University of California, Los Angeles
Department of Statistics

Statistics C183/C283

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Name: Solutions

Problem 1 (70 points)
For three stocks you are given the following data based on the single index model:

Stock	$\bar{R}$	β	σ^2_ϵ
A	0.0051	0.94	0.0033
B	0.0120	0.61	0.0038
C	0.0160	1.12	0.0046

Below you are given the solution to the problem (the point of tangency) when short sales are allowed and $R_f = 0.005$.

$$Z = \Sigma^{-1}R = \begin{pmatrix} 0.00489048 & 0.00103212 & 0.00189504 \\ 0.00103212 & 0.00446978 & 0.00122976 \\ 0.00189504 & 0.00122976 & 0.00685792 \end{pmatrix}^{-1} \begin{pmatrix} 0.0051 - 0.005 \\ 0.0120 - 0.005 \\ 0.0160 - 0.005 \end{pmatrix} = \begin{pmatrix} -0.883563202 \\ 1.327096101 \\ 1.610164293 \end{pmatrix}.$$

The sum of the z_i's is $\sum_{i=1}^3 z_i = 2.053697192$ and therefore the x_i's are:
$x_1 = -0.4302, x_2 = 0.6462, x_3 = 0.7840$.

The above is one way to solve the problem. We can also solve the problem by ranking the stocks based on the excess return to beta ratio.

a. Rank the three stocks based on the excess return to beta ratio and complete the table below that will allow you to find the C^*. You will also need $\sigma^2_m = 0.0018$. You can use the last page for extra calculations before you complete the table.

Stock i	$\hat{\beta}_i$	$\bar{R}_i$	$\hat{\sigma^2_{\epsilon i}}$	$\frac{\bar{R}_i - R_f}{\hat{\beta}_i}$	$\frac{(\bar{R}_i - R_f)\hat{\beta}_i}{\hat{\sigma^2_{\epsilon i}}}$	$\sum_{j=1}^i \frac{(\bar{R}_j - R_f)\hat{\beta}_j}{\hat{\sigma^2_{\epsilon j}}}$	$\frac{\hat{\beta}_i^2}{\hat{\sigma^2_{\epsilon i}}}$	$\sum_{j=1}^i \frac{\hat{\beta}_j^2}{\hat{\sigma^2_{\epsilon j}}}$	C_i
B	0.61	0.012	0.0038	0.011475	1.123684	1.123684	97.921053	97.921053	0.0017195
C	1.12	0.016	0.0046	0.009821	2.678261	3.801945	272.695652	370.616705	0.004105
A	0.94	0.0051	0.0033	0.000106	0.028485	3.830430	267.757576	638.374281	0.0032083

b. Assume short sales are allowed. Find C^* and use it to find the composition of the optimum portfolio (point of tangency). Your answer should be exactly the same as above.

$C^* = 0.0032083$

$$Z_B = \frac{0.61}{0.0038}\left[0.011475 - 0.0032038\right] = 1.3270961$$

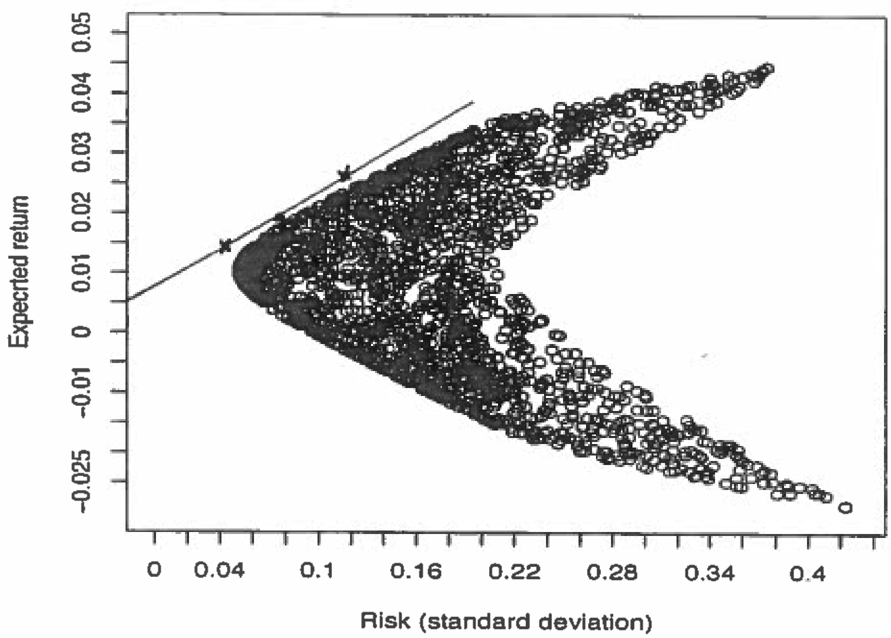
$$Z_C = \frac{1.12}{0.0046}\left[0.009821 - 0.0032038\right] = 1.61016429$$

