Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
D		2402006

Flow Regulation Model Based on Linear Regression

Summary

The five major lakes in North America are connected by two dams, the Compensating Works and Moses-Saunders Dam. As a network linking multiple lakes and rivers, the control of dams significantly influences water levels upstream and downstream, impacting the interests of various stakeholders downstream. In order to optimize the water levels of the five major lakes through dam operations and maximize stakeholder benefits, we first analyzed the importance of each stakeholder. Subsequently, we constructed a linear regression model, identifying different variable constraints to dynamically optimize the degree of dam opening and closing on a monthly basis.

Addressing the first question, we approached the subproblem of determining the optimal water level for Lake Ontario. By establishing linear relationships among the flow rates and water levels of various rivers and lakes without intervening dams, we derived the optimal water levels for other lakes. We obtained profit data for each stakeholder from official sources, and using softmax, we established the importance relationships among them. Considering the requirements of each stakeholder for the water level of Lake Ontario, we determined the monthly adjustments the dam should make. By adding these adjustments to the average monthly water level, we obtained the optimal water level for Lake Ontario each month.

Based on determining the optimal water levels, we employed a combination of linear regression and convex optimization methods to establish a dam control algorithm. Firstly, we used linear regression to identify the linear relationships between the flow rates of the river where the dam is located and the water levels of each lake, which were found to be generally linearly related. Subsequently, we divided the five Great Lakes into two separate regions, based on the water areas controlled by two dams, which have little impact on each other, and optimized them individually. We formulated the objective function by taking the dam-controlled outflow as a variable and the difference between the water level derived from the linear function and the optimal water level as the optimization goal, ensuring that the objective function is convex. For the constraint conditions of dam water volume, we applied linear regression to find the approximate relationship between lake water levels, neighboring lake water levels, and river water levels. We used a linear function with a bias term as an approximation to reflect the real values. By applying mathematical methods, we solved the bias term and used it to set upper and lower bounds for the constraint interval by adding or subtracting the assumed river flow rate. For the Morse-Saunders Dam on the St. Lawrence River, the requirements of stakeholders also need to be considered. We constructed constraint intervals based on the requirements of each stakeholder and took their intersection as the final constraint interval. After obtaining the constraint interval, we used gradient descent algorithm to find the optimal solution.

Lastly, we assessed the sensitivity of our algorithm by comparing it with extreme data from 2017 and analyzing the impact of extreme weather events. We focused on analyzing the stakeholders and natural factors related to Lake Ontario to optimize our algorithm.

Keywords: Optimal water level; Stakeholders; Softmax; Convex Optimization; Linear Regression; Multi-Objective Optimization; Genetic Algorithm; Sensitivity Analysis

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1 Introduction

1.1 Problem Background

The Great Lakes of the United States and Canada are the largest group of freshwater lake in the world, which contains many large urban areas in these two countries. Therefore, the lakes' water is used for many purposes like fishing, power generation, shipping, construction etc. Consequently, a vast variety of stakeholders have an interest in the management of the water that flows into and out of the lakes. Since the rates of rain, evaporation, erosion, ice jams, and other water flow Phenomena will greatly affect the water level of lake and the flow of river but a variance from normal of two to three feet of water level, especially St. Lawrence River and Ontario Lake, can dramatically affect some of the stakeholders, governments can use two dams – Compensating Works and Moses-Saunders Dam as control mechanisms that meet the acquirements of most of the stakeholders. The International Joint Commission (IJC) requests my team to develop a model for the control mechanisms and a management plan to implement the model.

1.2 Restatement of the Problem

Lakes often have a significant impact on the ecological and economic environment of a region. Through analysis and research on the background of the problem, combined with the specific constraints given, the restate of the problem can be expressed as follows:

- We should establish a model that can accurately describe the variations in river discharge and the water levels of the lake over different time periods. This model needs to incorporate both climatic factors and dam control factors.
- Based on the model, we can establish a function related to water levels in order to measure the satisfaction level of various stakeholders with regards to the water levels.
- Design an algorithm to schedule and operate the dam that the function could be optimized.
- We should understand the sensitivity of our algorithm by comparing the actual recorded water levels and new controls result from our algorithm in 2017 and find the sensitivity of our algorithm to the changes of the environment conditions.
- Focus the analysis on stakeholders and factors of Lake Ontario and improve our algorithm.

1.3 Our Work

This task requires us to convert the water resource allocation problem to an optimization problem. Our work includes the following:

1. We have derived the water level equation based on the hydrological conditions and water system characteristics of the Great Lakes.

