University of California, Los Angeles
Department of Statistics

Statistics C183/C283 Instructor: Nicolas Christou

Exam 1
07 May 2010

$S_e^2 = \frac{\sum e_i^2}{n-2}$ (MSE)

Name: SOLUTIONS

$\hat{\sigma}_{\epsilon_i}^2$ S_e^2

Problem 1 (20 points)
The betas for 10 stocks in two historical periods 2000-2004 and 2005-2009 are as follows:

```
        beta1     beta2
 [1,] 0.9072828 0.7333601
 [2,] 1.0874136 1.0096048
 [3,] 0.9871119 1.1143148
 [4,] 1.0084073 1.1011334
 [5,] 0.7606293 0.7711888
 [6,] 0.8047901 0.7834646
 [7,] 0.9533157 0.9914738
 [8,] 0.8036708 1.1083840
 [9,] 1.0867607 0.9524100
[10,] 1.0315184 0.8759303
```

$VAR(\hat{\beta}) = \frac{\sigma^2}{\sum (R_{mt} - \bar{R}_m)^2}$

a. Explain how you can obtain an estimate for the beta of stock 8 for the period 2010-2014 using the Blume's technique.

REGRESS BETA2 ON BETA1. GET THE FITTED LINE AND USE IT TO PREDICT β_8 FOR 2010-2014

$\boxed{\hat{\beta}_8 = \hat{\gamma}_0 + \hat{\gamma}_1 \, BETA2}$ $\hat{\gamma}_0, \hat{\gamma}_1$ FROM THE REGRESSION OF BETA2 ON BETA1.

b. Suppose that for the second period 2005-2009 the variance of the return of the S&P500 index is $\sigma_m^2 = 0.00217$. Assume that the single index model holds. Find the covariance between stocks 1 and 3 during the same period.

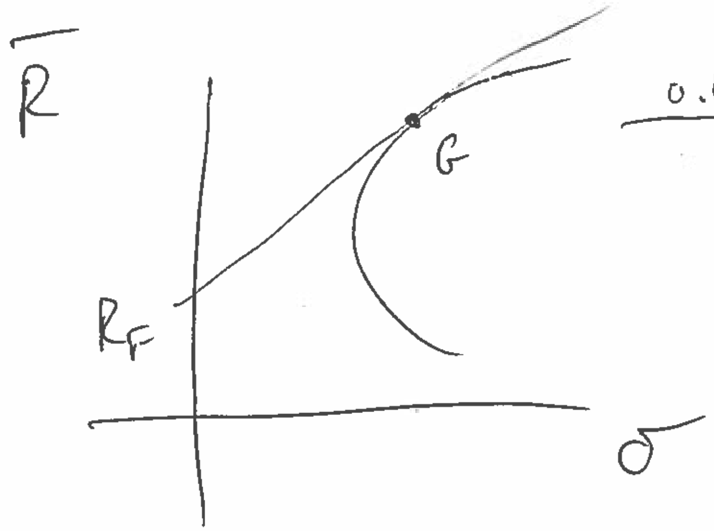
$\sigma_{13} = \beta_1 \beta_3 \sigma_M^2 = (0.733)(1.114)0.00217 \Rightarrow \boxed{\sigma_{13} = 0.00177}$

c. Explain how you can obtain an estimate for the beta of stock 8 for the period 2010-2014 using the Vacicek's technique.

$\frac{VAR(BETA2)}{VAR(BETA2) + VAR(\hat{\beta}_8)} 1.108 + \frac{VAR(\hat{\beta}_8)}{VAR(BETA2) + VAR(\hat{\beta}_8)} \cdot \overline{BETA2}$

d. Suppose that the correlation coefficient between stock A and S&P500 during the period 2005-2009 is 0.20. The variance of the return of stock A during the same period is 0.0143 and the variance of the return of the S&P500 index was $\sigma_m^2 = 0.00217$. Find the beta of stock A.

