Modeling and Forecasting Electricity Prices with Input/Output Hidden Markov Models

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***Abstract*—In competitive electricity markets, in addition to the uncertainty of exogenous variables such as energy demand, water inflows, and availability of generation units and fuel costs, participants are faced with the uncertainty of their competitors' behavior. The analysis of electricity price time series reflects a switching nature, related to discrete changes in competitors' strategies, which can be represented by a set of dynamic models sequenced together by a Markov chain. In this paper, an Input–Output Hidden Markov Model (IOHMM) is proposed for analyzing and forecasting electricity spot prices. The model provides both good predictions in terms of accuracy as well as dynamic information about the market. In this way, different market states are identified and characterized by their more relevant explanatory variables. Moreover, a conditional probability transition matrix governs the probabilities of remaining in the same state, or changing to another, whenever a new market session is opened. The model has been successfully applied to real clearing prices in the Spanish electricity market.**

***Index Terms*—Artificial neural networks, electricity markets, hidden Markov models, modeling competitors' behavior, price forecasting.**

I. Introduction

During the last decade, the electric industries of many countries all around the world have suffered profound regulatory changes, which gave rise to many international experiences. In Spain, the overall electricity business is organized as a sequence of markets. The day-ahead spot market consists of 24 hourly auctions that take place simultaneously one day in advance. The clearing of this market provides the provisional energy schedule of each bidding unit, and the hourly marginal price is found as the intersection of supply and demand curves. After the spot market, where the major part of the total energy is traded, subsequent short-term market mechanisms (intraday markets, ancillary reserves, and real-time markets) are available in order to guarantee the final balance between power generation and consumers' demand.

In this context, generating companies and wholesale buyers assume a more intense risk exposure than in the traditional framework. In order to hedge against this risk, participants typically carry out part of their transactions through other financial markets, such as future markets or bilateral contracts.

In this environment, participants need to develop efficient tools in order to obtain accurate energy spot price forecasts. Several models have been proposed in the literature for analyzing and forecasting electricity prices with different aims and time horizons. In this paper, a new classification of these models is presented. Subsequently, a novel approach for analyzing electricity prices with Input–Output Hidden Markov Models (IOHMM) [1] is proposed. These models are based on standard Hidden Markov Models (HMMs) [2]. In general HMMs are statistical methods that are extremely useful for modeling switching processes, such as handwriting or speech recognition. In particular, they are especially well suited to represent human behavior or processes in which human actions are involved [3].

In electricity markets, spot price series reflect a switching nature related to discrete changes in participants' strategies. IOHMMs allow the meeting of two fundamental objectives when dealing with electricity price time series. The first one is to interpret and to understand the evolution of the market. To achieve this, the model should be capable of discovering hidden market states and labeling them by their more relevant explanatory variables. The second one is to take advantage of all past information to forecast spot prices with an acceptable level of accuracy. In this paper, the success of IOHMMs in dealing with electricity prices time series has been proven in the particular case of the Spanish Spot Market.

The rest of the paper is organized as follows: Section II presents the taxonomy proposed to classify electricity prices models. Section III introduces the basic concepts of HMMs and their application to electricity markets. Section IV describes the IOHMM model proposed for modeling electricity prices. In Section IV, the proposed model is applied to real electricity prices using the Spanish Spot Market as an example, and finally, the main conclusions are presented in Section V.

II. Taxonomy of Electricity Price Models

In [4], electricity price models have been classified in three wide sets according to their origins: game theory models, time series models, and production costs models. In this paper, time series electricity models are comprehensively reviewed, and a more detailed classification is proposed. It is important to mention that fuzzy models have not been included in this review.

The first group of models is based on game theory. Under this approach, price evolution is studied, stressing the analysis of the strategic behavior of the agents. Among them, *equilibrium models* take the analysis of *strategic market equilibrium* (the set of strategies such that no player in the market can improve its position if its rivals maintain their strategies, i.e., *Nash equilibrium*) as a key point [5].

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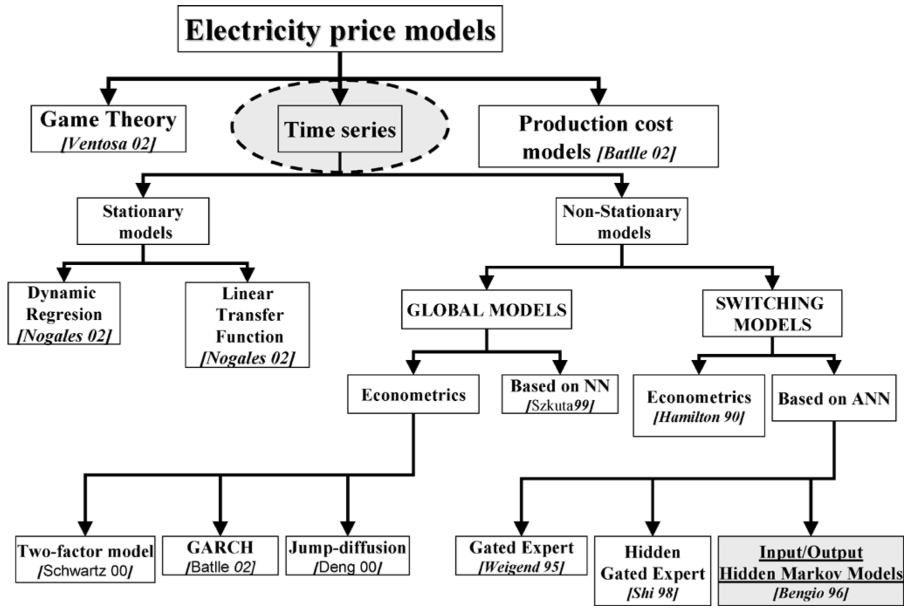

Fig. 1. Taxonomy of electricity price models.

The second group of models is based on time series analysis. These *black-box models* carry out price evolution analysis from a statistical point of view, without examining the underlying physical processes in detail.

Finally, *fundamental* or *structural models* based on traditional cost models developed for centralized systems and adapted to liberalized markets constitute the third choice. These models simulate the operation of power systems, considering not only production costs but also the agents' strategic behavior impact on market price. For example, the model proposed in [4] represents the commitment of the electricity production plants based on the strategic bids of the market agents in an oligopolistic market.

Next, an original taxonomy of time series models designed to tackle electricity price analysis is presented. This taxonomy (Fig. 1) clearly discriminates between stationary models and nonstationary models. A stochastic process $\{z_t\}_{t=1}^{T}$ is said to be *weakly stationary* if both the mean of z_t and the covariance between z_t and z_{t-l} are time invariant for all l [6] (the definition of strict stationarity can also be found in [6]).

