University of California, Los Angeles
Department of Statistics

Statistics C183/C283 **Instructor: Nicolas Christou**

Midterm
09 May 2008

Name: ______________________________

Problem 1 (70 points)
For three stocks you are given the following data based on the single index model:

Stock	$\bar{R}$	β	σ^2_ϵ
A	0.0051	0.94	0.0033
B	0.0120	0.61	0.0038
C	0.0160	1.12	0.0046

Below you are given the solution to the problem (the point of tangency) when short sales are allowed and $R_f = 0.005$.

$$\boldsymbol{Z} = \boldsymbol{\Sigma^{-1}R} = \begin{pmatrix} 0.00489048 & 0.00103212 & 0.00189504 \\ 0.00103212 & 0.00446978 & 0.00122976 \\ 0.00189504 & 0.00122976 & 0.00685792 \end{pmatrix}^{-1} \begin{pmatrix} 0.0051 - 0.005 \\ 0.0120 - 0.005 \\ 0.0160 - 0.005 \end{pmatrix} = \begin{pmatrix} -0.883563202 \\ 1.327096101 \\ 1.610164293 \end{pmatrix}.$$

The sum of the z_i's is $\sum_{i=1}^3 z_i = 2.053697192$ and therefore the x_i's are:
$x_1 = -0.4302, x_2 = 0.6462, x_3 = 0.7840$.

The above is one way to solve the problem. We can also solve the problem by ranking the stocks based on the excess return to beta ratio.

a. Rank the three stocks based on the excess return to beta ratio and complete the table below that will allow you to find the C^*. You will also need $\sigma^2_m = 0.0018$. You can use the last page for extra calculations before you complete the table.

Stock i	α_i	$\hat{\beta}_i$	$\bar{R}_i$	$\sigma^{\hat{2}}_{\epsilon i}$	$\frac{R_i - R_f}{\hat{\beta}_i}$	$\frac{(\bar{R}_i - R_f)\hat{\beta}_i}{\sigma^{\hat{2}}_{\epsilon i}}$	$\sum_{j=1}^{i} \frac{(\bar{R}_j - R_f)\hat{\beta}_j}{\sigma^{\hat{2}}_{\epsilon j}}$	$\frac{\hat{\beta}^2_i}{\sigma^{\hat{2}}_{\epsilon i}}$	$\sum_{j=1}^{i} \frac{\hat{\beta}^2_j}{\sigma^{\hat{2}}_{\epsilon j}}$	C_i
2	0.0059	0.61	0.0120	0.0038	0.0114754098	1.12368421	1.123684	97.92105	97.92105	0.001719548
3	0.0048	1.12	0.0160	0.0046	0.0098214286	2.67826087	3.801945	272.69565	370.61670	0.004105009
1	-0.0043	0.94	0.0051	70.0033	0.0001063830	0.02848485	3.830430	267.75758	638.37428	0.003208254

b. Assume short sales are allowed. Find C^* and use it to find the composition of the optimum portfolio (point of tangency). Your answer should be exactly the same as above.

c. Assume short sales are not allowed. Find C^* and use it to find the composition of the optimum portfolio.

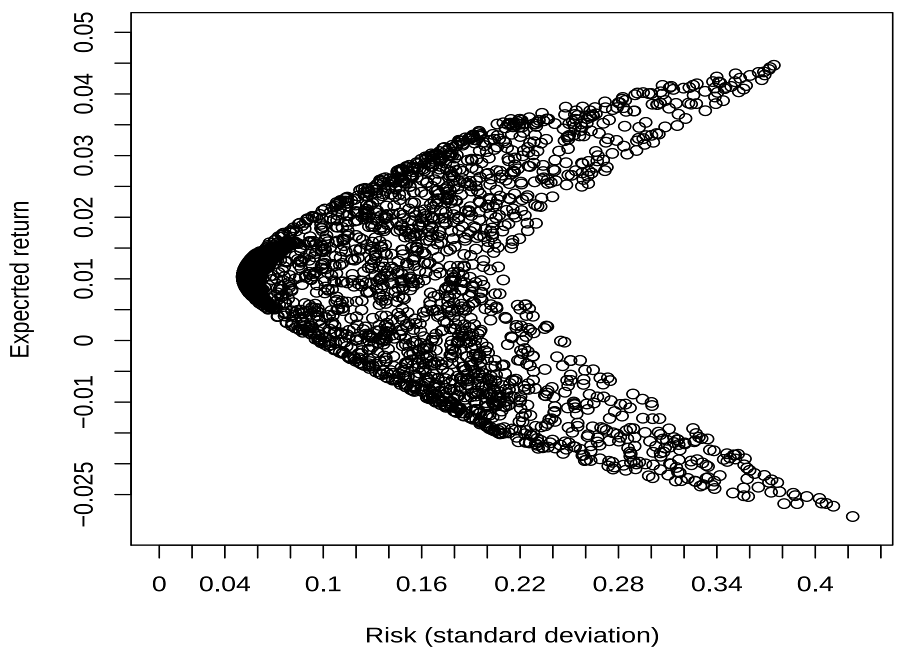
d. Compute the mean return and standard deviation of the portfolios in (b) and (c) and place them (approximately) on the graphs (opposite page). Your answer should be the point of tangency in both cases. Note: The first graph allows short sales, while the second graph does not.

e. Write down the expression in matrix form that computes the covariance between the portfolio of part (b) and the equally allocated portfolio $(\frac{1}{3}A, \frac{1}{3}B, \frac{1}{3}C)$. No calculations, just the expression!

