

University of California, Los Angeles
Department of Statistics

Statistics C183/C283

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Exam 1
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Name: SOLUTIONS

Please answer the following questions. Show all your work!

1. Consider the following scenario: Short sales allowed and risk free asset exists. For what value of R_f stock 2 will not be held? You are given below the mean returns of five stocks and inverse of the variance covariance matrix of the returns of these five stocks.

Mean returns:

IBM	WFC	JPM	LUV	XOM
0.013	0.014	0.009	0.007	0.004

Inverse of variance covariance matrix:

	IBM	WFC	JPM	LUV	XOM
IBM	512.13	91.18	-152.47	-99.17	-101.79
WFC	91.18	189.27	-160.23	-51.17	-41.30
JPM	-152.47	-160.23	267.73	-5.19	-28.06
LUV	-99.17	-51.17	-5.19	155.59	64.23
XOM	-101.79	-41.30	-28.06	64.23	469.27

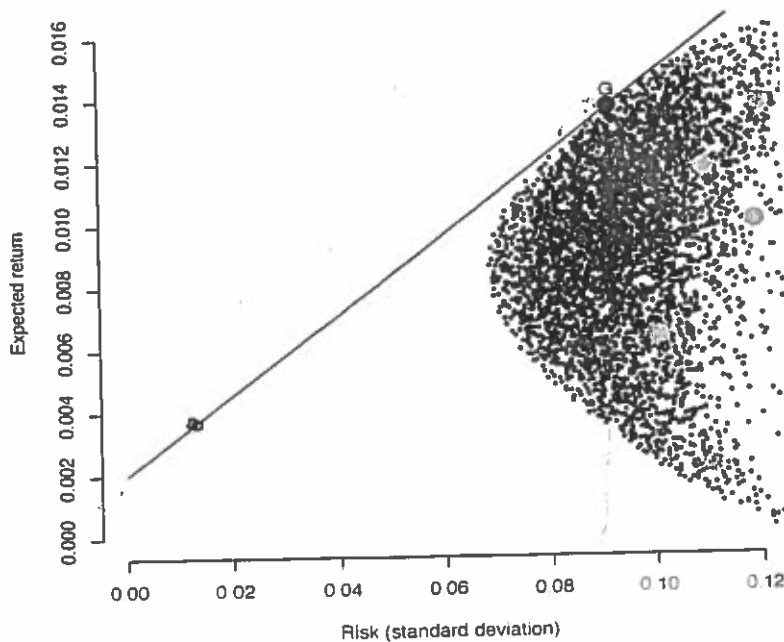
$$Z = \Sigma^{-1} R$$

$$Z_2 = \Sigma V_2, \bar{R}_i - R_F \Sigma V_2,$$

$$Z_2 = 0 \Rightarrow R_F = \frac{\Sigma V_2, \bar{R}_i}{\Sigma V_2}$$

$$= \frac{(91.18)(0.013) + \dots + (-41.30)(0.004)}{91.18 + \dots - 41.30} = 0.067$$

2. Locate on the graph below the portfolio that consists of 10% portfolio G and 90% risk free asset.



$$\sigma_C = 0.10 \sigma_G$$

3. Show that two random variables X and Y cannot possibly have the following properties: $E(X) = 3, E(Y) = 2, E(X^2) = 10, E(Y^2) = 29$, and $E(XY) = 0$.

$$\begin{aligned} \text{var}(X) &= E(X^2) - (E(X))^2 = 10 - 3^2 = 1 \Rightarrow \text{SD}(X) = 1 \\ \text{var}(Y) &= E(Y^2) - (E(Y))^2 = 29 - 2^2 = 25 \Rightarrow \text{SD}(Y) = 5 \\ \rho &= \frac{E(XY) - E(X)E(Y)}{\text{SD}(X)\text{SD}(Y)} = \frac{0 - 3 \cdot 2}{1 \cdot 5} = -1.2 \end{aligned}$$

BUT $-1 \leq \rho \leq 1$

4. Suppose that in a certain stock market, the average variance is 30 and the average covariance is 6. What is the variance of an equally weighted portfolio of 30 stocks?

$$\sigma_p^2 = \frac{\bar{\sigma}_i^2}{n} + \frac{n-1}{n} \bar{\sigma}_{ij} = \frac{30}{30} + \frac{29}{30} \cdot 6 = 1 + \frac{29}{5} = 6.8$$

5. Consider the following scenario: Short sales allowed and $R_f = 0.002$. It was found that the point of tangency has $\bar{R}_G = 0.016$ and $\text{var}(R_G) = 0.009$. Suppose that $z_3 = 0.28$. What percent of the investor's wealth will be placed in stock 3?