Although traditional time series models are based on the hypothesis of stationarity, many real-world time series violate this assumption. In particular, a significant nonstationarity component can be detected in electricity price time series, due to its multiple seasonalities (related to daily, weekly, and monthly periodicities) and to its switching nature related to discrete changes in participants' strategies. This last phenomenon gives rise to *piece-wise stationary time series* [7].

However, several stationary models have been applied in the literature to model electricity price time series. In [8], two models are presented. The first one is a *Dynamic Regression* model, where the price p_t at time t is estimated as a linear regression of different lags not only of the price but also of the demand variable. The second model is a *Linear Transfer Function model*, where the price p_t at time t is related to demand values through a polynomial function of the backshift operator and a disturbance term that follows an autoregressive moving average (ARMA) process [9] induced by a white noise process.

The main drawback of using stationary models is that nonstationarity should be removed before adjusting the models. This is a nontrivial process when working with electricity price time series. In fact, in many cases, it entails a key loss of information or, worse still, is an impossible task. Classical statistics provide different techniques for this aim. In order to stabilize the mean of the series, regular and seasonal differencing can be used. In terms of the variance, Box–Cox [9] mathematical transformations are commonly applied. However, this could not be enough to completely remove the seasonal component and the aforementioned piece-wise stationary characteristic of the series.

Intervention models try to solve this problem. The basic idea is to introduce into the model an "intervention variable" and to adjust separate models for the corresponding different time series dynamics. For example, working and nonworking days can be modeled independently. However, if long-term periods are considered, discrete changes in participants' strategies can appear (e. g., agents decide to switch from a conservative attitude to a more aggressive or risky one) and give rise to different stationary regimes in the time series. At this point, the analysis has to be focused separately on each of the stationary pieces of the time series. This fact supposes an important shortcoming, due to the loss of information about the overall evolution of the market.

In conclusion, a *stationary model* preceded by data preprocessing seems to be appropriate for tackling price modeling in the short term, assuming that market agents do not change their strategies during these shorter periods. Unfortunately, this assertion cannot be applied when it comes to modeling large time series, often characterized by discrete changes in the agents' strategies.

A more suitable alternative is to apply nonstationary models. *Nonstationary models* can be classified into two groups: *single global models*, where a unique model is proposed to cope with all series data, and *switching models*, which are first concerned with identifying different regimes in the time series and then with adjusting a different local model to each one. Single global models can be classified according to their origins. The first model comes from the economic and financial world. In this environment, models are concerned with price volatility, since they are applied in the context of physical or financial contracts, derivative evaluation, etc. An interesting model in this group is the mean-reversion model [10]. The basic idea is that the deviations of the price from its equilibrium level are corrected and subjected to random perturbations. However, the most important drawback of this model is that it fails to capture the cycles that characterize electricity price time series. Recently, Schwartz enounced in [11] a more complex version. In this paper, a two-factor model was proposed. The intuitive idea underlying this model is that whereas short-term deviations correspond to temporary changes in prices that are not expected to persist (resulting from, for example, unusual weather or supply disruptions), changes in the equilibrium level represent fundamental longer term changes that are expected to persist. In [12], this model is applied to the Nordic Power Exchange's spot, future, and forward prices.

One of the major drawbacks of the models just presented when applied to commodity markets such as electricity is the assumption of constant volatility [13].

In order to solve this problem, models that assume time dependent variance, named heterocedastic models, have been developed. The well-known Generalized Autoregressive Conditional Heterocedastic (GARCH) models were suggested by Bollerslev in [14]. In a GARCH(m,r) model, the basic idea is that the conditional variance depends not only on past residuals but also on its own previous values. The multivariate version of the GARCH model has been applied in [15] to generate fuel price scenarios for risk analysis in a wholesale electricity market.

In general, ARCH-type models [9] can take various other forms. Although they have been successfully applied to other commodity markets, their application to electricity prices requires the addition of other components, such as mean reversion or jump diffusion.

The last group of econometric global models is related to *jump-diffusion models*. These models are based on the hypothesis that abrupt and sporadic changes are common in price time series. In the case of electricity, the price exhibits sudden spikes mainly due to the impossibility of storing this commodity. See [16] for the application of this model to electricity spot prices.

The second group of *global models* is based on *artificial neural networks (ANNs)* [17]. An application to model electricity prices can be found, for example, in [18], where the authors propose a three-layered neural network for the short-term forecasting of system marginal price. A regression model taking into account different physical variables is trained. However, as it is wisely pointed out in [19], although neural networks are universal approximators and, therefore, can theoretically emulate any function, including regime switching, in practice, it is often very hard to extract such an unstructured global model from the data.

In order to solve this problem, *switching models* introduce a new perspective to deal with nonstationary processes (see Fig. 1). Instead of considering a single global model, switching models are concerned with adjusting several local models for the different time series regimes. As in global models, switching models can also be classified according to their origins: econometric models and ANN-based models.

The most important econometric switching model for the purpose of this paper is the one proposed in [20]. In this paper, the series is modeled through a Markovian switching process among autoregressive regimes, adapting to occasional discrete shifts in the level, variance, and autoregressive dynamics of the series. The probability law governing these shifts is fixed, time-invariant, and nonconditioned to exogenous variables. Therefore, this model presents a serious shortcoming in capturing the particular idiosyncrasy of electricity markets. A version in which transition probabilities are time-variant is presented in [21] for the Spanish wholesale electricity market. However, the main shortcoming of these models is that they are focused on point prediction instead of probabilistic density function estimation.

Finally, three types of switching models based on neural networks have been proposed: Gated Experts (GEs), Hidden Markov Experts (HMEs), and IOHMMs. These three models are focused not only on point prediction but also on conditional density estimation.

GE models, applied in [19] to discover different regimes in the electricity load time series, consist of a nonlinear gating network and several competing experts. Each expert learns to predict the conditional mean of the output variable and adapts its variance to match the noise level in its corresponding regime. The gating network learns to predict the conditional probability of each expert, given the inputs. Although in this approach exogenous variables are considered at each time step—and therefore, the probability of each regime is time-variant—the previous regime is not considered in the activation of the next expert.

We have not found any application of HMEs [22] to electricity markets, but they are pointed out because of their similarity to the model proposed in this paper. In the HME architecture, as in GE, separate feedforward neural networks (also called experts) perform different I/O mappings. The main difference between GE and HME resides in the way they estimate the next regime. Whereas GE models try to discover different time series regimes using input data only, without taking into account the previous regime, HME models adjust a transition probability matrix, as in standard HMM, using the Baum–Welch algorithm (see [2]), without taking into account the input variables.