2. We used convex functions to describe the interests of various parties, and combined it with the water level equation to ultimately obtain convex optimization problems.
3. We use algorithm to solve that optimization problem.
4. We should understand the sensitivity of this control algorithm by comparing the raw data from 2017 with the data obtained from our algorithm.
5. We should examine the sensitivity of the control algorithm by comparing various environmental conditions.
6. We need to analyze the situation considering only the stakeholders and factors related to Lake Ontario.
7. We need to write an article in the memorandum highlighting the key features of our algorithm and recommending it.

In order to avoid complicated description, intuitively reflect our work process, the flow chart is shown in Figure 3.

2 Assumptions and Explanations

Considering that practical problems always contain many complex factors, first of all, we need to make reasonable assumptions to simplify the model, and each hypothesis is closely followed by its corresponding explanation:

Assumption 1: **Morse-Saunders Dam have a high capacity for regulation**
Explanation: According to the data from the Excel sheet, it can be observed that even under extreme water levels of Lake Ontario, the flow rate of the St. Lawrence River remains stable.

Assumption 2: **Monthly river flows can be applied to the corresponding lakes in the same month**
Explanation: The rivers connecting two lakes have relatively short flow paths and higher water flow velocities.

Assumption 3: **The total amount of water entering the lake is the same as the total amount of water leaving the lake**
Explanation: the law of conservation of mass in physics.

Assumption 4: **The data can be considered reliable and can reflect the changing laws of the water level or the flow rate.**
Explanation: The historical lakes level or river flow data, lake's area, percentage of industrial economy and other data come from authoritative websites, such as the Statistic Canada and official website of IJC , with high accuracy.

Additional assumptions are made to simplify analysis for individual sections. These assumptions will be discussed at the appropriate locations.

3 Notations

Some important mathematical notations used in this paper are listed in Table 1.

Symbol	Description
Δ_j	The water level fluctuations in Lake Ontario caused by dam control during the jth month. $j \in [1, 12]$
Δ_j^i	In the jth month, the ith stakeholder wishes for the magnitude of water level changes in Lake Ontario. $i \in 1, 2, 3, 4, 5$, $j \in [1, 12]$
α_j^i	In the jth month, the influence of the ith stakeholder on the magnitude of water level changes in Lake Ontario. $i \in 1, 2, 3, 4, 5$, $j \in [1, 12]$
h_j^i	the water level the i-th stakeholder wishes to achieve in j-th month for Lake Ontario
$\bar{h}_j$	the average water level of Lake Ontario in j-th month
h_j^{opt}	the optimal water level of Lake Ontario in j-th month
h_j^{low}	the low water level of Lake Ontario in j-th month
h_j^{mid}	the middle water level of Lake Ontario in j-th month
h_j^{high}	the high water level of Lake Ontario in j-th month
$\hat{h}_j^l$	Used in Q2, the l-th lake's water level in j-th month before the dam's adjustment. $l \in [1, 5]$
h_j^l	Used in Q2, the l-th lake's water level in j-th month after the dam's adjustment. $l \in [1, 5]$
h_j^l	Used in Q2, the l-th lake's optimal water level in j-th month. $l \in [1, 5]$
f_j^1	The first target function to be optimized in j-th month. $j \in [1, 12]$
f_j^2	The second target function to be optimized in j-th month. $j \in [1, 12]$

Table 1: Notations used in this paper

*There are some variables that are not listed here and will be discussed in detail in each section.

4 Model

4.1 Data Overview

4.1.1 Data Preprocessing

This question provides mean water level data for the five Great Lakes in the basin from 2000 to 2022, on a monthly basis (measured in meters). Two lakes are merged together as they have the same water level, given that there is no difference in water surface elevation between them.

Additionally, flow data for six rivers is provided from 2000 to 2022, also on a monthly basis (measured in cubic meters per second). Most of the data for the lakes is complete over the 23 years, with occasional missing values. In such cases, the mean water level value for that month over the years is used to fill in the missing data.

For the rivers, most of the data is available only from 2009 onwards. Consequently, data before 2010 is discarded, and only the data from 2010 to 2022 is retained. For the missing values, the mean value of flow for that month over the years is used to fill in the missing data.

4.1.2 Data Collection

For the first question, we collected the economic information on Lake Ontario from Statistics Canada, where we found the profit proportion of the five shareholders. All data sources are shown in Table 2.