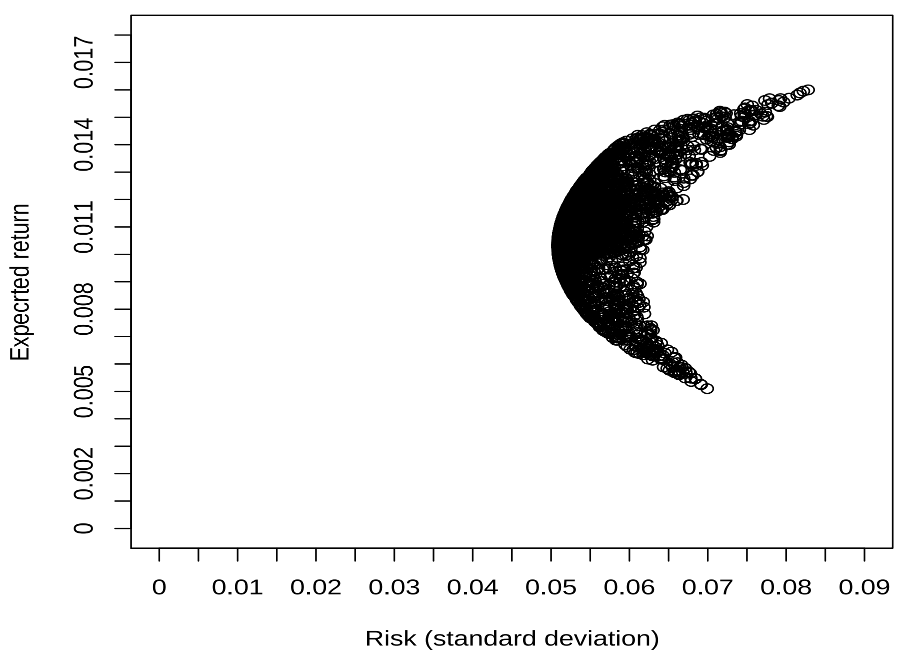
f. Consider the portfolio of part (b). Suppose that you want to place 60% of your funds in portfolio (b) and invest the other 40% in the risk free asset. Find the mean return and standard deviation of this new portfolio and show it on the first graph.

g. You have \$2000 to invest in portfolio (b). In addition you borrow another \$1000 to invest in portfolio (b). Show the position of this portfolio on the first graph (approximately). No calculations.

Short sales allowed:



Short sales not allowed:



Problem 2 (30 points)
Using the constant correlation model we completed the table below on 12 stocks. Assume $R_f = 0.05$ and average correlation $\rho = 0.45$.

Stock i	$\bar{R}_i$	σ_i	$\frac{R_i - R_f}{\sigma_i}$	$\frac{\rho}{1-\rho+i\rho}$	$\sum_{j=1}^{i} \frac{R_j - R_f}{\sigma_j}$	C_i
1	0.27	0.031	7.097	0.450	7.097	3.194
2	0.31	0.042	6.190	0.310	13.287	4.124
3	0.16	0.023	4.783	0.237	18.070	4.280
4	0.15	0.021	4.762	0.191	22.832	4.372
5	0.33	0.059	4.746	$a = ?$	$b = ?$	$c = ?$
6	0.27	0.061	3.607	0.138	31.184	4.318
7	0.19	0.039	3.590	0.122	34.774	4.229
8	0.13	0.029	2.759	0.108	37.532	4.070
9	0.16	0.051	2.157	0.098	39.689	3.883
10	0.12	0.038	1.842	0.089	41.531	3.701
11	0.08	0.022	1.364	70.082	42.895	3.510
12	0.06	0.028	0.357	0.076	43.252	3.271

a. Find the three missing numbers a, b, c in the table above.

b. Find the cut-off point C^* if short sales are not allowed.

c. Find the cut-off point C^* if short sales are allowed.

d. Write down the expression in matrix form that computes the variance of the portfolio when short sales are allowed. No calculations.

e. You are given a new stock with $\bar{R} = 0.055, \sigma = 0.025$. What will change when short sales are allowed and when short sales are not allowed in terms of the portfolio allocation. Briefly explain your answer without doing all the calculations.