$\boxed{\hat{\beta}_A = \rho_{AM} \frac{\sigma_A}{\sigma_M}} = 0.2 \frac{\sqrt{0.0143}}{\sqrt{0.00217}} \Rightarrow \boxed{\hat{\beta}_A = 0.513}$



$\frac{0.1}{}$

$0.1 \quad \frac{0.08}{0.2}$

$\frac{0.1}{1-0.1+0.1}$

0.1

$$\bar{R}_P = R_F + \left(\frac{\bar{R}_G - R_F}{\sigma_G}\right)\sigma_P$$

$$C^*\left(\frac{1}{0.08} - \frac{1}{0.2}\right)$$

$\bar{R}_G =$

$0.144 \neq 2$

2.144

$$Z_A = \frac{1}{0.9(0.2)}\left(\frac{0.08}{0.20} - C^*\right)$$

$$Z_B = \frac{1}{0.9(0.08)}\left(\frac{\bar{R}_B - 0.04}{0.08} - C^*\right)$$

$$\frac{1}{\cancel{0.9}(0.2)}\left[\frac{0.08}{0.20} - C^*\right] = \frac{1}{\cancel{0.9}(0.08)}\left(\frac{\bar{R}_B - 0.04}{0.08} - C^*\right)$$

$$\frac{0.08}{0.04} - \frac{C^*}{0.2} = \frac{\bar{R}_B - 0.04}{0.0064} - \frac{C^*}{0.08}$$

Problem 2 **(20 points)**
Use the following for questions (a) and (b) below:

$$\begin{pmatrix} 0.04 & 0.0016 \\ 0.0016 & 0.0064 \end{pmatrix}\begin{pmatrix} Z_A \\ Z_B \end{pmatrix} = \begin{bmatrix} 0.12-0.04 \\ \bar{R}_B - 0.04 \end{bmatrix}$$

Stock	$\bar{R}$	σ
A	0.12	0.20
B	???	0.08

$$\begin{pmatrix} Z_A \\ Z_B \end{pmatrix} = \begin{bmatrix} 25.25 & -6.31 \\ -6.31 & 157.828 \end{bmatrix}\begin{pmatrix} 0.08 \\ \bar{R}_B - 0.04 \end{pmatrix}$$

It is also given that $\rho_{AB} = 0.1$.

a. What expected return on stock B would result in an optimum portfolio of $\frac{1}{2}A$ and $\frac{1}{2}B$? Assume short sales are allowed and that $R_f = 0.04$.

since $X_A = X_B = \frac{1}{2} \Rightarrow Z_A = Z_B$

$$\left.\begin{aligned} 0.12 - 0.04 &= 0.04 Z_A + 0.0016 Z_B \\ \bar{R}_B - 0.04 &= 0.0016 Z_A + 0.0064 Z_B \end{aligned}\right\}$$

$0.08 = 0.0416 Z_A \Rightarrow Z_A = \frac{0.08}{0.0416} \Rightarrow Z_A = 1.9231$

$\therefore \bar{R}_B = 0.04 + (0.0016 + 0.0064)\, 1.9231 \Rightarrow \boxed{\bar{R}_B = 0.055385}$

b. What expected return on stock B would mean that stock B would not be held? Assume short sales are allowed and that $R_f = 0.04$.

B WILL NOT BE HELD $\Rightarrow X_B = 0 \Rightarrow Z_B = 0$

$$\left.\begin{aligned} 0.12 - 0.04 &= 0.04 Z_A + 0.0016 Z_B \\ \bar{R}_B - 0.04 &= 0.0016 Z_A + 0.0064 Z_B \end{aligned}\right\} \quad \text{BUT } Z_B = 0$$

$Z_A = \frac{0.08}{0.04} \Rightarrow Z_A = 2$

$\bar{R}_B = 0.04 + 0.0016(2) \Rightarrow \boxed{\bar{R}_B = 0.0432}$

c. Suppose X and Y represent the returns of two stocks. Show that these two random variables X and Y cannot possibly have the following properties: $E(X) = 0.3, E(Y) = 0.2, E(X^2) = 0.1, E(Y^2) = 0.29$, and $E(XY) = 0$. Reminder: $\sigma_{XY} = E(X - \mu_X)(Y - \mu_Y) = EXY - (EX)(EY)$.