Table 2: Data and Database Websites

Database Names	Database Websites
Statistics Canada	https://www.statcan.gc.ca/en/start
Problem D Great Lakes.XLSX	Provided by the problem

4.2 The Great Lakes Network

From the top view and local plane view, it can be seen that the water flow direction in the Five Great Lakes region is shown in the following flowchart:

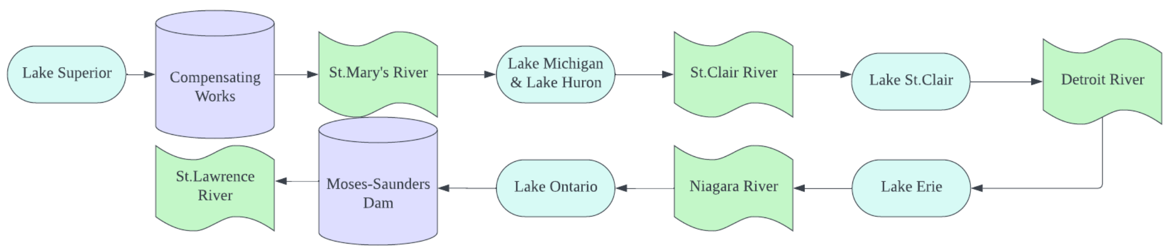


Figure 1: The Great Lakes Network

5 Question

In order to determine the optimal water levels for each lake every month, we aim to extrapolate the water level changes of Lake Ontario and from there, we can derive the water level change of other lakes upstream. Then, we can add these changes to the mean water levels in the history without the adjustment of the dam of each lake for the respective months. This process will help us calculate the best water levels for each lake for each month.

The analysis of Lake Ontario's water level changes requires a comprehensive consideration of the interests of six stakeholders. Therefore, we will first focus on modeling the sub-problem of Lake Ontario's water level changes.

5.1 Question 1

5.1.1 Subproblem: Water level change of Lake Ontario

The reason why we first consider the water level of Lake Ontario is that there are many more economic activities around Lake Ontario than the other lakes. Furthermore, according to the picture below, we can see that the fluctuation of Lake Ontario is the highest, which means the fluctuation may influence more on activities around Lake Ontario.

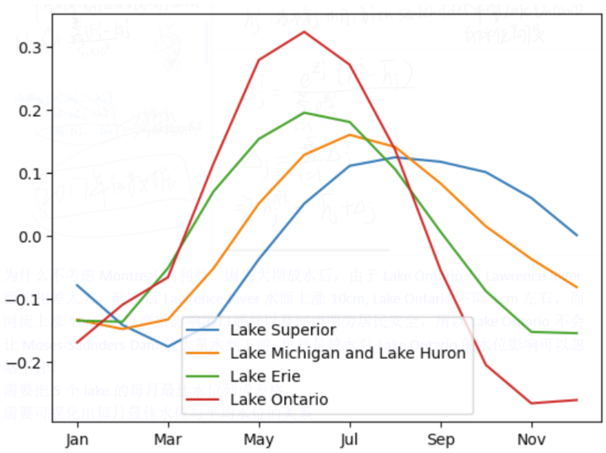


Figure 2: Fluctuation of lakes throughout the year

The question states that we need to consider six stakeholders:

- Ship company -> need high, static water in St Lawrence River.
- Shipping docks -> need low, static water in St Lawrence River.
- Environmentalist -> need seasonal high-level and low-level water on Lake Ontario.
- Property owners -> need mid-level, steady water on Lake Ontario.
- Recreational activities -> need mid-level, steady water on Lake Ontario.
- Hydro-power generation company -> needs high-level water on Lake Ontario.

Since according to International Lake Ontario-St. Lawrence River Board, after the Moses-Saunders Dam releases water, the significant difference in the surface areas of Lake Ontario and the St. Lawrence River leads to a distinct impact on water levels.

