University of California, Los Angeles
Department of Statistics

Statistics C183/C283 **Instructor: Nicolas Christou**

Homework 1

Exercise 1
Use the data presented in class (returns of IBM and TEXACO, $\bar{R}_1 = 0.010, \sigma_1^2 = 0.0061, \bar{R}_2 = 0.013, \sigma_2^2 = 0.0046$): Find the smallest value of the correlation coefficient above which all the portfolios would have risk larger than the risk of the least risky stock. In other words, with this value of ρ, all the resulting portfolios (from all combinations of x_1 and x_2, with $x_1 + x_2 = 1$ and $x_1 \geq 0, x_2 \geq 0$) will have variance larger than 0.0046. In this case diversification is not useful in reducing the risk and you should purchase only the stock with the least variance. [*Ans.* $\rho = 0.87$].

Exercise 2
Use R to construct the portfolio possibilities curve for the correlation coefficient ρ that you found in exercise 1.

Exercise 3
Suppose now that $\rho = -1$, and $\bar{R}_1 = 0.010, \sigma_1^2 = 0.0061, \bar{R}_2 = 0.013, \sigma_2^2 = 0.0046$ as given in the handout. Find x_1 and x_2 such that the risk is equal to zero.

Exercise 4
Assume that the average variance of the return for an individual security is 50 and that the average covariance is 10. What is the variance of an equally weighted portfolio of 5, 10, 20, 50, and 100 securities?

Exercise 5
Allow short sales (x_1 or x_2 can be negative, but always $x_1 + x_2 = 1$) to construct the portfolio possibilities curve using correlation coefficient $\rho = 0.95$. Use `R` to find the values of x_1 and x_2 that minimize the risk of the portfolio. Note: If short sales are not allowed there is no benefit from diversification because $\rho = 0.95 > 0.87$ - see exercise 1). But when short sales are allowed you will be able to find a combination of x_1, x_2 that gives variance less than the least risky asset.

Exercise 6
From *http://finance.yahoo.com* download the monthly data for two stocks for a period of at least 3 years (make sure that both stocks have data for the period that you choose).

a. From the two spreadsheets get the adjusted close prices and convert them into returns.
b. Compute the mean return, variance, and covariance of the two stocks.
c. Construct the portfolio possibilities curve using these two stocks.
d. What is the composition of the minimum risk portfolio?

Exercise 7
Please answer the following questions.

1. The mean returns and variance covariance matrix of the returns of three stocks (`C, XOM, AAPL` and the market `SP500`) are given below:

```
Mean returns:
         C         XOM        AAPL       ^GSPC
   0.005174    0.010617    0.016947    0.010846

Variance-covariance matrix:
             C      XOM     AAPL    ^GSPC
C     0.010025 0.000000 0.000000 0.000000
XOM   0.000000 0.002123 0.000000 0.000000
AAPL  0.000000 0.000000 0.005775 0.000000
^GSPC 0.000000 0.000000 0.000000 0.001217
```

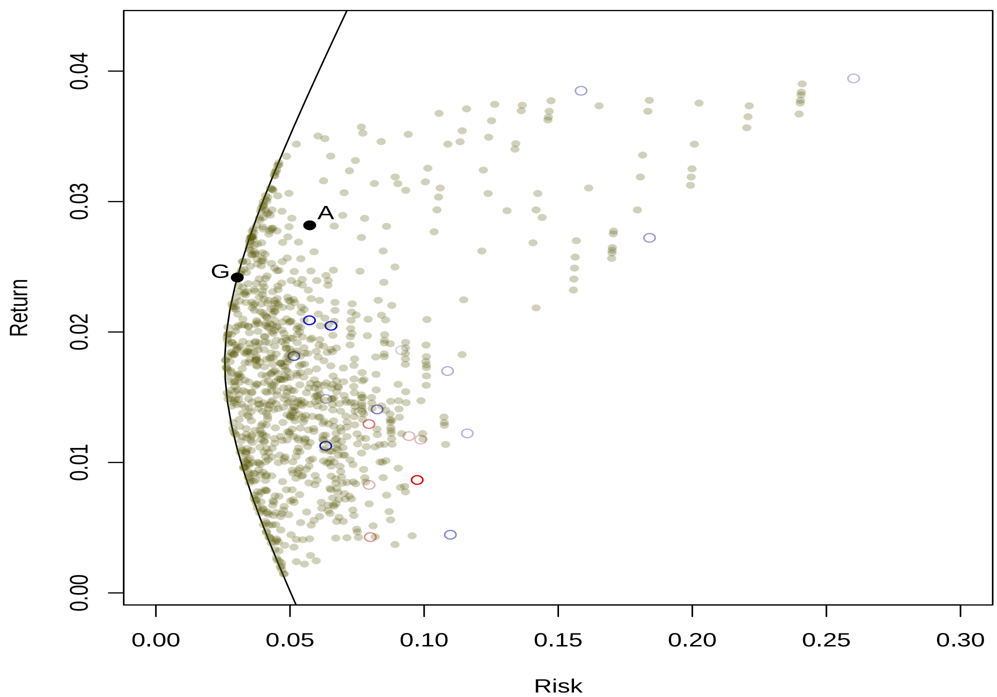
   Assume short sales are allowed. Compute the composition of the minimum risk portfolio using only the three stocks.