IOHMMs appeared in 1995 [23] as an attempt to solve the main drawbacks of these last two models. IOHMMs are probabilistic models with a fixed number of states corresponding to different conditional distributions of the output variables given the input variables and with Markovian transition probabilities between states that can also be conditioned to the input variables.

In conclusion, we maintain that nonstationary switching models based on neural networks are well suited to capture the dynamics of electricity prices time series.

III. Hidden Markov Models

Market participants have to build their bidding curve under uncertainty due not only to physical variables but also to their competitors' behavior. In order to hedge this uncertainty, they

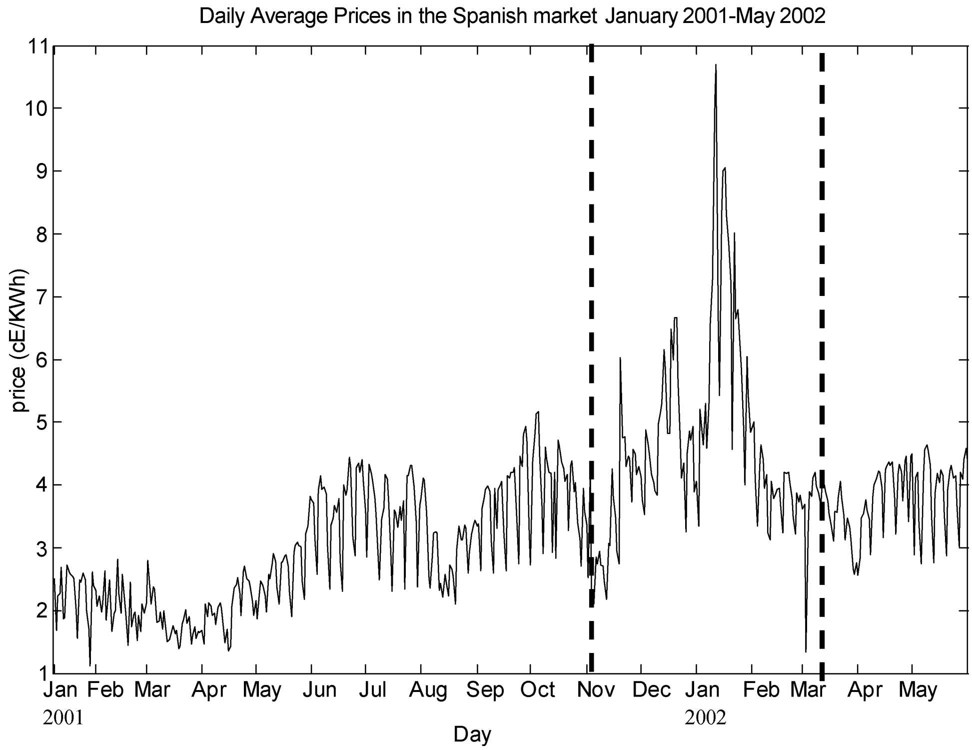

Fig. 2. Daily average prices in the Spanish Market Januay 2001–May 2002.

select their own strategy, which, in practice, is applied during a finite period of time.

Let us assume that all participants' strategies are fixed. Then, prices in periods of similar availability of resources and demand should be roughly the same. However, if an agent decides to deviate from his strategy—for example, from a conservative position to a more risky attitude—or if there is an important change in the availability of resources (e.g., due to nuclear unit maintenance), then the relationship between the available resources and prices will be broken. This change will be reflected as a switch in the price series dynamics.

In conclusion, the market evolves along time through different "market states," which are mainly characterized by the interaction among resources, demand, and participants' strategies. This sequence of "market states" is reflected in the electricity prices time series as different regimes in the dynamics of the prices.

Fig. 2 shows average daily prices for the Spanish spot market from January 2001 to May 2002. At first sight, several important changes can be observed in the series. For example, after November 2001, a significant increase in the spot price took place.

Finally, if the market is in a particular market state today, it is important to estimate both the probability that the market remains in the same state tomorrow and the probability that the market changes to others positions.

Therefore, in order to understand the evolution of the market, two sequences should be considered: the underlying market states sequence, which remains hidden, and the price sequence, which is visible for all participants.

Predicting market evolution in this way fits in directly with the observation generation process of an HMM.

HMM models were firstly introduced in the late 1960s [24]. An HMM is a double-embedded stochastic process (see [2]):

1) An underlying process defined by a Markov chain with a finite number of states $S = \{S_1, S_2, \ldots, S_N\}$, which is hidden from observation.

   A Markov chain [25] describes one of the mutually exclusive states that may characterize a system at any time step. Let $\{s_n\}_{n\geq 0}$ be a sequence of random variables taking values in a finite set $S = \{S_1, S_2, \cdots, S_N\}$. These random variables are said to be a Markov chain if the following equality holds:

$$P(s_{t+1} = S_i | s_0 = S_j, \cdots, s_t = S_k) = P(s_{t+1} = S_i | s_t = S_k). \quad (1)$$

   Equation (1) is referred to as the *Markovian property*. The Markovian property implies that given the present state, the future probabilistic behavior is independent of its history.

   The conditional probabilities $P(s_{t+1} = S_i | s_t = S_k)$ are referred to as the *transition probabilities*, and when they are time-invariant, the Markov chain is said to be homogeneous.

2) An observable process that is determined by the underlying Markov chain. At each state, a unique probability density function (pdf) governs the emission of observations.

Mathematically, an HMM is defined by the following elements:

— the number of states in the model N;
— an initial state probability distribution:

$$\Pi = (\pi_i = P(s_0 = S_i)) \in [0, 1]^N$$

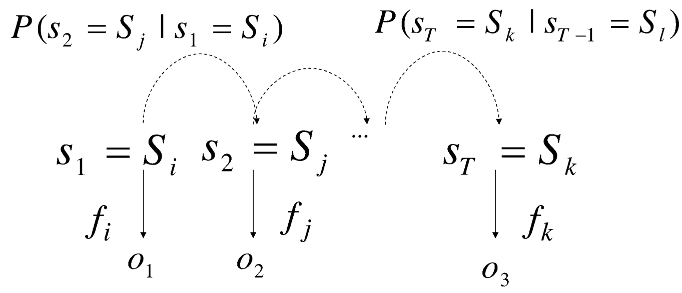

Fig. 3. Hidden Markov model process.

— the transition state probability matrix:

$$A = (a_{ij} = P(s_{t+1} = S_i | s_t = S_j)) \in [0,1]^{N \times N}$$

— a set of N emission probability density functions:

$$(f_i)_{i=1,2,\ldots,N}.$$

The HMM process of generating observations is as follows (see Fig. 3). The first state is selected according to probability vector Π, which contains the probabilities of starting from each state $S_i, \forall i \in \{1, \cdots, N\}$. For each time step t, the system shifts from the state $s_{t-1} = S_k$ to the state $s_t = S_j$, according to the transition probability matrix. At state S_j, a new observation o_t^j is emitted according to its corresponding pdf f_j. This process evolves until the last stage T is reached.