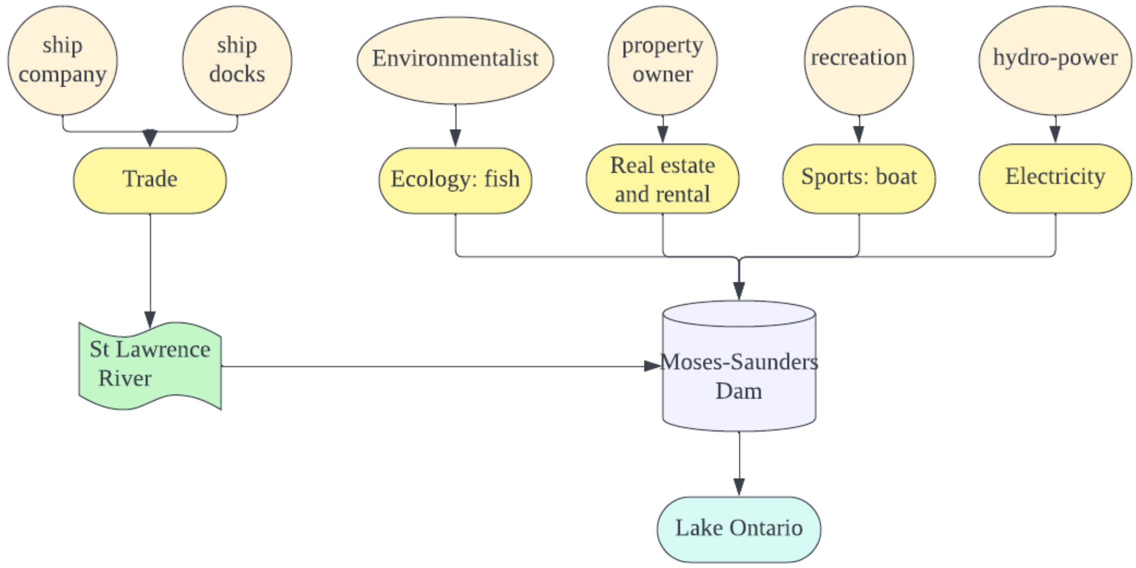


Figure 3: Factors influenced the water level of Lake Ontario

Following the water release, the St. Lawrence River experiences an increase of approximately 10cm, while Lake Ontario sees a decrease of around 1cm. However, given the vast difference in their surface areas, the rise of 10cm in the river poses a substantial risk to port navigation and the safety of residents along the riverbanks. Consequently, in order to protect the interests of people near Montreal Port, Lake Ontario strategically refrains from allowing the Moses-Saunders Dam to release a large volume of water downstream so that the influence of the interests of Montreal on Lake Ontario's water level is considered negligible.

According to the data on Canada Statistics, we found the profits of all five stakeholders all year round as the table below:

Stakeholder	**Year Profit(billion)**
Ship company	515
Environment	100
Property Owner	1045
Recreation	39
Hydro-Power	69

Table 3: Stakeholders and their year profits

We will begin by dividing the entire year into two phases: December to April and May to November. The period from December to April represents the ice-covered season, during which the shipping company, a stakeholder, cannot engage in shipping activities. Therefore, the revenue for this period is set to zero. Conversely, from May to November, when the ice melts, the shipping company can resume shipping, and we assign the revenue for this period as 515 billion.

Next, we aim to determine the importance of each stakeholder's impact on water levels during these two phases. We employ the softmax method, where Z_j^i represents the revenue of the i-th stakeholder in the j-th month. The variable α_i^j denotes the softmax result, indicating the importance of that stakeholder in that month.

$$\alpha_j^i = \frac{e^{Z_j^i}}{\sum_{i=1}^{5} e^{Z_j^i}} \tag{1}$$

We denote Δ_j^i as the amount of water level the i-th stakeholder wishes to adjust on Lake Ontario in j-th month; h_j^i as the water level the i-th stakeholder wishes to achieve in j-th month; $\bar{h_j}$ is the average water level of Lake Ontario in j-th month; Δ_j denotes the amount of water level to be adjusted after considering all stakeholders' wishes; h_j^{opt} denotes the optimal water level of Lake Ontario in j-th month; h_j^{high} denotes the high water level of Lake Ontario in j-th month; h_j^{low} denotes the low water level of Lake Ontario in j-th month; h_j^{mid} denotes the middle water level of Lake Ontario in j-th month.

$$\Delta_j^i = h_j^i - \bar{h_j} \tag{2}$$

$$\Delta_j = \sum_{i=1}^{5} \alpha_j^i * \Delta_j^i \tag{3}$$

$$h_j^{opt} = \bar{h_j} + \Delta_j \tag{4}$$

$$h_j^{high} = \bar{h_j} + \sigma_{h_j} \tag{5}$$

$$h_j^{low} = \bar{h_j} - \sigma_{h_j} \tag{6}$$

$$h_j^{mid} = \bar{h_j} \tag{7}$$

The table below indicates the water level each stakeholder wants:

Month	Shipping company	Environment	Property Owner	Recreation	Hydro-Power
12-4	None	High	Mid	Mid	High
5-11	High	Low	Mid	Mid	High

Table 4: Stakeholders and their preference for water level on Lake Ontario

After applying the formula above, we got the optimal water level of Lake Ontario:

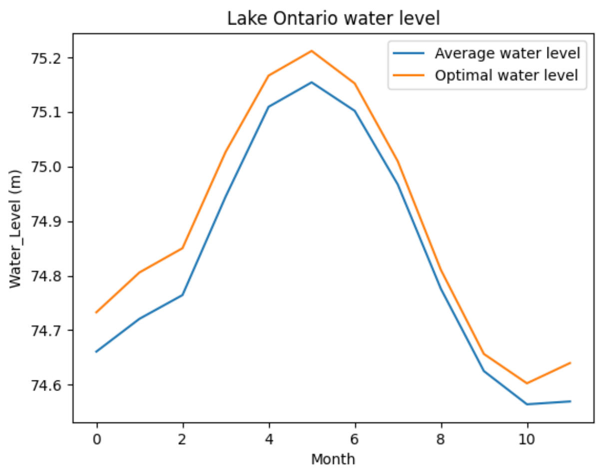


Figure 4: Comparison of optimal and real water level of Lake Ontario

According to the relationship between rivers and lakes, we can derive the optimal water level of Lake Erie, Lake St. Clair, Lake Huron and Michigan with the linear equations. However, there is no obvious relationship between the water level of Lake Superior and Lake Huron and Michigan. It can be understood because there is Compensating Works between them. So for Lake Superior, we found its optimal in j-th month by looking at all the water level data in j-th on Lake Ontario and picking the several data which are close to the optimal water level of Lake Ontario and then using the corresponding years to find the water level on Lake Superior. Finally, we averaged those data and came up with the optimal water level of Lake Superior.

