285E+03(2.6E+02) <sup>+</sup>	<b>2.551E+03(3.1E+02)</b>
KP (3, $D = 250$ )	3.808E+03(1.9E+02) <sup>≈</sup>	<b>3.866E+03(1.5E+02)</b>	3.848E+03(3.3E+02) <sup>≈</sup>	<b>3.856E+03(2.6E+02)</b>
KP (3, $D = 500$ )	4.634E+03(1.0E+02) <sup>≈</sup>	<b>4.682E+03(1.0E+02)</b>	4.699E+03(2.2E+02) <sup>≈</sup>	<b>4.711E+03(9.4E+02)</b>
NK (2, $K = 2$ )	<b>2.882E-01(1.1E-02)</b> <sup>≈</sup>	2.837E-01(6.2E-03)	2.777E-01(3.8E-03) <sup>+</sup>	<b>2.926E-01(3.1E-03)</b>
NK (2, $K = 3$ )	2.358E-01(7.7E-03) <sup>+</sup>	<b>2.632E-01(3.2E-02)</b>	2.389E-01(6.0E-03) <sup>≈</sup>	<b>2.484E-01(8.4E-03)</b>
NK (2, $K = 5$ )	<b>2.863E-01(5.2E-02)</b> <sup>≈</sup>	2.780E-01(3.5E-02)	2.558E-01(7.7E-03) <sup>≈</sup>	<b>2.632E-01(3.2E-02)</b>
NK (3, $K = 2$ )	4.759E-01(4.1E-03) <sup>+</sup>	<b>4.912E-01(5.6E-03)</b>	4.922E-01(3.5E-03) <sup>≈</sup>	<b>4.940E-01(4.5E-03)</b>
NK (3, $K = 3$ )	4.851E-01(7.6E-03) <sup>≈</sup>	<b>4.892E-01(8.5E-03)</b>	<b>4.951E-01(6.3E-03)</b> <sup>≈</sup>	4.914E-01(2.7E-03)
NK (3, $K = 5$ )	4.956E-01(6.1E-03) <sup>≈</sup>	<b>4.964E-01(8.2E-03)</b>	4.951E-01(7.9E-03) <sup>≈</sup>	<b>5.012E-01(1.4E-02)</b>
TSP (2, $\rho = 0.2$ )	6.767E+00(4.6E+00) <sup>+</sup>	<b>7.792E+00(3.8E+00)</b>	6.335E+00(1.8E+00) <sup>+</sup>	<b>7.284E+00(1.2E+00)</b>
TSP (2, $\rho = 0$ )	9.101E+00(4.2E+00) <sup>≈</sup>	<b>9.122E+00(3.5E+00)</b>	9.266E+00(6.2E+00) <sup>≈</sup>	<b>9.268E+00(4.5E+00)</b>
TSP (2, $\rho = -0.2$ )	1.643E+01(5.7E+00) <sup>≈</sup>	<b>1.692E+01(4.7E+00)</b>	1.897E+01(1.3E+00) <sup>+</sup>	<b>1.950E+01(1.4E+00)</b>
TSP (3, $\rho = 0.2$ )	1.669E+01(1.2E+02) <sup>≈</sup>	<b>1.688E+01(1.1E+02)</b>	1.620E+01(1.8E+02) <sup>≈</sup>	<b>1.626E+01(1.3E+02)</b>
TSP (3, $\rho = 0$ )	2.393E+01(1.0E+02) <sup>+</sup>	<b>2.404E+01(1.0E+02)</b>	2.909E+01(1.4E+02) <sup>≈</sup>	<b>2.922E+01(2.1E+02)</b>
TSP (3, $\rho = -0.2$ )	2.999E+01(1.4E+02) <sup>≈</sup>	<b>3.001E+01(1.3E+02)</b>	2.611E+01(2.3E+02) <sup>≈</sup>	<b>2.624E+01(3.4E+02)</b>
+ / ≈ / -	5/11/0		5/11/0	

The symbols +, ≈ and - indicate that MOEA/D-DRA-UT-CWI (MOEA/D-LdEA-CWI) is statistically better than, equivalent to and worse than MOEA/D-DRA-UT (MOEA/D-LdEA) at a 0.05 level by Wilcoxon's rank sum test. The better mean for each case is highlighted in boldface.

Table 5: MS results (mean and SD) of compared algorithms on the multi-objective NRP.

Problem	MOEA/D-DRA-UT	MOEA/D-DRA-UT-CWI	MOEA/D-LdEA	MOEA/D-LdEA-CWI
NRP ( $c = 15, D = 40$ )	0.493(0.018) $\approx$	<b>0.507(0.027)</b>	0.550(0.029) $\approx$	<b>0.552(0.021)</b>
NRP ( $c = 50, D = 80$ )	0.484(0.027) $^+$	<b>0.530(0.066)</b>	0.587(0.047) $\approx$	<b>0.589(0.028)</b>
NRP ( $c = 2, D = 200$ )	0.540(0.016) $\approx$	<b>0.542(0.020)</b>	0.587(0.040) $^+$	<b>0.596(0.023)</b>
NRP ( $c = 100, D = 20$ )	0.509(0.010) $^+$	<b>0.522(0.042)</b>	0.541(0.018) $^+$	<b>0.564(0.011)</b>
NRP ( $c = 100, D = 25$ )	0.475(0.016) $^+$	<b>0.497(0.037)</b>	0.576(0.018) $\approx$	<b>0.587(0.027)</b>
NRP ( $c = 100, D = 140$ )	0.540(0.022) $\approx$	<b>0.548(0.011)</b>	0.552(0.032) $^+$	<b>0.577(0.017)</b>
$+ / \approx / -$	3/3/0		3/3/0	

The symbols  $+$ ,  $\approx$  and  $-$  indicate that MOEA/D-DRA-UT-CWI (MOEA/D-LdEA-CWI) is statistically better than, equivalent to and worse than MOEA/D-DRA-UT (MOEA/D-LdEA) at a 0.05 level by Wilcoxon's rank sum test. The better mean for each case is highlighted in boldface.