$$Z_A = \frac{0.94}{0.0033}\left[0.000106 - 0.0032038\right] = -0.8835632$$

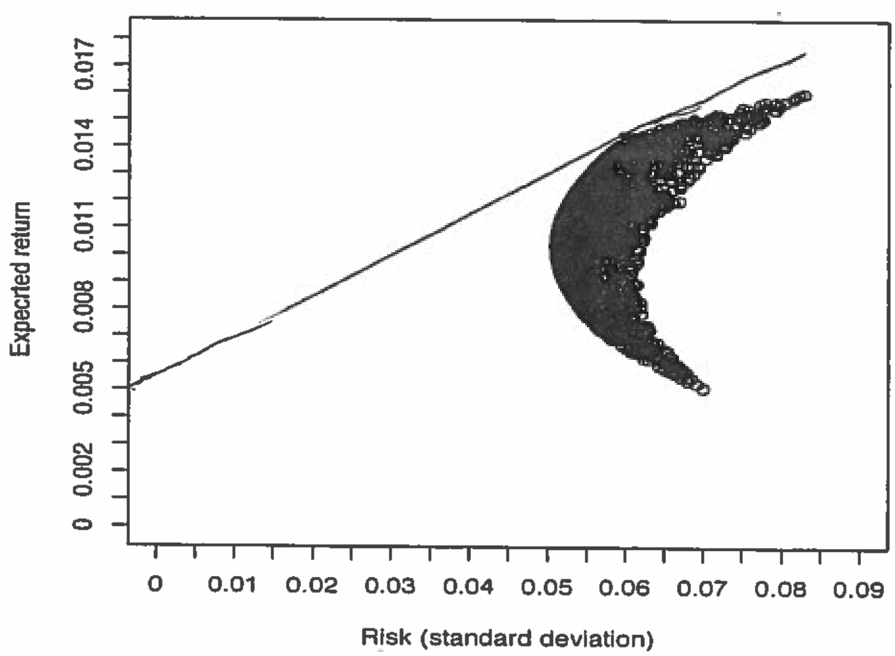
$\sum Z_i = 2.0536972$

$$X_A = \frac{1.3270961}{2.0536972} \Rightarrow \boxed{\begin{array}{l} X_A = 0.6462 \\ X_B = 0.7840 \\ X_C = -0.4302 \end{array}}$$

Short sales allowed:



Short sales not allowed:



c. Assume short sales are not allowed. Find C^* and use it to find the composition of the optimum portfolio.

$$C^* = 0.604105$$

$$Z_B = \frac{0.61}{0.0038}\left[0.011475 - 0.004105\right] = 1.183143$$

$$Z_C = \frac{1.12}{0.0046}\left[0.009821 - 0.004105\right] = 1.391824$$

$$\sum Z_i = 2.574967$$

$$X_A = \frac{1.183143}{2.574967} \Rightarrow \boxed{\begin{array}{l} X_A = 0.45948 \\ X_B = 0.54052 \end{array}}$$

d. Compute the mean return and standard deviation of the portfolios in (b) and (c) and place them (approximately) on the graphs (opposite page). Your answer should be the point of tangency in both cases. Note: The first graph allows short sales, while the second graph does not.