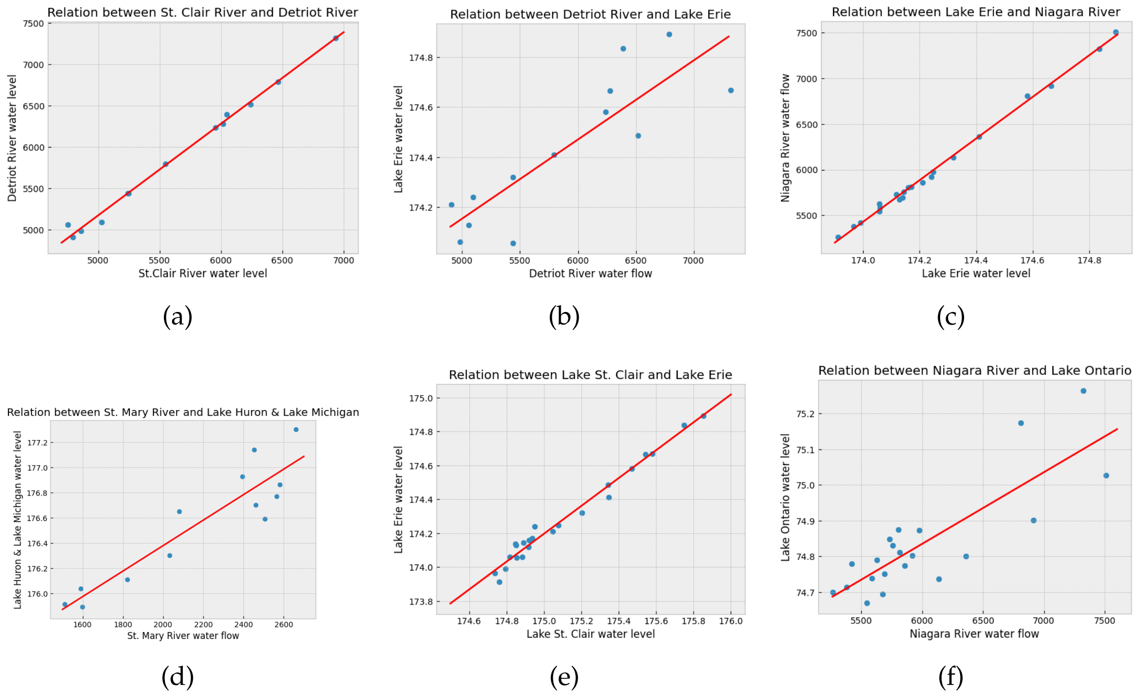


Figure 5: Linear relationship between rivers and lakes

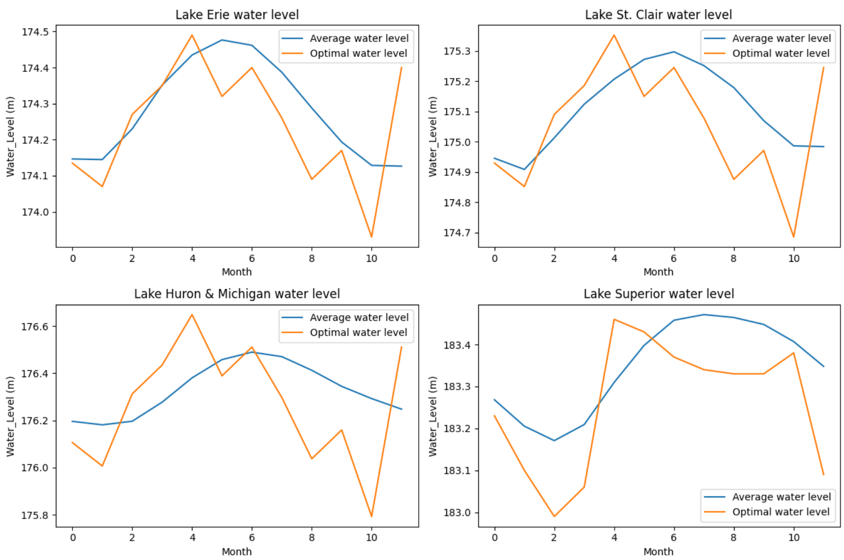


Figure 6: Comparasion of four lakes' optimal water level and their average water level

