University of California, Los Angeles
Department of Statistics

Statistics C183/C283 **Instructor: Nicolas Christou**

Homework 3

Exercise 1:
Read the data and run the regression of the returns of each stock on the returns of the market.
Note: The data below are already returns. The data used in homework 3 represent prices for 10 stocks and therefore needed to be converted into returns, but they are from the same stocks and period as with the data below.

```
data1 <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c183_c283/
         stocks_10.txt", header=TRUE)

b <- as.matrix(data1)

#Initialize the vectors and matrices:
x <- rep(0, 50)
xx <- matrix(x, ncol=10, nrow=5)

alpha <- rep(0,10)
beta <- rep(0,10)
mse <- rep(0,10)
mse1 <- rep(0,10)
Rbar <- rep(0,10)
var <- rep(0,10)
q1 <- rep(0,10)
beta_adj <- rep(0,10)
Ratio <- rep(0,10)

col1 <- rep(0,10)
col2 <- rep(0,10)
col3 <- rep(0,10)
col4 <- rep(0,10)
col5 <- rep(0,10)

#Perform regression of each stock on the index and record alpha, beta, Rbar, var, mse, Ratio:
for(i in 1:10){
alpha[i] <- lm(data=data1,formula=data1[,i] ~ data1[,11])$coefficients[1]
beta[i] <- lm(data=data1,formula=data1[,i] ~ data1[,11])$coefficients[2]
Rbar[i] <- alpha[i]+beta[i]*mean(b[,11])
mse1[i] <- summary(lm(data=data1, formula=data1[,i] ~ data1[,11]))$sigma^2
var[i] <- beta[i]^2*var(data1[,11]) + mse[i]
Ratio[i] <- (Rbar[i]-0.001)/beta[i]
              }
```

a. Estimate β, α, and residual risk (σ_ϵ^2) for each stock.

```
> cbind(alpha, beta, mse)
            alpha      beta         mse
 [1,] -0.002152648 0.5571742 0.005168777
 [2,] -0.005787947 1.0622912 0.007098190
 [3,] -0.007134964 1.5595751 0.011186323
 [4,] -0.007749413 0.9299682 0.007797070
 [5,] -0.002997322 0.8575829 0.007718417
 [6,] -0.006964282 1.1161600 0.007402733
 [7,] -0.003923698 1.3169270 0.008880664
 [8,]  0.008377507 1.1269555 0.017827338
 [9,] -0.007951761 1.4606240 0.007492516
[10,] -0.022777860 1.1861733 0.045632803
```

b. Estimate the mean and variance for each stock and the market:

```
> cbind(Rbar, var)
             Rbar         var
r1   0.0027625075 0.005827313
r2   0.0035831363 0.009491973
r3   0.0066229478 0.016345853
r4   0.0004543727 0.009631637
r5   0.0045679106 0.009278507
r6   0.0028820096 0.010045449
r7   0.0076936743 0.012559589
r8   0.0183190310 0.020521422
r9   0.0049332466 0.012018099
r10 -0.0123139407 0.048617456

> mean(data1[,11])
[1] 0.008821577

> var(data1[,11])
[1] 0.002121278
```

c. Adjustment of betas based on the Vasicek's method:

```
#Vasicek's method:
> for(i in 1:10){
q <- lm(data=data1, formula=data1[,i] ~ data1[,11])
q1[i] <- vcov(q)[2,2]
beta_adj[i] <- q1[i]*mean(beta)/(var(beta)+q1[i]) +
var(beta)*beta[i]/(var(beta)+vcov(q)[2,2])
               }

> cbind(beta, beta_adj)
           beta  beta_adj
 [1,] 0.5571742 0.6224087
 [2,] 1.0622912 1.0707286
 [3,] 1.5595751 1.4614250
 [4,] 0.9299682 0.9610444
 [5,] 0.8575829 0.9003011
 [6,] 1.1161600 1.1163479
 [7,] 1.3169270 1.2800751
 [8,] 1.1269555 1.1239514
 [9,] 1.4606240 1.4055578
[10,] 1.1861733 1.1491554
```