$VAR(X) = 0.1 - 0.3^2 = 0.01 \Rightarrow SD(X) = 0.1$

$VAR(Y) = 0.29 - 0.2^2 = 0.25 \Rightarrow SD(Y) = 0.5$

$Cov(X,Y) = EXY - (EX)(EY) = 0 - (0.3)(0.2) \Rightarrow \sigma_{XY} = 0.06$

$$\rho = \frac{-0.06}{0.1 \quad 0.5} = -1.2$$

$-1 \le \rho \le 1$

Data for problem 3:

```
#Create the ticker vector for the two stocks plus the S&P500:
> ticker <- c("ibm", "xom", "^GSPC")
> data <- getReturns(ticker, start="2005-01-31", end="2009-12-31")

#Get the summary statistics:
> summary(data$R)
      ibm                  xom                  ^GSPC
 Min.   :-0.205144   Min.   :-0.116543   Min.   :-0.1694245
 1st Qu.:-0.013633   1st Qu.:-0.028956   1st Qu.:-0.0184670
 Median : 0.009613   Median : 0.003498   Median : 0.0099800
 Mean   : 0.009026   Mean   : 0.008107   Mean   : 0.0001331
 3rd Qu.: 0.050789   3rd Qu.: 0.045487   3rd Qu.: 0.0277094
 Max.   : 0.129405   Max.   : 0.233054   Max.   : 0.0939251

#Get the variance covariance matrix:
> cov(data$R)
                ibm          xom        ^GSPC
ibm    0.0039985797 0.0004087865 0.0017214346
xom    0.0004087865 0.0035546519 0.0009878328
^GSPC  0.0017214346 0.0009878328 0.0021726295

#Run the regression of the returns of IBM on the returns of S&P500
#and obtain alpha, beta, mse:
> reg1 <- lm(data$R[,1] ~ data$R[,3])
> summary(reg1)$coef[1]
```

$\longrightarrow \alpha$

```
[1] 0.008920891
> summary(reg1)$coef[2]
```

$\longrightarrow \beta$

```
[1] 0.7923277
> summary(reg1)$sigma^2
```

$\longrightarrow \sigma_\varepsilon^2$

```
[1] 0.002680861

#Run the regression of the returns of EXXON-MOBIL on the returns of S&P500
#and obtain alpha, beta, mse:
> reg2 <- lm(data$R[,2] ~ dara$R[,3])
> summary(reg2)$coef[1]
[1] 0.008046374
> summary(reg2)$coef[2]
[1] 0.4546716
> summary(reg2)$sigma^2
[1] 0.003159995
```

Problem 3 (20 points)

Using the package `stockPortfolio` we have obtained the returns of IBM, Exxon-Mobil, and the S&P500 index for the period 2005-01-31 to 2009-12-31. The summary statistics, variance-covariance matrix of the returns, and the regressions of the returns of IBM and Exxon-Mobil on the index are shown on the previous page.

a. Using the single index model compute the variance of the returns of IBM.

$$\sigma^2_{IBM} = \beta^2_{IBM}\sigma^2_m + \sigma^2_\epsilon = 0.792^2(0.00217) + 0.00268 \Rightarrow \boxed{\sigma^2_{IBM} = 0.004}$$

Also $\sigma^2_{XOM} = 0.00361$

b. What is the beta of a portfolio that consists of 80% IBM and 20% Exxon-Mobil?

$$\beta_p = \sum x_i\beta_i = 0.8(0.792) + 0.2(0.455) \Rightarrow \boxed{\beta_p = 0.7246}$$

c. Using the historical variance-covariance matrix of the returns and assuming $R_f = 0.008$ we get the following:

```
> z
             [,1]
ibm 0.256622417
xom 0.000559498
```

1. Explain how these z values were computed. No calculations, but please be very specific!

$$\underset{\sim}{Z} = \underset{\sim}{\Sigma}^{-1}\underset{\sim}{R} \qquad \Sigma \text{ 2x2 VAR-COVAR MATRIX} \qquad R = \begin{pmatrix} \bar{R}_1 - R_F \\ \bar{R}_2 - R_F \end{pmatrix}$$