When a sequence of length T is generated, the result of the HMM process will be both the observations sequence $o_1, o_2, \ldots, o_T$ and the states sequence $s_1, s_2, \ldots, s_T$. However, whereas the observations sequence is visible, the states sequence remains hidden, and no information about it is available to an observer.

Given an observation sequence $o_1, o_2, \ldots, o_T$ (or several ones defined for the same temporal scope), the problem of adjusting an HMM is concerned with finding the set of parameters $\Theta = \{\Pi, A, (f_i)_{i=1}^N\}$ that best fits the training data. Consequently, the Expectation–Maximization (EM) algorithm [26], [27] and its simplified version, due to Baum and Welch [2], are widely proposed in the related literature.

It is important to remark that in HMMs, the probability law governing the transitions between states is fixed, time-invariant, and nonconditioned to exogenous variables. The pdf of the output variable is also nonconditioned to input variables. Therefore, this model presents a serious shortcoming in capturing the idiosyncrasy of electricity markets. As a generalization of HMMs, IOHMMs consider at each time step a set of input variables. The main difference between HMMs and IOHMMs is that in IOHMMs, both the distribution of the output variable in each regime and the transition probabilities among regimes are conditioned to a set of explanatory variables. In the next section, IOHMMs are proposed as a way of characterizing electricity markets.

A. Applying IOHMMs to Electricity Markets

In the case of electricity markets, the observations and states of IOHMMs can be interpreted as follows:

TABLE I
ANALOGY BETWEEN HMM AND ELECTRICITY MARKETS

IOHMM	Electricity Markets
States $S=\{S_1,S_2,\ldots,S_N\}$	*"Market state"* $M=\{M_1,M_2,\ldots,M_N\}$
Observations o_t	Market clearing prices P_t
pdf at state s_i **(conditioned to a set of input variables)**	Probability of emitting a particular price from the *Market State* M_i given the current explanatory variables
Initial Probability	Probability of selecting the market state M_i at time *t=1*
Transition matrix (conditioned to a set of input variables)	Probability of selecting market state M_i at time *t* given the market state M_j at time *t-1* and the set of current explanatory variables

Observations: Wholesale electricity prices will be considered as the observations of the electricity market bidding process treated in this paper. The electricity marginal prices are published by the Market Operator after market clearance.

Market states: Each market state is characterized by a particular electricity price emission probability density function, conditioned by a set of input variables such as load, hydro, thermal, and nuclear resources. In that sense, different states correspond to different functional relationships between the input variables and the marginal price. These functional relationships are associated with the interaction of participants' strategies.

Table I shows the analogy between IOHMMs and electricity markets.

IV. MODEL DESCRIPTION

A. Architecture

IOHMMs were introduced in 1996. They have been successfully applied in different fields, such as grammatical inference [1], financial return series [28], and handwriting recognition [29]. In general, HMMs are well suited to represent behavior or processes in which human actions are involved [3]. The architecture is based on standard HMMs, including a set of input variables at each time step. Whereas HMMs are focused on estimating the pdf $f(o_1^T)$ of the output sequence $o_1^T = o_1, o_2, \cdots, o_T : o_i \in R^r$, IOHMMs are trained to fit the output sequence pdf $f(o_1^T | u_1^T)$ conditioned to the input variable sequence $[u]_1^T = u_1, u_2, \cdots, u_T : u_i \in R^m$. (Note: In Fig. 4, a distinction has been made between input variables for the state network u_t^s and input variables for the output network u_t^o. However, for the sake of simplicity, in the rest of the paper, input variables have been expressed as u_t for both state and output networks.)

Using the Markovian property (1), the conditional distribution can be expressed as

$$f(o_t | u_t) = \sum_{s=S_1}^{S_N} f(o_t | s_t, u_t) P(s_t | s_{t-1}, u_t) \tag{2}$$

that is, the sum of the products of the conditional output distribution and the conditional transition probabilities.

The basic idea of the IOHMM architecture (see Fig. 4) relies on taking advantage of the above expression in order to distribute the learning tasks. In this sense, a modular architecture is