The trend of change in the water level of Lake Erie, Lake St.Clair, Lake Huron and Michigan is similar since there is no dam between those lakes and the water level difference between those lakes is small. However, Lake Superior has a different trend as for the change in water level, since there is a dan – Compensating Works between it and Lake Huron and Michigan. The optimal water levels of each lake around the year are depicted in the following table:

Lake	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Lake Ontario	74.73	74.81	74.85	75.03	75.17	75.21	75.15	75.01	74.81	74.66	74.60	74.64
Lake Erie	174.14	174.07	174.27	174.35	174.49	174.32	174.40	174.26	174.09	174.17	173.93	174.4
Lake St. Clair	174.93	174.85	175.09	175.19	175.35	175.15	175.24	175.08	174.88	174.97	174.69	175.24
Lake Huron and Michigan	176.05	176.01	175.93	176.08	176.31	176.16	176.25	176.23	176.12	176.05	175.97	176.04
Lake Superior	183.23	183.10	182.99	183.06	183.46	183.43	183.37	183.34	183.33	183.33	183.38	183.09

Table 5: Optimal water level for each lake around the year

5.2 Question 2

5.2.1 Question Analysis

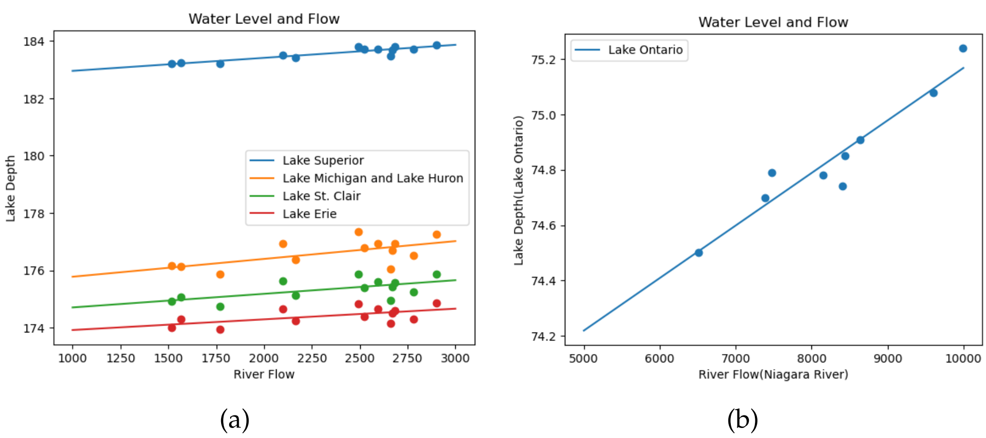
Question 2 asks to make an algorithm to control the flow of two dams to maintain the optimal water level based on the inflow and outflow of the lake. These are actually two convex optimization problems.

We believe that there is a linear relationship between lake water level and river flow rate. The amount of water entering the lake is equal to the amount of water leaving it, indicating a balance between the change in water level and the inflow and outflow rates.

Although there are factors beyond surface runoff that can affect lake water level, analyzing the data indicates that non-surface runoff factors such as precipitation and evaporation generally exhibit a linear relationship with lake water level. Thus, we can consider that there is a linear relationship between the water level of a lake and the inflow and outflow rates of the associated river.

Before solving the problem, we will define two variables that we use in the problems below. We suppose G_j^1 as Soo Locks Dam's flow in month j and G_j^2 as Morse Saunders Dam's flow in month j ($j \in [1, 12]$).

We suppose l as a representation of lakes or rivers. For lakes, $l = 1$ represents Lake Superior or St. Mary's River; $l = 2$ represents Lake Michigan or Lake Huron or St. Clair River; $l = 3$ represents Lake St Clair or Detroit River; $l = 4$ represents Lake Erie



(a) (b)

Figure 7: Linear relationship between water level and water flow

or Niagara River; $l = 5$ represents Lake Ontario or St. Lawrence River; $l = 6$ represents Ottawa River. We suppose S_l as the area of Lake l. We let $c_1 = 10^6$ and $c_2 = 3600*24*30$ as two constant numbers to consistently control the unit.

We suppose $\bar{h_j^l}$ as the optimal water level of Lake l in month j from Question 1 and h_j^l as the water level of Lake l in month j after algorithmic regulation. We hope that $|h_j^l - \bar{h_j^l}|^2$ is small enough for each $j \in [1, 12]$.