2. Compute the proportion of the investor's wealth that goes into each stock.

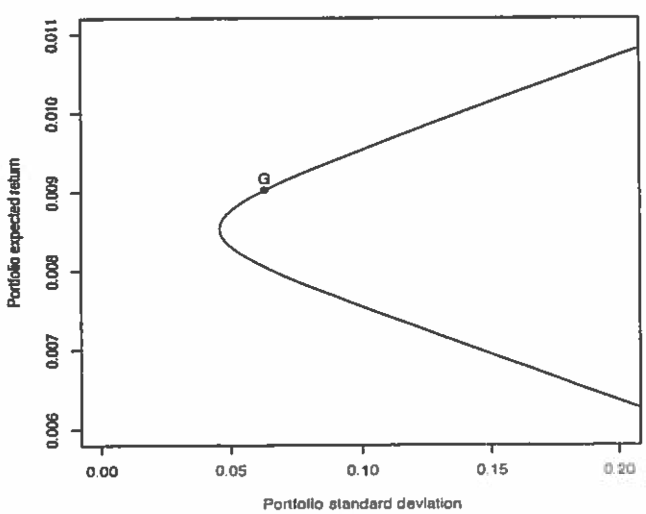
$$x_{IBM} = \frac{0.25622}{0.256622 + 0.000559} \Rightarrow \boxed{\begin{array}{l} x_{IBM} = 0.9978 \\ x_{XOM} = 0.0022 \end{array}}$$

3. Compute the expected return and standard deviation of the point of tangency. This point should be point G on the graph below.

$$\bar{R}_G = 0.9978(0.009026) + 0.0022(0.008107) \Rightarrow \boxed{\bar{R}_G = 0.00902}$$

$$\sigma_G = \left[0.9978^2(0.004) + 0.0022^2(0.00361) + 2(0.9978)(0.0022)(0.000781)\right]^{1/2} \Rightarrow \boxed{\sigma_G = 0.0631}$$

$$\sigma_{IBM,XOM} = 0.7923 \times 0.45467 \times 0.00217 = 0.000781$$



Problem 4 (20 points)
Using the single index model three stocks x, y, z were ranked based on the excess return to beta ratio as follows:

Stock i	$\frac{\bar{R}_i - R_f}{\beta_i}$	$\frac{(\bar{R}_i - R_f)\beta_i}{\sigma^2_{\epsilon i}}$	$\sum_{j=1}^{i} \frac{(\bar{R}_j - R_f)\beta_j}{\sigma^2_{\epsilon j}}$	$\frac{\beta_i^2}{\sigma^2_{\epsilon i}}$	$\sum_{j=1}^{i} \frac{\beta_j^2}{\sigma^2_{\epsilon j}}$	C_i
y	0.0395	13.7170	13.7170	342.2115	347.2115	C_1
x	0.0080	4.3307	18.0477	538.4952	885.7067	C_2
z	0.0067	1.9733	20.0210	294.4627	1180.1694	C_3

Assume $R_f = 2\%$ and that the variance of the returns of the market is $\sigma^2_m = 0.0023$.

a. Find C_1, C_2, C_3.

$$C_1 = \frac{0.0023 \times 13.7170}{1 + 0.0023 \times 347.2115} \Rightarrow C_1 = 0.01754$$

$$C_2 = \frac{0.0023 \times 18.0477}{1 + 0.0023 \times 885.7067} \Rightarrow C_2 = 0.01367$$