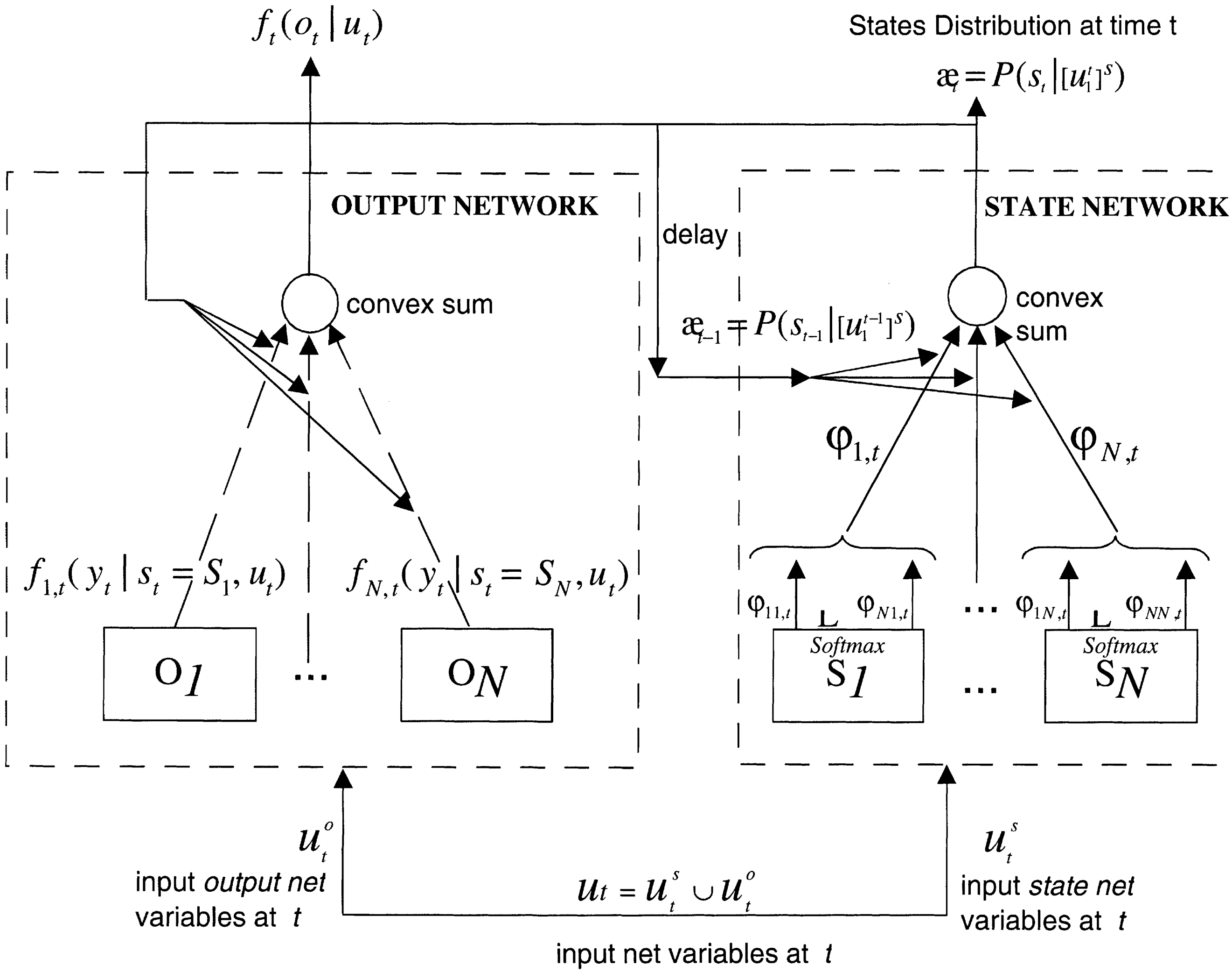

Fig. 4. IOHMM architecture.

composed by a state network and an output network. Both networks share the learning problem, but each of them is devoted to the estimation of just one of the product factors in (2).

The state network is composed of a set of state subnetworks $\{\mathcal{S}_j\}_{j=1}^N$, one for each state defined in the model. The aim of each state subnetwork $\mathcal{S}_j$ is to propose a prediction of the next state distribution, assuming that the previous state was its own state, bearing in mind the current input variables values, i.e.,$\varphi_{jt} = P(s_t|s_{t-1} = S_j, u_t)$. Hence, at each time step, every state subnetwork has m inputs $u_t \in R^m$ and N outputs (one for each state), $\varphi_{jt} = [\varphi_{1j,t}, \cdots, \varphi_{Nj,t}]$, where $\varphi_{ij,t} = P(s_t = S_i|s_{t-1} = S_j, u_t)$.

In this paper, each state subnetwork is implemented as a multilayer perceptron with a single hidden layer and sigmoidal activation functions [30]. As proposed in the original architecture, the softmax function [30] has also been implemented at the output layer as a normalization function, which guarantees that output states subnetworks are nonnegative and summing one. It is important to highlight that transition probabilities "$\varphi_{ij,t}$" are estimated by the set of state subnetworks.

Also, by combining the candidates of each subnetwork and the previous state distribution, the current state distribution is computed as $\zeta_t = [\zeta_{1,t}, \cdots, \zeta_{N,t}]$, where $\zeta_{j,t} = \sum_{j=1}^N \zeta_{j,t-1}\varphi_{j,t}$

The output network is composed of a set of output subnetworks $\{\mathcal{O}_j\}_{j=1}^N$, each one associated with a unique state of S. The task of the output subnetworks deals with predicting the expected output value, given the current state and the current input variables. Thus, a conditional probability density function f_j is implemented in each output subnetwork $\mathcal{O}_j$. Typically, the output of $\mathcal{O}_j$ is computed as the expectation $f_{j,t} = E[f_j(o_t|, u_t)] \in R^r$. Several distributions seem to be well suited [31] for the output subnetworks.

In this paper, a conditional normal distribution $G^j(\mu^j, \sigma^j)$ has been adopted. The mean of this distribution is time-variant and dependent on the input variables. In particular, it has been implemented following a dynamic regression process of the input variables. The covariance matrix has been implemented with a diagonal structure, and it is independent of the input variables.

The combination of the output subnetworks generates the output distribution $f(o_t|, u_t)$, which is a mixture of probabilities [32], in which each component is conditional on a particular state and the mixing proportions are the current state prob-

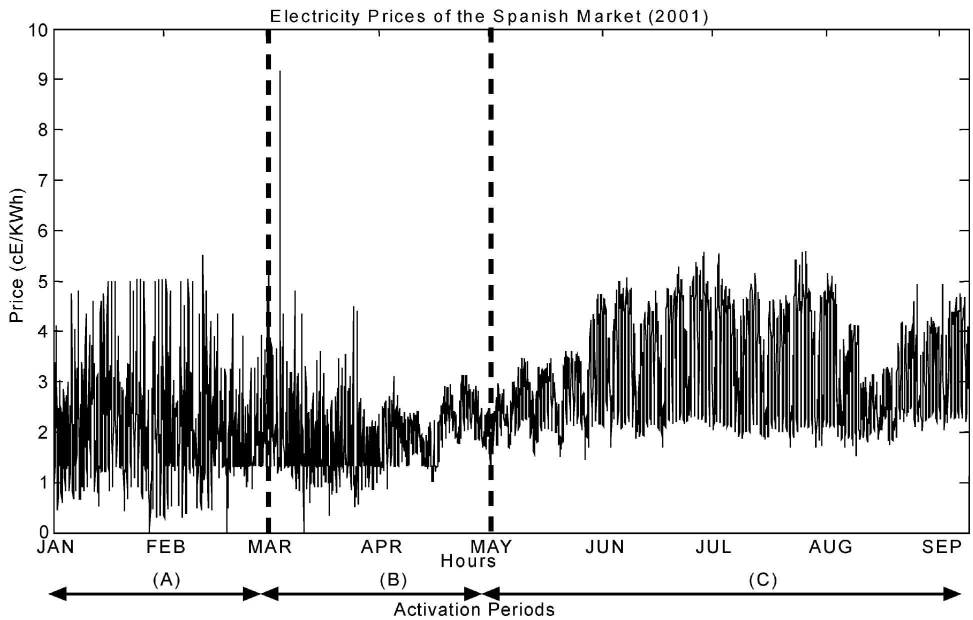

Fig. 5. Spanish electricity market spot prices.

abilities (states network outputs) conditioned to the input. This output can be expressed as follows: $f_t = \sum_{j=1}^{n} \zeta_{j,t} f_{j,t}$.

With this architecture, the IOHMM can be seen as a switching model in which the system evolves through different states, where a particular dynamic regression model is adjusted in each one. This model allows, on the one hand, weighting of the importance of input variables in each state and, on the other hand, weighting of the output by the probability of each state.

