Homework 5

Exercise 1:
You want to find the value of a European call option for the following data: $S_0 = \$50, E = \$60, u = 1.2, d = \frac{1}{u}, r = 0.10$ (for each period), and number of periods to expiration $n = 10$. Using the binomial option pricing model:

a. Find the value of k, the number of up movements of the stock, so that the call is "in the money" at the end of the 10_{th} period.

b. Draw the binomial tree diagram and place the price of the stock at each node of the binomial tree (only at the end of 10_{th} period).

c. What is the intrinsic value of the call at each node of the 10_{th} period?

d. Find the price of the call at $t = 0$ by:

   1. Using the binomial formula:
   $$C = S_0 \sum_{j=k}^{n} \binom{n}{j} p'^j (1 - p')^{n-j} - \frac{E}{(1 + r)^n} \sum_{j=k}^{n} \binom{n}{j} p^j (1 - p)^{n-j}$$

   2. Discounting the expected value of the call at the end of the 10_{th} period.

Exercise 2:
A stock price is currently $50. Over each of the next two 3-month periods it is expected to go up by 6% or down by 5%. The risk-free interest rate is 5% per year with continuous compounding. What is the value of a 6-month European call option with strike price of $51? Complete the entire binomial tree diagram for the 2 periods. Place the price of the stock and the price of the call at each node on the binomial tree diagram.

Exercise 3:
Refer to exercise 2. What is the value of a 6-month European put option with strike price of $51? Complete the entire binomial tree diagram for the 2 periods. Place the price of the stock and the price of the put at each node on the tree diagram. Verify that the European call and the European put prices satisfy the put-call parity.

Exercise 4:
Refer to exercise 3. If the put option were American, would it ever be optimal to exercise it early at any of the nodes on the binomial tree diagram? Find the value of this American put option.