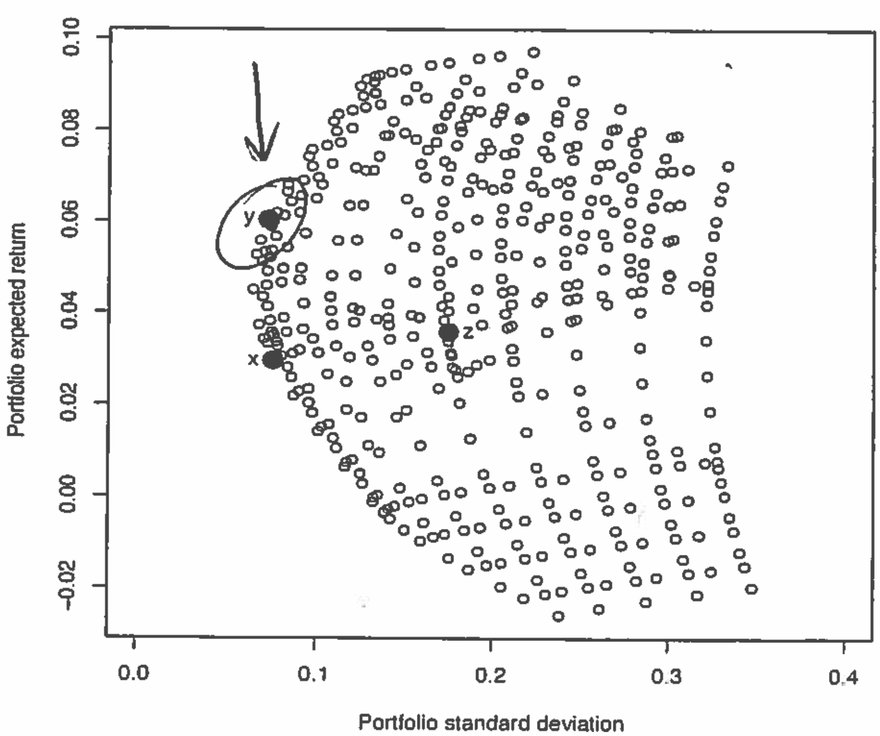
$$C_3 = \frac{0.0023 \times 20.0210}{1 + 0.0023 \times 1180.1694} \Rightarrow C_3 = 0.01240$$

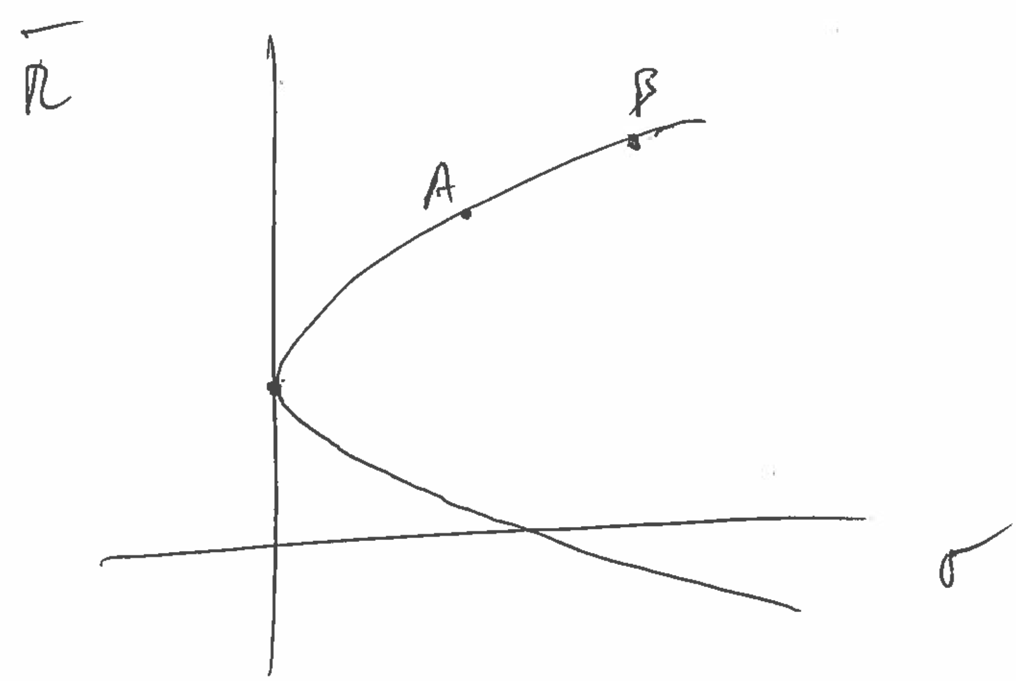
b. What is the composition of the optimal portfolio when short sales are not allowed?

