University of California, Los Angeles
Department of Statistics

Statistics C183/C283 **Instructor: Nicolas Christou**

Homework 2

Exercise 1
Access the following data:

`http://www.stat.ucla.edu/~nchristo/statistics_c183_c283/statc183c283_5stocks.txt.`

In `R` you can access the data from the command line as follows:

```
a <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c183_c283/statc183c283_5stocks.txt", header=T)
```

These are closing monthly prices from January 1986 to December 2003. The first column is the date and P_1, P_2, P_3, P_4, P_5 represent the close monthly prices for the stocks Exxon-Mobil, General Motors, Hewlett Packard, McDonalds, and Boeing respectively.

a. Convert the prices into returns for all the 5 stocks. Please do not print these numbers!

b. Compute and print the mean return for each stock and the variance-covariance matrix.

c. Use only Exxon-Mobil and Boeing stocks: For these 2 stocks find the composition, expected return, and standard deviation of the minimum risk portfolio.

d. Plot and print the portfolio possibilities curve and identify the efficient frontier on it (no short sales).

e. Use only Exxon-Mobil, McDonalds and Boeing stocks and assume short sales are allowed to answer the following question: For these 3 stocks compute the expected return and standard deviation for many combinations of x_a, x_b, x_c with $x_a + x_b + x_c = 1$ and plot and print the cloud of points. Reminder: Short sales are allowed. In case you need many combinations you can get them from here:

```
a <- read.table("http://www.stat.ucla.edu/~nchristo/datac183c283/statc183c283_abc.txt", header=T)
```

f. Assume $R_f = 0.001$ and that short sales are allowed. Find the composition, expected return and standard deviation of the portfolio of the point of tangency G and draw the tangent to the efficient frontier of question (e). Print the graph.

g. Find the expected return and standard deviation of the portfolio that consists of 60% G 40% risk free asset. Show this position on the capital allocation line (CAL).

h. Now assume that short sales allowed but risk free asset does not exist.

1. Using $R_{f1} = 0.001$ and $R_{f2} = 0.002$ find the composition of two portfolios A and B (tangent to the efficient frontier - you found the one with $R_{f1} = 0.001$ in exercise f).
2. Compute the covariance between portfolios A and B?
3. Use your answers to (1) and (2) to trace out the efficient frontier of the stocks Exxon-Mobil, McDonalds, Boeing. Submit a printout of the graph that shows this efficient frontier on top of the cloud of points from question (e).
4. Find the composition of the minimum risk portfolio (how much of each stock), its expected return, and standard deviation.

Exercise 2
Given the following:

Stock	$\bar{R}$	σ
Stock A	0.12	0.20
Stock B	???	0.08

It is also given that $\rho_{AB} = 0.1$.

a. What expected return on stock B would result in an optimum portfolio of $\frac{1}{2}A$ and $\frac{1}{2}B$? Assume short sales are allowed and that $R_f = 0.04$.

b. What expected return on stock B would mean that stock B would not be held? Assume short sales are allowed and that $R_f = 0.04$.

Exercise 3
Answer the following questions:

a. Assume that the variance of security A is 0.16 and the variance of security B is 0.25. The variance of a portfolio consisting of 50%A and 50%B is 0.0525. Find the covariance between securities A and B.

b. Assume $R_f = 0.05$ and two stocks A, B with $\bar{R}_A = 0.14, \bar{R}_B = 0.10$. Suppose the point of tangency to the efficient frontier (the one constructed using the two stocks), consists of 60%A and 40%B. Let's say that you want to build a portfolio by combining the risk free asset and portfolio G to obtain expected return 11%. Determine the percentages of your investment in the risk free asset and in portfolio G.

c. Consider the data from part (b). Suppose you want to build a portfolio with expected return 0.10 by combining the risk free asset and portfolio G. What is the composition of this portfolio in the risk free asset, in A, and in B?

Exercise 4
Answer the following questions:

a. Assume the single index model holds. Find the covariance between an equally weighted portfolio and the market.

b. Consider the following variation of the single index model. We are still using a similar model, $R_i = \theta_i + \delta_i R_m + \epsilon_i$, but here we will assume that R_m in non-random. In addition, $var(\epsilon_i) = \sigma^2, i = 1, \ldots, n$, $cov(\epsilon_i, \epsilon_j) = k\sigma^2$ (that is, the covariance between the error terms of two stocks is equal to $k\sigma^2$, and θ_i, δ_i are non-random parameters associated with stock i. Assume an equally weighted portfolio. Approximately, what is the variance of this portfolio when n gets large?

c. You are given the variance covariance matrix of the returns of three stocks. Can you find the composition of the minimum risk portfolio? What assumption do you need to make?

d. Consider the single index model discussed in class. Find the covariance between a portfolio that consists of n stocks and stock i. Note: stock i is one of the stocks in the portfolio.