B. Learning Algorithm

The parameter estimation of IOHMMs can be approached as a maximum likelihood estimation (MLE) problem. This problem can be successfully solved by means of the iterative EM algorithm [27]. For a detailed description of the IOHMM learning algorithm, see [1].

V. Example

A. Studied Data and Proposed Architecture

The time series analyzed in this study corresponds to the hourly electricity price on the Spanish spot market from January to September 2001 (see Fig. 5). This period was split into the training set, from January to June, and the test set, from July to September.

The physical explanatory variables considered as inputs include past values of hourly production by technology and the hourly system demand. In addition, taking advantage of the autoregressive component of the price series, different lags of the price have also been considered as input variables. The set of explanatory variables for the IOHMM model is summarized in Table II and shown in Fig. 6.

It is important to note that the original IOHMM architecture proposed in [1] considers the same input variables for the state and output networks. In contrast, in the proposed model, different variables for each type of network are allowed. This small improvement makes it easier to interpret and to understand

TABLE II
Explanatory Variable Definitions

Variable	Description
$D_{d,h}$	Load for the day d and hour h
$H_{d,h}$	Hydro generation for the day d and hour h
$N_{d,h}$	Nuclear generation for the day d and hour h
$T_{d,h}$	Thermal generation for the day d and hour h
$P_{d,h}$	Price value for the day d and hour h

model results. For instance, the input variables of the states capture the market states and switches among them, whereas input variables for the output network accurately fix the price values given the probability states distribution. With this approach, the user can intentionally split input variables using his knowledge and experience. Table III presents the input data used for state and output networks.

Note that only physical variables related to electricity demand and available resources have been included as inputs to the state network in the final model. The selection in this particular case generates a more intuitive segmentation of states and does not significantly affect the accuracy of the forecasts.

In a general framework of electricity price forecasting, other input variables related to participants' pricing strategies and production costs should be considered: lagged values of electricity prices, fuel costs, aggregated supply functions, generation companies' shares, etc. These variables were evaluated in the case of study, but no significant improvement of accuracy was observed. This fact confirms the stability of the production costs and pricing strategies applied by the agents in that period of time, and it reinforces the role of the availability of resources in the identification of states.

B. Discovering and Analyzing Market States

The first step in the application of IOHMM is the selection of the number of states. Increasing the number of states gives place to models with not only higher approximation capabilities but

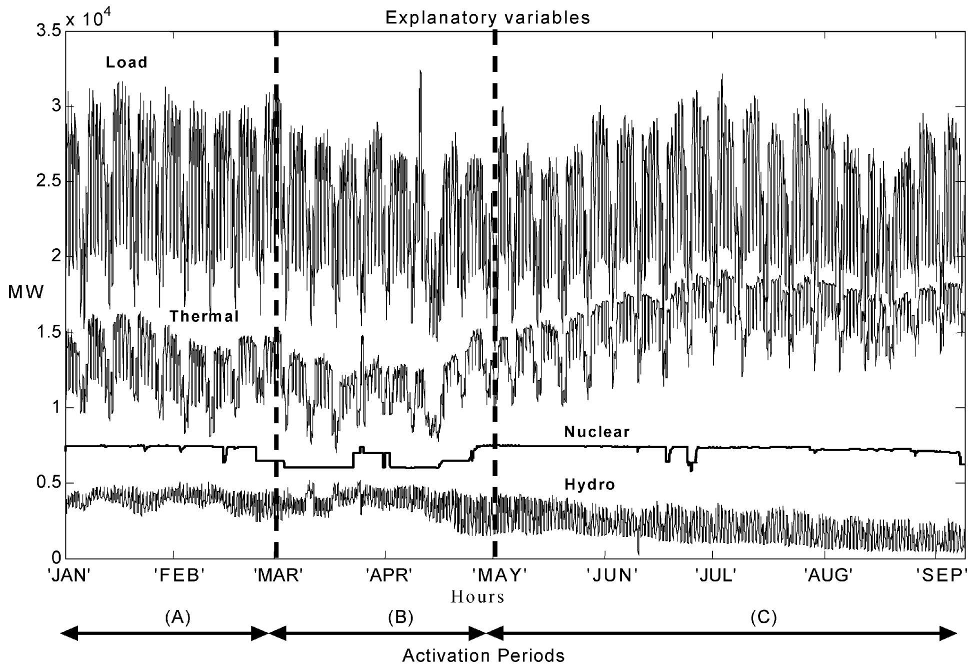

Fig. 6. Explanatory variables for the studied period.

TABLE III
INPUT VARIABLES FOR EACH TYPE OF NETWORK

Network	Input Variables
States	$D_{d,h}, D_{d,h-1}, H_{d-1,h}\ T_{d-1,h}\ N_{d-1,h}$
Output	$D_{d,h},\ D_{d,h-1},\ H_{d-1,h}\ P_{d,h-1}$

also with higher risk of over-fitting and higher computational effort. Moreover, models with too many states are also more difficult to interpret. In this example, different architectures have been adjusted in order to select the size of the model. It is important to stress that in a search for increasing accuracy and interpretability and diminishing the risk of overfitting, models have been evaluated in the test set; finally, a *four states model* was selected that was inspired in the well-known *standard error rule* [33]. In the original paper [1], the authors apply cross validation for building different architectures before selecting the correct model.

In Fig. 7, the activation of the most probable state for each hour is represented. Each row corresponds to one day and each column to 1 hr. States are represented in different colors. This figure provides useful information since it can be interpreted from two standpoints.

- The first analysis consists of fixing the y-axis values (days) and moving along the abscissa axis (hours). A first glance at Fig. 7 reveals a state activation pattern related to off-peak, peak, and medium-load hours. For instance, the hours from 3:00 to 6:00 during the May–September period correspond to state I. Hour 20:00 during the January–February period is activated with state III. This fact can be justified since the criterion followed when participants build their offers for similar hours should not be very different in the same market session.
- The opposite point of view consists of fixing the x-axis values and moving along the y-axis. The evolution of the different market states along the temporal scope shows discrete changes related to the different patterns in the behavior of the participants. Although different clustering techniques could be applied for identifying daily state activation patterns, in this example, three activation periods have been distinguished by visual inspection. The first one (A) goes from January to the beginning of March. The four states are activated according to the type of hour. For example, medium hours are more likely to be activated with state II, whereas state III is the most probable during peak hours. The second period (B) begins in March and finishes at the beginning of May. This period is clearly identified since state IV is activated during almost all hours of the day. This period corresponds with the horizontal stripe of clear colors in the figure. Finally, the third activation period (C) goes from the beginning of May to the end of the period studied. This activation area is characterized by state I for off-peak hours, while state II seems to exhibit the highest probability in the rest of the hours. Therefore, it may be observed that specific patterns are assigned to weekends. For example, in the last period from 8:00 to 24:00, Saturdays and Sundays are activated, with most probability, with state I, whereas working days are activated with state II.