5.2.2 First problem's solution

The first problem is about how to control Soo Locks Dam. Based on the aforementioned assumptions, it can be inferred that if we treat the last four lakes as a whole (due to their nearly similar water levels), the average water level of the last four lakes and the water level of Lake Superior have a linear relationship with the flow rate of the St. Mary's River. Additionally, from the data, The water levels of these four lakes have a good linear relationship with the flow rates of the three rivers between them. Therefore, it can be observed that the water levels of each of the last four lakes also exhibit a linear relationship with the flow rate of the St. Mary's River.

Therefore, we can use data in the Excel sheet to fit a straight line. h_j^l can be expressed as

$$h_j^l = k_l G_j^l + b_l \tag{8}$$

k_l and b_l are coefficients of linear equations of Lake l ($l = 1, 2, 3, 4$).

Since we hope $|h_j^l - \bar{h_j^l}|^2$ can be small enough, we construct our objective function for each month f_j^1 as

$$f_j^1 = \sum_{l=1}^{4} |h_j^l - \bar{h_j^l}|^2 \tag{9}$$

We want to find its minimum for month j and the corresponding G_j^1 ($j \in [1, 12]$) with constraints of G_j^1.

Next, we will discuss the constraints of G_j^1. The dam's function is to regulate the river flow, and its regulation is limited and closely related to the original river flow. Therefore, we would like to obtain the deviation amount that Soo Locks dam can regulate based on the river flow in month j, denoted as Δh_j^1.

We suppose $\hat{h_j^l}$ as the water level of Lake l in month j from the Excel sheet and F_j^l as the flow rate of l River in month j from the Excel sheet. We assume that

$$\hat{h_j^2} = m_1 \hat{h_j^1} + n_1 + l_1 \Delta h_j^1 \tag{10}$$

$$\hat{h_j^2} = m_2 F_j^1 + n_2 + l_2 \Delta h_j^1 \tag{11}$$

as the real relationship between the water level of Lake Superior, the water level of Lake Michigan and Lake Huron , and the flow rate of St. Mary's River ($\hat{h_j^1}$, $\hat{h_j^2}$ and F_j^1). Among them, m_1 m_2 n_1 n_2 l_1 l_2 are coefficients.

Based on the data in the Excel sheet, we can also fit two curves to roughly represent the relationship among these three variables.

$$h_{j1}^2 = m_1^* \hat{h_j^1} + n_1^* \tag{12}$$

$$h_{j2}^2 = m_2^* F_j^1 + n_2^* \tag{13}$$

m_1^* m_2^* n_1^* n_2^* are coefficients. Since the change in river flow caused by the dam is relatively small. Therefore, $m_1^* \approx m_1$ $m_2^* \approx m_2$ $n_1^* \approx n_1$ $n_2^* \approx n_2$. Using the concept of linear regression, we can obtain

$$\frac{h_{j1}^2 - \hat{h_j^2}}{h_{j2}^2 - \hat{h_j^2}} = a \tag{14}$$

a is a constant number.

Using a simple mathematical method, we can get Δ_j^1 for any $j \in [1, 12]$. We define $\Delta h_{max}^1 = max[\Delta h_1^1\ \Delta h_2^1\ ...\Delta h_{12}^1]$. Then, we define $[F_j^1 - \Delta h_{max}^1, F_j^1 + \Delta h_{max}^1]$ as a constraint of G_j^1.

5.2.3 Second problem's solution

The second problem is about how to control the Morse-Saunders Dam. Based on the fitting of the data from the Excel sheet, it can be observed that there is a linear relationship between the water levels of Lake Ontario and the flow rates of the St. Lawrence River.

Therefore, we can use data in the Excel sheet to fit a straight line. h_j^5 can be expressed as

$$h_j^5 = k_5 G_j^2 + b_5 \tag{15}$$

k_5 and b_5 are coefficients of linear equations of Lake Ontario.

Since we hope $|h_j^5 - \bar{h_j^5}|^2$ can be small enough, we construct our objective function for each month f_j^2 as

$$f_j^2 = |h_j^5 - \bar{h_j^5}|^2 \tag{16}$$

We want to find its minimum for month j and corresponding G_j^2 ($j \in [1, 12]$) with constraints of G_j^2.

Next, we will discuss the constraints of G_j^2. We have already know that Morse-Saunders Dam has a deviation amount that when regulating the water based on the river flow for every month, is denoted as Δh_j^2. We assumed that

$$\hat{h_j^5} = m_5 F_j^5 + n_5 + l_3 \Delta h_j^2 \tag{17}$$

as the real relationship between the water level of Lake Ontario and the flow rate of the St. Lawrence River from the Excel sheet. Among them, m_5 n_5 are coefficients. $l_3 = \frac{c_2}{c_1 S_5}$ is a constant number to control the unit.

Based on the data in the Excel sheet, we can also fit a curve to roughly represent the relationship between these two variables

$$h_j^5 = m_5^* F_j^5 + n_5^* \tag{18}$$

m_5^* n_5^* are coefficients. Since the change in river flow caused by the dam is relatively small. Therefore, $m_5^* \approx m_5$ $n_5^* \approx n_5$.