$$\begin{aligned} X_3 &= \frac{z_3}{\lambda} \text{ where } \lambda = \frac{\bar{R}_G - R_f}{\sigma_G^2} = \frac{0.016 - 0.002}{0.009} = 1.556 \\ \therefore X_3 &= \frac{0.28}{1.556} = 0.18 \text{ or } 18\% \end{aligned}$$

6. Suppose short sales are allowed. The inverse of the variance covariance matrix of three stocks is given below.

Inverse of variance covariance matrix:

	IBM	AAPL	C
IBM	890.27	-155.98	-144.02
AAPL	-155.98	218.77	-7.78
C	-144.02	-7.78	119.65

What percent of the investor's wealth will be placed in AAPL at the minimum risk portfolio.

$$\frac{-155.98 + 218.77 - 7.78}{547} = 0.0897 \approx 9\%$$

7. Suppose short sales are allowed and risk free asset exists. Show that the z_k value of stock k can be computed using the following expression

$$z_k = \sum_{j=1}^n a_{kj} \bar{R}_j - R_f \sum_{j=1}^n a_{kj},$$

where the a values are elements of the inverse of the variance covariance matrix of the returns of n stocks.

$$\underline{z} = \underline{\Sigma}^{-1} \underline{R} = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ a_{21} & \dots & a_{2n} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{pmatrix} \begin{pmatrix} \bar{R}_1 - R_f \\ \vdots \\ \bar{R}_n - R_f \end{pmatrix}$$

$$z_k = \sum a_{kj} \bar{R}_j - R_f \sum a_{kj}$$

(kth row) $\cdot \underline{R}$

8. Suppose we want to adjust the betas using Blume's technique. Data for the betas of a number of stocks for the period 2003-2007 and for the period 2008-2012 gave the following:

$$\bar{b}_1 = 0.78, sd(b_1) = 0.32, \bar{b}_2 = 0.82, sd(b_2) = 0.60, cov(b_1, b_2) = 0.08.$$

Note: 1 refers to period 2003-2006 and 2 refers to period 2008-2012. For example, \bar{b}_1 is the average of the 50 betas in period 2003-2006. What is the equation that will adjust the betas for the forecasting period 2013-2017?

$$b_2 = \gamma_0 + \gamma_1 b_1 + \epsilon \quad \text{ESTIMATE } \gamma_0, \gamma_1$$

$$\hat{\gamma}_1 = \frac{cov(b_1, b_2)}{var(b_1)} = 0.78 \quad \hat{\gamma}_0 = \bar{b}_2 - \hat{\gamma}_1 \bar{b}_1 = 0.82 - 0.78(0.78) = 0.21$$

$$\hat{b}_2 = 0.21 + 0.78 b_1$$

9. Consider the single index model. When is $\sigma_p^2 \approx \beta_p^2 \sigma_m^2$?

EQUAL ALLOCATION

10. Suppose you work at an investment institution and you are responsible for 50 stocks. If you use the classical Markowitz approach how many inputs you need compute for building portfolios using these 50 stocks.

$$VAR-COVAR: \frac{50 \times 49}{2} \quad 50 \bar{R}_i$$

1325

11. Refer to question (10). If instead you use the single index model, how many inputs you need to compute now?

$$\left. \begin{array}{l} \alpha: 50 \\ \beta: 50 \\ \sigma_{\epsilon_i}^2: 50 \\ \bar{R}_m, \sigma_m^2: 2 \end{array} \right\} 152$$

12. Consider the single index model and assume you have a portfolio that consists of two stocks (1 and 2). Find an expression of the covariance between the return of the portfolio and the return of stock 1, $cov(R_p, R_1)$.

$$cov(R_1, R_p) = cov(x_1 + b_1 \bar{R}_m + \epsilon_1, \text{EXACT EXPLICIT } \sum x_i \epsilon_i)$$

$$= b_1 b_p \sigma_m^2$$

13. Based on the single index model write the entries of the variance covariance matrix of three stocks.

$$\begin{pmatrix} b_1^2 \sigma_m^2 + \sigma_{\epsilon_1}^2 & b_1 b_2 \sigma_m^2 & b_1 b_3 \sigma_m^2 \\ b_2 b_1 \sigma_m^2 & b_2^2 \sigma_m^2 + \sigma_{\epsilon_2}^2 & b_2 b_3 \sigma_m^2 \\ b_3 b_1 \sigma_m^2 & b_3 b_2 \sigma_m^2 & b_3^2 \sigma_m^2 + \sigma_{\epsilon_3}^2 \end{pmatrix}$$