$C^* = C_1 = 0.01754$

ONLY STOCK 1. $\Rightarrow$ 100% IN STOCK Y.

c. Show the optimal portfolio of part (b) on the graph below. On this graph x, y, z are the three stocks.





$\bar{R}$
A
B
σ

Problem 5 (20 points)

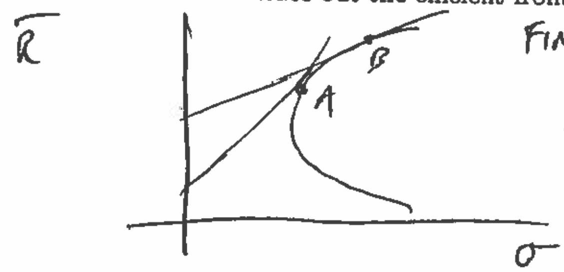
Part A:

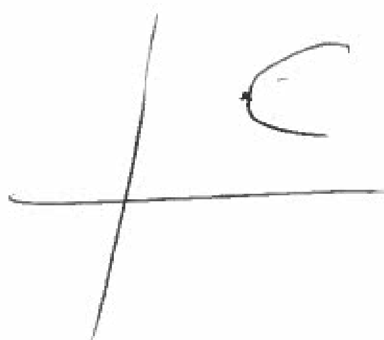
A portfolio manager wants to present to his clients the efficient frontier using 25 stocks. Explain clearly and in detail how you would help this portfolio manager to trace out the efficient frontier when short sales are allowed but no riskless lending and borrowing exists. You must show a graph, the inputs you are using, the vectors and matrices you are multiplying, etc. One should be able to follow step by step your procedure and be able to trace out the efficient frontier.

ASSUME TWO RF VALUES TO FIND: $\bar{R}_A, \sigma_A, \bar{R}_B, \sigma_B, \sigma_{AB}$.

TREAT A, B AS TWO "STOCKS" AND USING MANY COMBINATIONS, $X_A + X_B = 1$ ALLOWING SHORT SALES

TRACE OUT THE PORTFOLIO POSSIBILITIES CURVE AND THE EFFICIENT FRONTIER





$\frac{2}{100}$ $\frac{2}{1}$ $\frac{0.02}{0.01 \quad 0.2}$

Part B:

Suppose short sales are allowed and three stocks X, Y, Z are used to construct the efficient frontier. Let A and B be two portfolios on the efficient frontier with: $\bar{R}_A = 0.006, \sigma_A = 0.1, \bar{R}_B = 0.01, \sigma_B = 0.2$ and $\sigma_{AB} = 0.02$. The composition of portfolio A is $0.53X, -0.50Y, 0.97Z$. The composition of portfolio B is $0.53X, -1.80Y, 2.27Z$. Find the composition of the minimum risk portfolio in terms of the two portfolios and in terms of the three stocks X, Y, Z. On the previous page draw the graph of the expected return against standard deviation and show approximately the portfolio possibilities curve, identify the efficient frontier, and place the two portfolios A, B, and the minimum risk portfolio on the graph.

IN TERMS OF A, B:

$$X_A = \frac{\sigma_B^2 - \sigma_{AB}}{\sigma_A^2 + \sigma_B^2 - 2\sigma_{AB}} = \frac{0.2^2 - 0.02}{0.1^2 + 0.2^2 - 2(0.02)} \Rightarrow \boxed{\begin{matrix} X_A = 2.0 \\ X_B = -1 \end{matrix}}$$

IN TERMS OF X, Y, Z:

$$X: \ 2(0.53) - 1(0.53) \Rightarrow 0.53 \text{ IN } X$$

$$Y: \ 2(-0.50) - 1(-1.80) \Rightarrow 0.80 \text{ IN } Y$$

$$Z: \ 2(0.97) - 1(2.27) \Rightarrow -0.37 \text{ IN } Z$$

$$\bar{R} = 2(0.006) - 1(0.01) \Rightarrow \bar{R} = 0.002$$

$$\sigma^2 = 2^2(0.1)^2 + 1^2 0.2^2 - 2 \cdot 2 \cdot 1 \cdot 0.02$$

$$\sigma^2 = 0$$