C. States and Explanatory Variables

The identification of three activation periods (A, B, and C in Fig. 7) is related to the occurrence of causal episodes in the states network input variables. In the first activation period (A), explanatory variables do not suffer abrupt changes. At the beginning of March (A $\rightarrow$ B), nuclear production is drastically reduced due to the maintenance of several units. The joint effect of this reduction in base production and the decreasing tendency of the load are reflected as a shift in the variance of the

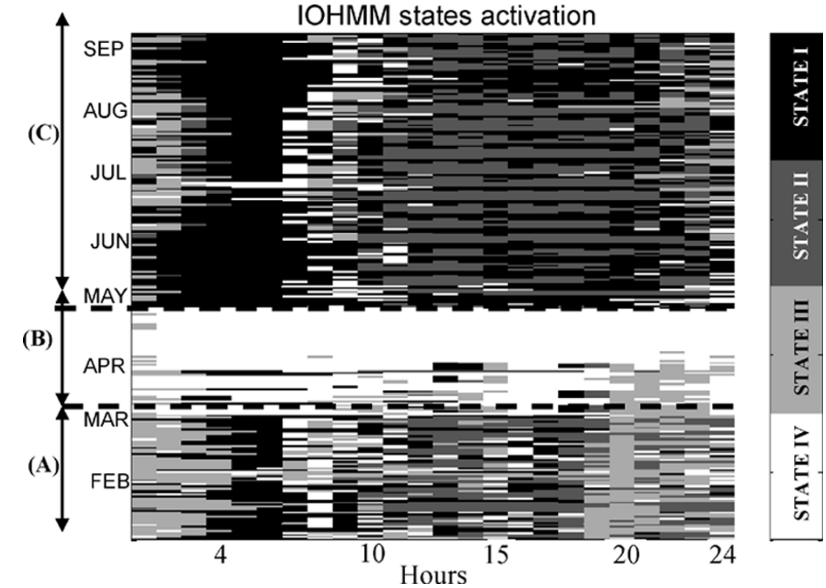

Fig. 7. Activation of IOHMM states.

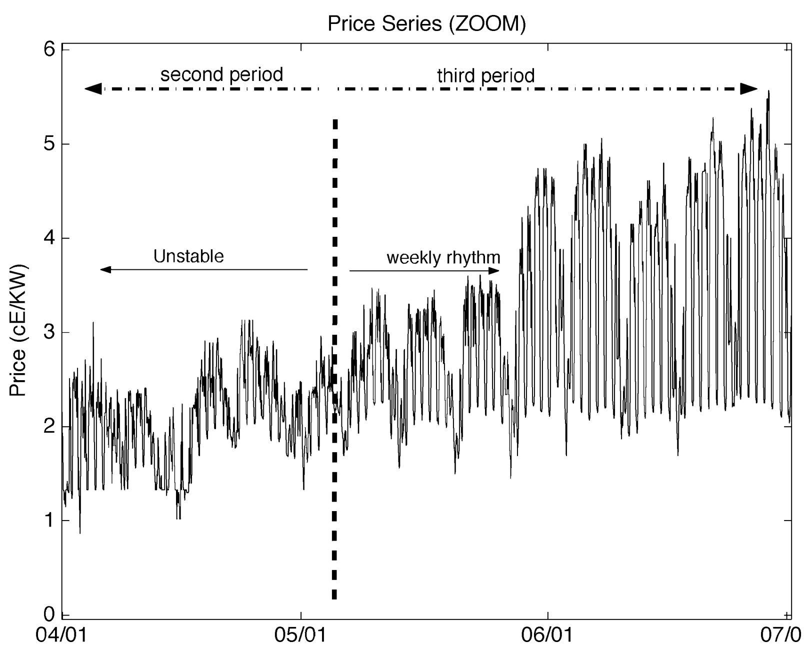

Fig. 8. Spanish electricity market spot prices (zoom).

electricity price time series. At the beginning of May (B $\rightarrow$ C), some of the nuclear units in maintenance are recovered, but water resources present an important drop. This new situation results in a thermal production increase. Regarding the behavior of the prices time series, during period (A), price values are between 0.5 c Euro/KWh and 5 c Euro/KWh. In the second period (B), minimum prices increase while maximum prices decrease. In the third period (C), minimum prices were around 2 c Euro/KW, but price variance increased, which led to maximum prices around 5 c Euro/KWh.

It is important to note that in this context, market states are not related to price levels but rather to a functional relationship between the set of input variables and the marginal price. This fact can be observed in Fig. 8. This figure shows a zoom of price series from April to July. Although, during May, minimum prices were around 1.5 c(B $\rightarrow$ C) and maximum prices close to 3.5 c Euros, as in the second period, the series has already recovered the stability and weekly rhythm. The model is able to capture this switch, and therefore, May is classified in the third period.

Fig. 9 provides information about the normalized significance of each explanatory variable in the outputs subnetworks. For this aim, coefficients of the regression model for the conditioned mean of the Gaussian model, as well as standard deviation values, are presented. The analysis of the signs of the coefficients (positive for $D_{d,h}$, negative for $D_{d,h-1}$, and positive

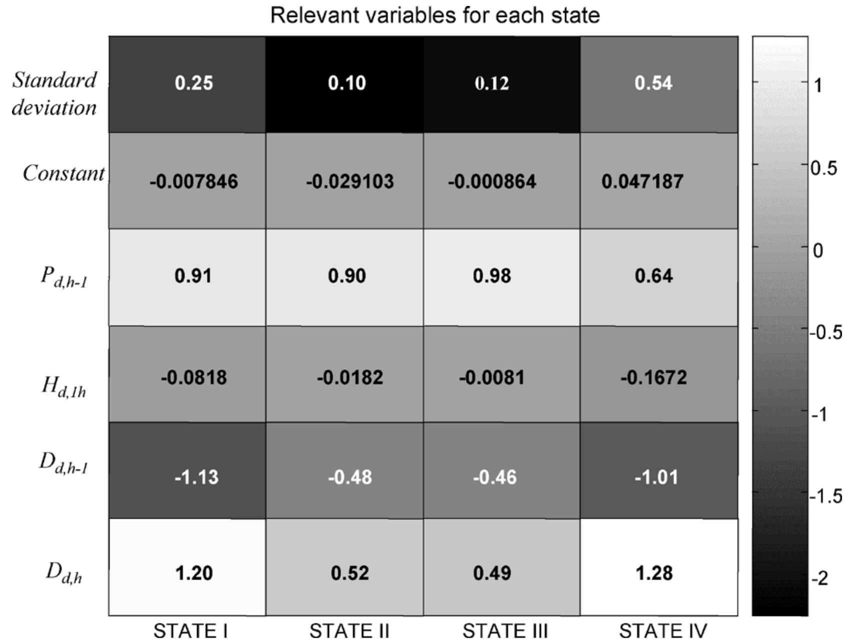

Fig. 9. Explanatory variables coefficient for each state.