<u>FOR (b)</u>:

$$E\bar{R}_b = x'\bar{R} = 0.018105$$

$$SD(R_b) = \sqrt{x'\Sigma x} = \sqrt{0.006381037} = 0.079881$$

<u>FOR (c)</u>:

$$E\bar{R}_c = x'\bar{R} = 0.0141621$$

$$SD(R_c) = \sqrt{x'\Sigma x} = \sqrt{0.0035581} = 0.059650$$

e. Write down the expression in matrix form that computes the covariance between the portfolio of part (b) and the equally allocated portfolio $(\frac{1}{3}A, \frac{1}{3}B, \frac{1}{3}C)$. No calculations, just the expression!

$$Cov(R_b, R_e) = (X_A, X_B, X_C)\,\Sigma \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$$

f. Consider the portfolio of part (b). Suppose that you want to place 60% of your funds in portfolio (b) and invest the other 40% in the risk free asset. Find the mean return and standard deviation of this new portfolio and show it on the first graph.

$$\bar{R}_p = 0.60\,\bar{R}_b + 0.40\,R_F = 0.60(0.018105) + 0.40(0.005) = 0.012863$$

$$SD(R_p) = \sqrt{0.60^2\,VAR(R_b)} = 0.60(0.079881) = 0.0479$$

g. You have \$2000 to invest in portfolio (b). In addition you borrow another \$1000 to invest in portfolio (b). Show the position of this portfolio on the first graph (approximately). No calculations.

BEYOND THE POINT OF TANGENCY

$$\bar{R}_p = 1.5\,\bar{R}_b - 0.5\,R_F = 1.5(0.018105) - 0.5(0.005) = 0.0246575$$

$$\sigma_p = 1.5^2\,\sigma_b = 1.5^2(0.079881) = 0.1198$$

Problem 2 (30 points)
Using the constant correlation model we completed the table below on 12 stocks. Assume $R_f = 0.05$ and average correlation $\rho = 0.45$.

Stock i	$\bar{R}_i$	σ_i	$\frac{\bar{R}_i - R_f}{\sigma_i}$	$\frac{\rho}{1-\rho+i\rho}$	$\sum_{j=1}^{i} \frac{\bar{R}_j - R_f}{\sigma_j}$	C_i
1	0.27	0.031	7.097	0.450	7.097	3.194
2	0.31	0.042	6.190	0.310	13.287	4.124
3	0.16	0.023	4.783	0.237	18.070	4.280
4	0.15	0.021	4.762	0.191	22.832	4.372
5	0.33	0.059	4.746	$a=?$	$b=?$	$c=?$
6	0.27	0.061	3.607	0.138	31.184	4.318
7	0.19	0.039	3.590	0.122	34.774	4.229
8	0.13	0.029	2.759	0.108	37.532	4.070
9	0.16	0.051	2.157	0.098	39.689	3.883
10	0.12	0.038	1.842	0.089	41.531	3.701
11	0.08	0.022	1.364	70.082	42.895	3.510
12	0.06	0.028	0.357	0.076	43.252	3.271

a. Find the three missing numbers a, b, c in the table above.

$\underline{a}$: $\frac{0.45}{1-0.45+5(0.45)} = 0.161$, $\underline{b}$: $4.746 + 22.832 = 27.578$

c: $(0.161) \times (27.578) = 4.432$

b. Find the cut-off point C^* if short sales are not allowed.

$C^* = 4.432$

c. Find the cut-off point C^* if short sales are allowed.

$c^* = 3.271$

d. Write down the expression in matrix form that computes the variance of the portfolio when short sales are allowed. No calculations.

$$VAR(R_s) = (X_1, \ldots, X_{12}) \Sigma \begin{pmatrix} X_1 \\ \vdots \\ X_{12} \end{pmatrix}$$

e. You are given a new stock with $\bar{R} = 0.055, \sigma = 0.025$. What will change when short sales are allowed and when short sales are not allowed in terms of the portfolio allocation. Briefly explain your answer without doing all the calculations.

$$RATIO = \frac{0.055 - 0.05}{0.025} = 0.2$$

IT IS RANKED LAST.

SHORT SALES NOT ALLOWED : NOTHING CHANGES

SHORT SALES ALLOWED : C^* CHANGES

∴ ALLOCATION CHANGES