Note: The variance of each $\hat{\beta}$ is extracted from the `lm` object with the `vcov` function which gives the variance covariance matrix of the estimated parameters of the model (here 2×2). Here they are:

```
> q1
 [1] 0.01138614 0.01563638 0.02464200 0.01717592 0.01700266
 [6] 0.01630725 0.01956294 0.03927129 0.01650503 0.10052308
```

Exercise 2:
Using the single index model with $R_f = 0.001$.

a. Short sales not allowed (unadjusted betas): $c^* = 0.003113105$.

```
> aaaa_no
           Ratio      col1     col2      col3     col4
[1,] 0.015367982 1.0948228 1.094823  71.24051  71.2405
[2,] 0.005082798 0.9926150 2.087438 195.28909 266.5296
[3,] 0.004160427 0.3964257 2.483864  95.28487 361.8145
[4,] 0.003605436 0.7839404 3.267804 217.43291 579.2474
[5,] 0.003163297 0.1899915 3.457795  60.06124 639.3086
            col5        z_no        x_no
[1,] 0.002017532 0.774692242 0.616205426
[2,] 0.002828723 0.292088864 0.232333220
[3,] 0.002981012 0.116366543 0.092560234
[4,] 0.003110235 0.068639838 0.054597475
[5,] 0.003113105 0.005410534 0.004303645
```

Short sales not allowed (adjusted betas): $c^* = 0.003136239$.

```
> aaaaa
           Ratio      col1     col2     col3     col4
[1,] 0.015367982 1.0919044 1.091904  70.8612  70.8612
[2,] 0.005082798 0.9648385 2.056743 184.5124 255.3736
[3,] 0.004160427 0.4161726 2.472915 105.0140 360.3876
[4,] 0.003605436 0.7346040 3.207519 190.9263 551.3139
            col5       z_no       x_no
[1,] 0.002013561 0.77116860 0.62568886
[2,] 0.002829909 0.28058057 0.22764949
[3,] 0.002972963 0.11946451 0.09692772
[4,] 0.003136239 0.06129764 0.04973393
```

b. Short sales allowed (unadjusted betas): $c^* = 0.002404819$.

```
> aaaa
              Ratio        col1     col2      col3      col4
 [1,]  0.0153679818  1.09482280 1.094823  71.24051   71.2405
 [2,]  0.0050827984  0.99261505 2.087438 195.28909  266.5296
 [3,]  0.0041604267  0.39642572 2.483864  95.28487  361.8145
 [4,]  0.0036054357  0.78394035 3.267804 217.43291  579.2474
 [5,]  0.0031632968  0.18999154 3.457795  60.06124  639.3086
 [6,]  0.0026928535  0.76676436 4.224560 284.74046  924.0491
 [7,]  0.0024316650  0.38658348 4.611143 158.97892 1083.0280
 [8,]  0.0016861467  0.28376327 4.894907 168.29097 1251.3190
 [9,] -0.0005867161 -0.06507779 4.829829 110.91870 1362.2377
[10,] -0.0112242793 -0.34608089 4.483748  30.83324 1393.0709
             col5            z            x
 [1,] 0.002017532  0.819466548  0.908971335
 [2,] 0.002828723  0.397121654  0.440496565
 [3,] 0.002981012  0.195063236  0.216368673
 [4,] 0.003110235  0.167387673  0.185670296
 [5,] 0.003113105  0.081761036  0.090691241
 [6,] 0.003027353  0.056150791  0.062283762
 [7,] 0.002966430  0.004017734  0.004456564
 [8,] 0.002841361 -0.108359000 -0.120194320
 [9,] 0.002633995 -0.356804838 -0.395776216
[10,] 0.002404819 -0.354273050 -0.392967898
```