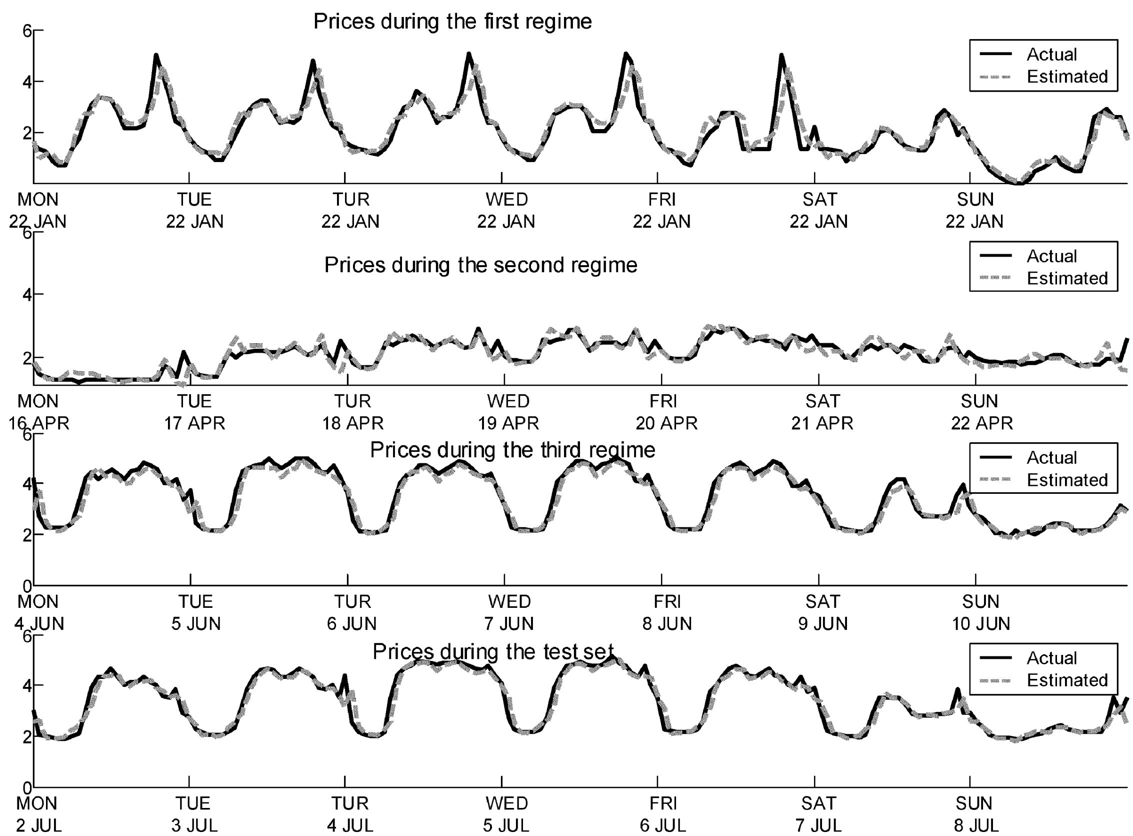

Fig. 10. IOHMM forecasted prices (ZOOM).

for $P_{d,h-1}$) reveals an autoregressive "integrated"[1] component of the form

$$P_{d,h} - a_1 P_{d,h-1} = b_1 D_{d,h} + b_2 D_{d,h-1}.$$

[1]In the sense that the model tries to capture the incremental evolution in the price instead of the price value.

The comparative analysis of the coefficients of each state reveals similarities between states I and IV and between II and III. States I and IV highlight a coefficient for the $D_{d,h}$ variable of around 1.2 and for the $D_{d,h-1}$ close to -1.

The difference between these two states comes from the first autoregressive component. Whereas state I exhibits a coefficient value for $P_{d,h-1}$ close to 0.9, this value for the state IV is around

0.6. Regarding states II and III, these two states reveal a coefficient for the $D_{d,h}$ variable close to 0.5, whereas the coefficient for the demand in the previous hour $D_{d,h-1}$ is approximately -0.48. The value of the autoregressive component in state III $P_{d,h-1}$ is slightly more important ($\simeq$0.98) than in state II, where the autoregressive coefficient is close to 0.9.

As expected, all coefficients affecting $H_{d-1,h}$ are negative. This is justified since the greater the hydro resources, the lower the prices. Finally, state II has been characterized as being the least noisy state (the minimum standard deviation corresponds to this state).

D. Forecasting Prices

Fig. 10 shows four examples of 1-hr ahead forecasting performance. The three first plots correspond to training set data and illustrate the differences among the three activation periods. The fourth plot illustrates the forecasting performance with testing data, which also belongs to the third activation period. The IOHMM model has been successfully adjusted, reacting on time to changes of states and predicting accurate values of prices. The model produces a mean absolute percentual error (MAPE) value of 15.83% for the testing set.

The model has been run in a PC with 512 MB of RAM memory and 1.2-GHz clock frequency. The training phase consumed 320 s of CPU time.

VI. Conclusions

In this paper, a new taxonomy for electricity prices models has been presented. This taxonomy discriminates between stationary and nonstationary time series models. Although stationary models can be applied to short-term electricity time series, they present serious problems when it comes to applying them to large series. In contrast, nonstationary models appear to be as more suitable for electricity prices.

In this paper, a nonstationary model based on neural networks (the IOHMM model) has been developed and applied to the Spanish Electricity Spot Market price time series. The proposed IOHMM not only provides price predictions but also dynamic information about the market. In this sense, different market states have been identified. Each market state is characterized by a particular pdf, which represents the relationship between explanatory variables and the spot price through a dynamic regression model. The activation of these states gives rise to the identification of three fundamental regimes. The switches among these regimes have been explained according to the evolution of the considered explanatory variables.

IOHMMs have some crucial advantages over other prediction models. First, as the model estimates the conditional output pdf, not only is a point price prediction provided at each time step, but also, the agent is faced with an estimation of the uncertainty. This pdf can be used, for example, to evaluate different risk measures and for the generation of scenario trees. Second, the IOHMM can be interpreted as a piecewise dynamic regression model where the most suitable model is activated automatically by the underlying Markov chain.

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