2. Refer to questions (1). Assume $R_f = 0.001$ and short sales allowed. Find the composition of the optimal portfolio (point of tangency).

Exercise 8

Please answer the following questions.

1. An investor wants to hold portfolio `A` as shown on the plot below. Suppose that short sales are allowed and $R_f = 0.001$. Suggest a better investing strategy. Note: Point `G` on the plot is the point of tangency.



2. Which one of the following portfolios cannot lie on the efficient frontier as described by Markowitz? Please explain your answer.

Portfolio	Expected return (%)	Standard deviation (%)
W	15	36
X	12	15
Y	5	7
Z	9	21

3. Suppose all stocks have $E(R) = 15\%$, $\sigma = 60\%$, and common correlation coefficient $\rho = 0.5$. What are the expected return and standard deviation of an equally weighted portfolio of $n = 25$ stocks?

4. Refer to question (3). What is the smallest number of stocks necessary to generate a portfolio with standard deviation of at most 43%?

5. Refer to question (3). As n gets larger, is it true that $\sigma_p = \sigma\sqrt{\rho}$? Please explain your answer.

Exercise 9

Consider two stocks A and B with expected returns $\bar{R}_1, \bar{R}_2$, variances σ_1^2, σ_2^2, and covariance σ_{12}. Suppose short sales are allowed and risk free asset R_f exists. Show that the composition of the optimal portfolio is

$$
\begin{aligned}
x_1 &= \frac{\bar{R}_A \times \sigma_2^2 - \bar{R}_B \times \sigma_{12}}{\bar{R}_A \times \sigma_2^2 + \bar{R}_B \times \sigma_1^2 - (\bar{R}_A + \bar{R}_B) \times \sigma_{12}} \\
x_2 &= 1 - x_1
\end{aligned}
$$

Note: $\bar{R}_A = \bar{R}_1 - R_f$ and $\bar{R}_B = \bar{R}_2 - R_f$.

Exercise 10

Consider a portfolio consisting of $n+1$ assets: n risky assets, and the $(n+1)$st asset is the risk free asset with guaranteed return R_f. When short sales allowed, the efficient frontier of all feasible portfolios which can be constructed from these $n+1$ assets is defined as the locus of feasible portfolios that have the smallest variance for a prescribed expected return E is determined by solving the problem

$$\begin{aligned} \min \quad & \frac{1}{2}\mathbf{x}'\boldsymbol{\Sigma}\mathbf{x} \\ \text{subject to} \quad & E = R_f + (\bar{\mathbf{R}} - R_f\mathbf{1})'\mathbf{x} \end{aligned}$$

Definitions:

$\mathbf{x}$	Vector of the weight of the n risky assets.
$\boldsymbol{\Sigma}$	Variance covariance matrix of the n risky assets.
$\bar{\mathbf{R}}$	Vector of the expected returns of the n risky assets.
$\mathbf{1}' = (1, 1, \ldots, 1)$	$n \times 1$ vector of ones.
R_f	Risk free rate.
E	Required expected return (combination of the n risky assets and the risk free asset).

Answer the following questions:

a. Show that the weights of the n risky assets is given by

$$\mathbf{x} = \frac{E - R_f}{(\bar{\mathbf{R}} - R_f\mathbf{1})'\boldsymbol{\Sigma}^{-1}(\bar{\mathbf{R}} - R_f\mathbf{1})}\boldsymbol{\Sigma}^{-1}(\bar{\mathbf{R}} - R_f\mathbf{1}).$$

b. Does $\mathbf{x}$ represent the point of tangency as discussed in class? Where is the portfolio that corresponds to $\mathbf{x}$ located on the expected return against risk (standard deviation) space? Show that for the $\mathbf{x}$ found in question (a) the efficient locus is linear in σ, i.e.

$$E = R_f + \sigma\sqrt{(\bar{\mathbf{R}} - R_f\mathbf{1})'\boldsymbol{\Sigma}^{-1}(\bar{\mathbf{R}} - R_f\mathbf{1})}$$