Short sales allowed (adjusted betas): $c^* = 0.002408639$.

```
> aaaa
              Ratio        col1     col2      col3      col4
 [1,]  0.0153679818  1.09190436 1.091904  70.86121   70.8612
 [2,]  0.0050827984  0.96483846 2.056743 184.51236  255.3736
 [3,]  0.0041604267  0.41617264 2.472915 105.01404  360.3876
 [4,]  0.0036054357  0.73460395 3.207519 190.92630  551.3139
 [5,]  0.0031632968  0.21223591 3.419755  74.94860  626.2625
 [6,]  0.0026928535  0.73785697 4.157612 263.67547  889.9380
 [7,]  0.0024316650  0.38965398 4.547266 161.51439 1051.4524
 [8,]  0.0016861467  0.28381103 4.831077 168.34763 1219.8000
 [9,] -0.0005867161 -0.06725246 4.763825 118.45558 1338.2556
[10,] -0.0112242793 -0.33528044 4.428544  28.93879 1367.1944
             col5            z            x
 [1,] 0.002013561  0.817041234  0.918657427
 [2,] 0.002829909  0.385458165  0.433397962
 [3,] 0.002972963  0.204334134  0.229747364
 [4,] 0.003136239  0.156354156  0.175800071
 [5,] 0.003115449  0.090873601  0.102175636
 [6,] 0.003054032  0.053317117  0.059948217
 [7,] 0.002985992  0.003473296  0.003905273
 [8,] 0.002856574 -0.108953423 -0.122504064
 [9,] 0.002632428 -0.369198929 -0.415116549
[10,] 0.002408639 -0.343313155 -0.386011337
```

Exercise 3:
Constant correlation model with $R_f = 0.001$.
First we compute the average correlation coefficient:

```
> rho <- (sum(cor1)-10)/90
> rho
[1] 0.2100567
```

Therefore, the average correlation is 0.2100567.

a. The cut-off point C^* when short sales are allowed is 0.02183670 and when short sales are not allowed is 0.03402333 (see table below).

b. Short sales not allowed:

```
> aaaa
        Ratio      col1      col2       col3       z_no
r8 0.12114435 0.2100567 0.1211443 0.02544718 0.77144785
r7 0.05982677 0.1735924 0.1809711 0.03141522 0.29195277
r3 0.04405100 0.1479154 0.2250221 0.03328424 0.09944797
r5 0.03711254 0.1288557 0.2621347 0.03377754 0.04067784
r9 0.03593079 0.1141472 0.2980654 0.03402333 0.02205851
         x_no
r8 0.62945278
r7 0.23821504
r3 0.08114327
r5 0.03319055
r9 0.01799836
```

c. Short sales allowed:

```
> aaaa
           Ratio       col1      col2       col3           z
r8   0.121144346 0.21005669 0.1211443 0.02544718  0.87935924
r7   0.059826767 0.17359244 0.1809711 0.03141522  0.42983830
r3   0.044051002 0.14791544 0.2250221 0.03328424  0.22030710
r5   0.037112537 0.12885569 0.2621347 0.03377754  0.20114804
r9   0.035930795 0.11414718 0.2980654 0.03402333  0.16298849
r2   0.026560070 0.10245251 0.3246255 0.03325870  0.06148059
r1   0.023136557 0.09293145 0.3477621 0.03231803  0.02160069
r6   0.018809887 0.08502953 0.3665720 0.03116944 -0.03829604
r4  -0.005570178 0.07836610 0.3610018 0.02829030 -0.35419021
r10 -0.060515275 0.07267115 0.3004865 0.02183670 -0.47384520
               x
r8   0.79193658
r7   0.38710535
r3   0.19840498
r5   0.18115064
r9   0.14678477
r2   0.05536841
r1   0.01945322
r6  -0.03448879
r4  -0.31897792
r10 -0.42673725
```