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Department of Statistics

Statistics C183/C283 Instructor: Nicolas Christou

Exam 2
16 May 2016

Name: SOLUTIONS UCLA ID: ____________

Answer the following questions:

a. Consider the constant correlation model with n stocks. You are given the historical variance covariance matrix Σ, the expected return vector $\bar{\mathbf{R}}$, the optimal portfolio weights vector $\mathbf{x}$, and the risk free rate R_f. Explain how to find the cut-off rate C^* when short sales are allowed. Show all your work.

$$C^* = \rho \sum z_j \sigma_j = \rho \lambda \sum x_j \sigma_j$$

$$\lambda = \frac{\bar{R}_p - R_f}{\sigma_p^2} \qquad \mathbf{x} \text{ is given}$$

OR

$$C^* = \frac{\rho}{1-\rho+n\rho} \sum_{i=1}^{n} \frac{\bar{R}_i - R_f}{\sigma_i}$$

b. Consider the single index model with $n = 3$ stocks. The following table was constructed in order to find the optimal portfolio (point of tangency) when short sale are not allowed and risk free interest rate $R_f = 0.001$. Please find the missing numbers.

	stock	alpha	beta	Rbar	mse	Ratio
[1,]	3	-0.007134964	1.5595751	0.006622948	0.011186323	0.003605436
[2,]	1	-0.002152648	0.5571742	0.002762508	0.005168777	0.003163297
[3,]	2	-0.005787947	1.0622912	0.003583136	0.007098190	??????????? → 0.002431665

col1	col2	col3	col4	col5	z_noshort	x_noshort
?????????	0.7839403	217.43290	217.4329	0.001138047	0.2937377	0.4793076
0.1899915	0.9739319	60.06124	????????	0.001300469	0.1794532	0.2928234
0.3865835	1.3605154	158.97892	436.4731	0.001498551	?????????	0.2278690

0.7839403

$$\frac{(\bar{R} - R_f)\beta_i}{\sigma_{\epsilon_i}^2}$$

277.

0.1396466

$$\lambda = \frac{0.2937377}{0.4793076}$$

$$z_3 = x_3 \lambda$$

$$\frac{0.2937377}{0.29 + 0.179 + ?} = 0.479$$

3 industries
2 stocks / ind

(c). $\bar{R}_1 - R_f$

$$= z_1\left[\beta_1^2\left(b_1^2\sigma_m^2+\sigma_{c_1}^2\right)+\sigma_{\epsilon_1}^2\right)$$

$$+ z_2\left(\beta_1\beta_2\left[b_1^2\sigma_m^2+\sigma_{c_1}^2\right]\right)$$

$$+ z_3\beta_1\beta_3 b_1 b_2\sigma_m^2 + z_4\beta_1\beta_4 b_1 b_2\sigma_m^2$$

$$+ z_5\beta_1\beta_5 b_1 b_3\sigma_m^2 + z_6\beta_1\beta_6 b_1 b_3\sigma_m^2$$

(d) $M(3,3) =$

$$1+\left(\sigma_{c_3}^2+b_3^2\sigma_m^2\right)\sum_{i\in c}\frac{\beta_i^2}{\sigma_{\epsilon_i}^2}$$

(e) Multigroup 3×3

Z_3

1
2
3
4
5
6
7
6
5

$$Z_3 = \frac{1}{\sigma_3(1-\rho_{11})}\left[\frac{\bar{R}_3-R_f}{\sigma_3} - \sum_{g=1}^{3}\rho_{1g}\Phi_g\right]$$

ρ_{11} = Avg corr. in industry 1

ρ_{12}, ρ_{13} avg cross- corr

$\Phi_1 = Z_1\sigma_1 + Z_2\sigma_2$

$\Phi_2 = Z_3\sigma_3 + Z_4\sigma_4$

$\Phi_3 = Z_5\sigma_5 + Z_6\sigma_6$

(f) Treynor for portfolio A:

$$\frac{\bar{R}_A - R_f}{\beta_A} = \frac{0.16 - 0.08}{1.2} = 0.067$$

Sharpe for portfolio B:

$$\frac{\bar{R}_B - R_f}{\sigma_B} = \frac{0.22 - 0.08}{0.16} = 0.875$$

(g) $\bar{R}_p = R_f + (\bar{R}_m - R_f)\beta_p$

$R - \bar{R}'$

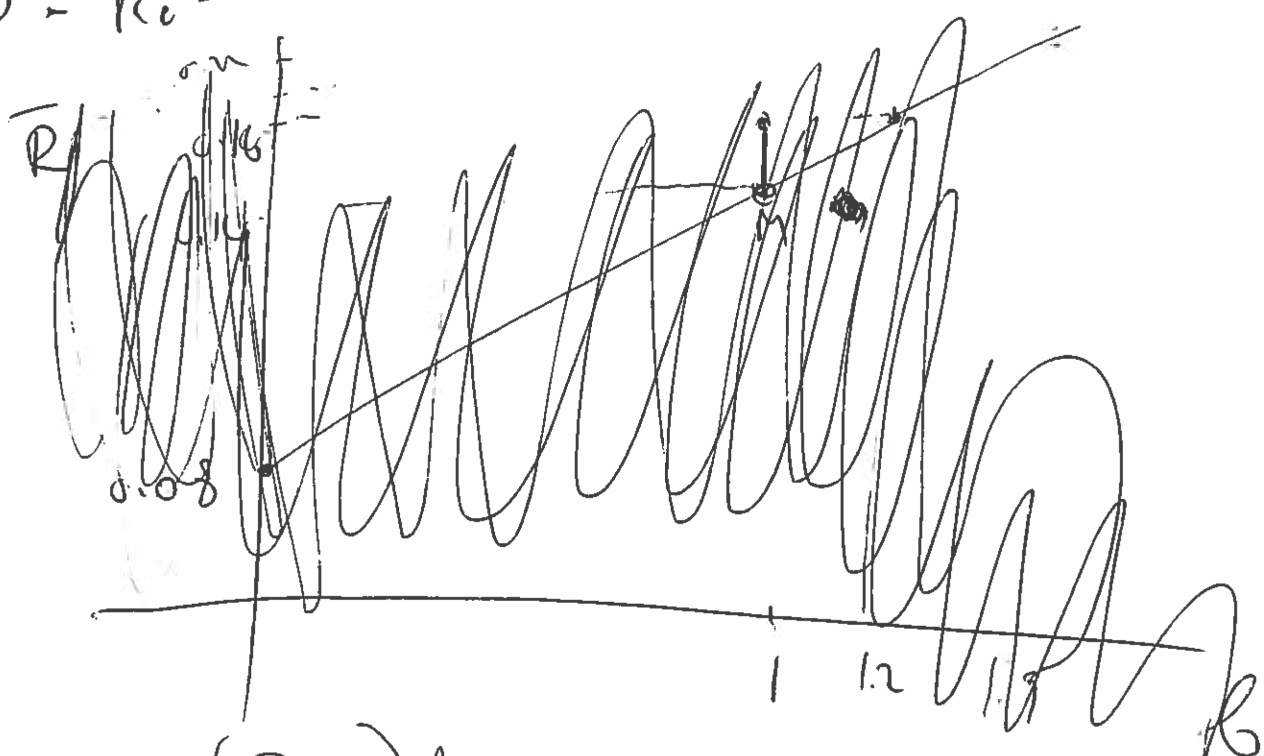
A: $\bar{R}_i = 0.08 + (0.14 - 0.08)1.2 = 0.152$ $\quad 0.16 - 0.152 = 0.00$

