University of California, Los Angeles
Department of Statistics

Statistics C183/C283 **Instructor: Nicolas Christou**

Exam 2
10 May 2006

Name: _____

Problem 1 (20 points)
Answer the following questions:

a. Suppose the single index model holds. Show that the cut-off point C^* can be written as:

$$C^* = (\bar{R}_p - R_f)\beta_p \frac{\sigma_m^2}{\sigma_p^2}.$$

b. Assume the constant correlation model holds and that short sales are allowed. Under what condition the cut-off point C^* is approximately equal to:

$$C^* \approx \frac{1}{N}\sum_{i=i}^{N} \frac{\bar{R}_i - R_f}{\sigma_i}$$

Problem 2 (25 points)
You are given the following data:

Stock i	R_i	σ_i
1	0.29	0.03
2	0.19	0.02
3	0.08	0.15

a. Assume short sales are allowed, $R_f = 0.05$, and $\bar{\rho} = 0.5$. Rank the stocks based on the excess return to standard deviation ratio, find the cut-off point C^*, and find the optimum portfolio.

b. The above solution could have been found using the techniques that discussed earlier in class through the following:

$$\boldsymbol{Z} = \boldsymbol{\Sigma^{-1}R} = \begin{pmatrix} 0.00090 & 0.00030 & 0.00225 \\ 0.00030 & 0.00040 & 0.00150 \\ 0.00225 & 0.00150 & 0.02250 \end{pmatrix}^{-1} \begin{pmatrix} 0.29 - 0.05 \\ 0.19 - 0.05 \\ 0.08 - 0.05 \end{pmatrix} = \begin{pmatrix} 280.00 \\ 320.00 \\ -48.00 \end{pmatrix}.$$

Explain what you see here and verify that the solution is the same as with part (a).

Problem 3 (30 points)
Suppose the single index model holds. Also short sales are allowed and there is a risk free rate $R_f = 0.002$. For 3 stocks the following were obtained based on monthly returns for a period of 5 years:

Stock i	α	β	σ_ϵ^2
1	0.01	1.08	0.003
2	0.04	0.80	0.006
3	0.08	1.22	0.001

The expected return and variance of the market are $\bar{R}_m = 0.10$ and $\sigma_m^2 = 0.002$ for the same period.

a. Suppose that the optimum portfolio consists of 30% of stock 1, 50% of stock 2, and 20% of stock 3. What is the β of this portfolio.

b. Suppose that you are a portfolio manager and you have \$500000 to invest in this optimum portfolio on behalf of a client. In addition this client wants to invest another \$300000 by borrowing this amount at the risk free rate $R_f = 0.002$. What is the expected return and standard deviation of this portfolio. Show it on the expected return standard deviation space.

c. What is the covariance between the portfolio of part (a) and the market?

d. If the client wants to allocate 60% of his initial funds in the optimum portfolio and the remaining 40% in the risk free asset, what would be the expected return and standard deviation of this position?

e. What is the covariance between stock 1 and the market?

Problem 4 (25 points)
Assume that $\sigma_m^2 = 10, R_f = 0.05$. You are also given $\beta_1 = 1, \beta_2 = 1.5, \beta_3 = 1, \beta_4 = 2, \beta_5 = 1, \beta_6 = 1.5, \beta_7 = 2, \beta_8 = 0.8, \beta_9 = 1, \beta_{10} = 0.6$. The table below shoes the procedure for finding the cut-off point C^*.

Stock i	$\frac{\bar{R}_i - R_f}{\beta_i}$	$\frac{(\bar{R}_i - R_f)\beta_i}{\sigma_{\epsilon i}^2}$	$\sum_{j=1}^{i} \frac{(\bar{R}_j - R_f)\beta_j}{\sigma_{\epsilon j}^2}$	$\frac{\beta_i^2}{\sigma_{\epsilon i}^2}$	$\sum_{j=1}^{i} \frac{\beta_j^2}{\sigma_{\epsilon j}^2}$	C_i
1	10.0	0.20	(a)	0.02000	0.02000	1.67
2	8.0	0.45	0.65	0.05625	0.07625	3.69
3	7.0	0.35	1.00	0.05000	0.12625	4.42
4	6.0	2.40	3.40	0.40000	0.52625	5.43
5	6.0	0.15	3.55	0.02500	0.55125	(c)
6	4.0	0.30	3.85	0.07500	0.62625	5.30
7	3.0	0.30	4.15	0.10000	(b)	5.02
8	2.5	0.10	4.25	0.04000	0.76625	4.91
9	2.0	0.10	4.35	0.05000	0.81625	4.75
10	1.0	0.06	4.41	0.06000	0.87625	4.52

a. Find the three missing values, (a), (b), (c) in the table above.

b. If short sales are not allowed find the cut-off point C^* and the value of z_1.

c. If short sales are allowed find the cut-off point C^* and the value of z_1.

d. Find the correlation coefficient between stock 1 and the market.