Problem 2 (25 points)
You are given the following data on 12 stocks:

Stock i	$\bar{R}_i$	σ_i	$\frac{R_i - R_f}{\sigma_i}$	$\frac{\rho}{1-\rho+i\rho}$	$\sum_{j=1}^{i} \frac{R_j - R_f}{\sigma_j}$	C_i
1	0.27	0.031	7.097	0.450	7.097	3.194
2	0.31	0.042	6.190	0.310	13.287	4.124
3	0.16	0.023	4.783	0.237	18.070	4.280
4	0.15	0.021	4.762	0.191	22.832	4.372
5	0.33	0.059	4.746	0.161	27.578	4.432
6	0.27	0.061	3.607	0.138	31.184	4.318
7	0.19	0.039	3.590	0.122	34.774	4.229
8	0.13	0.029	2.759	0.108	37.532	4.070
9	0.16	0.051	2.157	0.098	39.689	3.883
10	0.12	0.038	1.842	0.089	41.531	3.701
11	0.08	0.022	1.364	70.082	42.895	3.510
12	0.06	0.028	0.357	0.076	43.252	3.271

Problem 3 (25 points)
Based on the single index model three stocks have the following:

Stock i	$\bar{R}_i$	$\hat{\beta}_i$	$\sigma^2_{\epsilon_i}$
A	0.010	0.95	0.0044
B	0.013	0.58	0.0040
C	0.019	1.14	0.0051

Assume $\bar{R}_m = 0.011$ and $\sigma^2_m = 0.0019$.

Answer the following questions:

a. Write down the 3×3 variance-covariance matrix of the returns of the three stocks.

b. Assume short sales are allowed and $R_f = 0.005$. Use Microsoft Excel (or other programs) to find the composition, expected return, and standard deviation of the portfolio of the point of tangency (G) using $\boldsymbol{Z} = \boldsymbol{\Sigma}^{-1}\boldsymbol{R}$.

c. Find the mean return and standard deviation of a portfolio that consists of $\frac{1}{3}A$, $\frac{1}{3}B$, $\frac{1}{3}C$ (equal allocation).

d. Find the covariance between the portfolio of part (b) and the portfolio of part (c).

e. Download the file *http://www.stat.ucla.edu/˜nchristo/exam1_abc_with_short_sales.xls*. This file contains 5000 combinations of the stocks A, B, C (short sales are allowed). Use Microsoft Excel (or other programs) to compute the expected return and standard deviation for each one of these combinations. Plot the expected return against the standard deviation and submit a printout of the plot. Assuming $R_f = 0.005$ draw the tangent to the efficient frontier (approximately) and verify that indeed the point of tangency is the point that you found in part (b).

Problem 4 (25 points)
You want to purchase 2 puts and 1 call. The call option costs \$5 and the put option costs \$6. The exercise price for the call or the put is \$50. Use Microsoft Excel (or other programs) to plot the profit against the stock price at the expiration date:

a. For the 2 puts (together).

b. For the call.

c. For the combination of the 2 puts and 1 call.

Problem 2 (25 points)
Answer the following questions:

a. Assume that the variance of security A is 0.16 and the variance of security B is 0.25. The variance of a portfolio consisting of 50%A and 50%B is 0.0525. What is the covariance between securities A and B?

b. Assume $R_f = 0.05$ and two stocks A, B with $\bar{R}_A = 0.14, \bar{R}_B = 0.10$. Suppose the point of tangency to the efficient frontier (the one constructed using the two stocks), consists of 60%A and 40%B. Let's say that you want to build a portfolio by combining the risk free asset and portfolio G to obtain expected return 11%. Determine the percentages of your investment in the risk free asset and in portfolio G.

c. Consider the data from part (b). Suppose you want to build a portfolio with expected return 0.10 by combining the risk free asset and portfolio G. What is the composition of this portfolio in the risk free asset, in A, and in B?

d. Suppose two stocks have the following: $\bar{R}_A = 0.14, \sigma_A = 0.06, \bar{R}_B = 0.08, \sigma_B = 0.03$. What value of ρ_{AB} will force you to invest everything in the least risky asset?

e. Consider the data from part (d). If short sales are allowed, what composition of A and B will minimize the risk of the resulting portfolio if $\rho_{AB} = 0.80$?

Problem 3 (25 points)
Answer the following questions:

a. On the opposite page the graph represents the tangent to the efficient frontier (point G) using 2 stocks A, B which have the following mean returns and standard deviations.

Stock	$\bar{R}$	σ
Stock A	0.05	0.02
Stock B	0.03	0.01

The correlation coefficient between A, B is $\rho_{AB} = 0.20$. Using $R_f = 0.025$ it was found that the composition of the portfolio at the point of tangency is approximately $70\%A$ and $30\%B$.

1. Find the expected return and standard deviation of a combination of 50% in G and 50% in the risk free asset, and place it on the graph.

2. Suppose that initially you have \$10000 invested in G. You borrow another \$5000 at the risk free rate $R_f = 0.025$ to invest it in G. What is the expected return and standard deviation of this combination. Show it on the graph.

b. You are given the following:

Stock	$\bar{R}$	σ
Stock A	0.20	0.10
Stock B	0.15	0.06

The correlation coefficient is ρ_{AB}. What value of ρ_{AB} would result in an optimum portfolio of $\frac{1}{3}A$ and $\frac{2}{3}B$? Assume short sales are allowed and that $R_f = 0.05$.