~~[illegible]~~

B: $\bar{R}_i = 0.08 + (0.14 - 0.08)1.9 = 0.194$ $\quad 0.22 - 0.194 = 0.026$

C: $\bar{R}_i = 0.08 + (0.14 - 0.08)0.8 = 0.128$ $\quad 0.10 - 0.128 = -0.028$

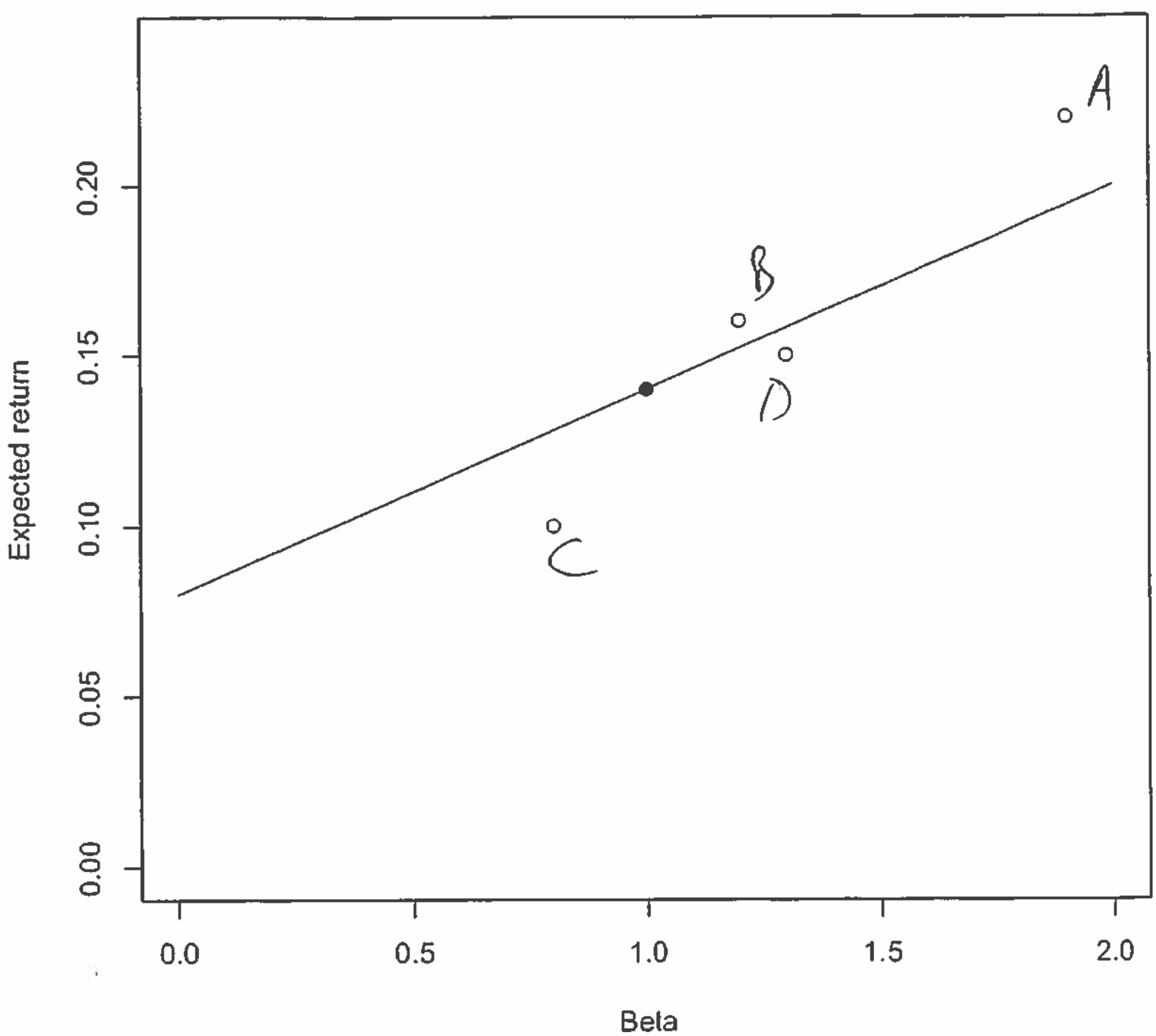
D: $\bar{R}_i = 0.08 + (0.14 - 0.08)1.3 = 0.158$ $\quad 0.15 - 0.158 = -0.008$



$\bar{R}_i = R_f + (\bar{R}_p - R_f)\beta_i$

$0.08 + (0.14 - 0.08)$

(g)



(h) $P + S_0 = C + \frac{E}{1+r}$

$5 + \cancel{1}110 \overset{?}{=} 17 + \frac{105}{1+0.05}$

$115 \neq 117$

Strategy: Sell call
(Borrow 115) to Buy $P + S_0$

$$\begin{array}{r} 117 \\ -115 \\ \hline +\ 2 \end{array}$$

Need to return ~~102.5~~ 102.9

$S_1 > 105$ Sell at 105
$S_1 < 105$ Sell at 105

in Both ~~~~ CASES

$105 - 102.9 = 2.1$ profit

(i)

	Long Stock	Short Call, $E = 50$	Short call $E = 60$	Total
$S_1 < 50$	S_1	0	0	S_1
$50 < S_1 < 60$	S_1	$50 - S_1$	0	50
$S_1 > 60$	S_1	$50 - S_1$	$60 - S_1$	$110 - S_1$
$S_1 > 110$	S_1	$50 - S_1$	$60 - S_1$	0

(j).

$$P + S = C + \frac{E}{1+r}$$

At the money $\rightarrow$ $E = S$

$$P + E = C + \frac{E}{1+r}$$

$$C = P + E - \frac{E}{1+r}$$

$$C > P$$