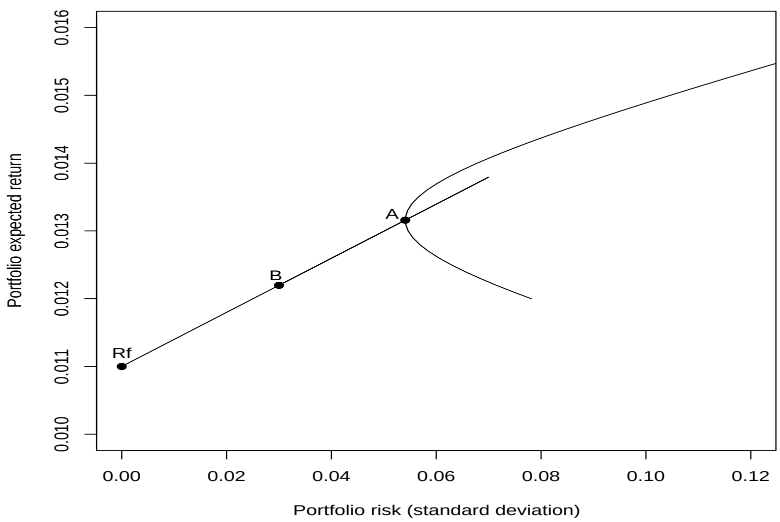
Exercise 5
The covariance matrix $\mathbf{Q}$ of the returns of two stocks has the following inverse:

```
> solve(Q)
          [,1]      [,2]
[1,] 166.21139 -22.40241
[2,] -22.40241 220.41076
```

Answer the following questions:

a. Find the composition of the minimum risk portfolio.

b. It is given that the minimum risk portfolio (point A on the graph below) has standard deviation equal to 0.05408825 and expected return equal to 0.01315856. Portfolio B (see graph below) has expected return equal to 0.01219724. What is the composition of portfolio B in terms of portfolio A and the risk free asset? Assume $R_f = 0.011$.



c. The standard deviation of portfolio B is equal to 0.03. Given this level of risk, can you do better than the expected return of portfolio B? Please explain.

Exercise 6
Answer the following questions:

a. Show that two random variables X and Y cannot possibly have the following properties: $E(X) = 3, E(Y) = 2, E(X^2) = 10, E(Y^2) = 29$, and $E(XY) = 0$.

b. Assume that the single index model holds. The characteristics of two stocks A and B are the following:

Stock	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}^2_\epsilon$
A	0.0082	0.79	0.027
B	0.0099	1.12	0.006

If $\sigma^2_m = 0.0022$ find the correlation coefficient between stocks A and B.

c. The data in part (c) were obtained using the returns from the adjusted close prices for 60 months. If $\sum_{i=1}^{60}(R_{mt} - \bar{R}m)^2 = 0.13$, construct a 95% confidence interval for the beta of stock B. Use $t_{0.025;58} = 2.000$.

d. Use the data from parts (c) and (d). Consider the simple regression of R_A on R_m. Compute the coefficient of determination R^2. *Hint:* $SSR = \hat{\beta}^2 \sum_{i=1}^{n}(R_{mt} - \bar{R}_m)^2$.

Exercise 7

Suppose the single index model holds. Also short sales are allowed and there is a risk free rate $R_f = 0.002$. For 3 stocks the following were obtained based on monthly returns for a period of 5 years:

Stock i	α	β	σ^2_ϵ
1	0.01	1.08	0.003
2	0.04	0.80	0.006
3	0.08	1.22	0.001

The expected return and variance of the market are $\bar{R}_m = 0.10$ and $\sigma^2_m = 0.002$ for the same period.

a. Suppose that the optimum portfolio consists of 30% of stock 1, 50% of stock 2, and 20% of stock 3. What is the β of this portfolio.

b. Suppose that you are a portfolio manager and you have $500000 to invest in this optimum portfolio on behalf of a client. In addition this client wants to invest another $300000 by borrowing this amount at the risk free rate $R_f = 0.002$. What is the expected return and standard deviation of this portfolio. Show it on the expected return standard deviation space.

c. What is the covariance between the portfolio of part (a) and the market?

d. If the client wants to allocate 60% of his initial funds in the optimum portfolio and the remaining 40% in the risk free asset, what would be the expected return and standard deviation of this position?

e. What is the covariance between stock 1 and the market?