Using a simple mathematical method, we can get Δ_j^2 for any $j \in [1, 12]$. We define $\Delta h_{max}^2 = max[\Delta h_1^2\ \Delta h_2^2\ ...\Delta h_{12}^2]$. Then, we define $B1_j = [F_j^5 - \Delta h_{max}^2, F_j^5 + \Delta h_{max}^2]$.

Since the Morse-Saunders Dam is built on the St. Lawrence River, according to the subproblem of Lake Ontario, Shipping companies want high and static water levels while people who manage shipping docks or live near Montreal harbor want the water level to be steady and slow. Therefore, they want high water in the upper and middle St. Lawrence River, and low water near the Port of Montreal, and both want smooth water. The level of water in a river can be judged by the amount of flow. Therefore, we need a large amount of water in the upper and middle. We suppose y_{ij} as data of St. Lawrence River in year i month j from the Excel sheet ($i \in [2012, 2022]$). We expressed its average and variance as

$$y_j = \frac{1}{11} \sum_{i=2012}^{2022} y_{ij} \tag{19}$$

$$\sigma_{yj}^2 = \frac{1}{11} \sum_{i=2012}^{2022} (y_{ij} - y_j)^2 \tag{20}$$

We denote this constraint interval as $B2_j = [y_j, y_j + \sigma_{yj}]$

Near the Port of Montreal, due to the confluence of the Ottawa River, the flow should be the sum of the St. Lawrence and Ottawa Rivers, and we need a smaller flow. We suppose z_{ij} as data of the Ottawa River in year i month j from the Excel sheet ($i \in [2000, 2022]$).Then we have

$$z_j = \frac{1}{23} \sum_{i=2000}^{2022} z_{ij} \tag{21}$$

$$\sigma_{zj}^2 = \frac{1}{23} \sum_{i=2012}^{2022} (z_{ij} - z_j)^2 \tag{22}$$

$$sum_j = \frac{1}{11} \sum_{i=2012}^{2022} (y_{ij} + z_{ij}) \tag{23}$$

$$\sigma_{sum_j}^2 = \frac{1}{11} \sum_{i=2012}^{2022} (y_{ij} + z_{ij} - y_j)^2 \tag{24}$$

We denote this constraint interval as $B3_j = [sum_j - \sigma_{sum_j} - (z_j - \sigma_{zj}), y_j - (z_j - \sigma_{zj})]$. Then $G_j^2 \in B1_j \cap B2_j \cap B3_j$.

5.3 Question 3

5.3.1 Comparison between actual result in 2017 and new control result

The water levels compared to the long-term average were significantly higher for many lakes in 2017, with Lake Ontario experiencing the most pronounced increase. Apart from January and December, the water levels in Lake Ontario were consistently higher than the average. However, in our optimized data, there was a noticeable decrease in the water levels of Lake Ontario for each month. Although they may still be higher than the average, the deviation from the optimal water levels was controlled within a range of 2 feet.

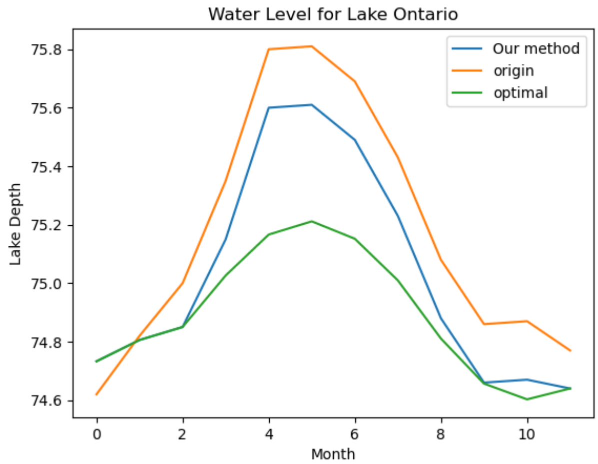


Figure 8: Our method vs Real water level for Lake Ontario in 2017

5.3.2 Sensitivity analysis

This extreme situation does have an impact on our algorithm. We still observe a significant difference between the optimized values and the optimal water levels, with a maximum difference of up to 0.4 meters. However, our algorithm does effectively reduce the extreme water levels and keeps them within 2 feet of the optimal water levels. Therefore, our algorithm is considered superior, as it minimizes the impact of outliers and exhibits lower sensitivity to extreme values.

5.4 Question 4

In the linear function describing the relationship between lake water level and river flow rate, the slope can be interpreted as the relationship between the change in lake water level and the change in river flow rate. The y-intercept can roughly indicate the current water level of the lake.

5.4.1 Intense precipitation

Intense precipitation are only considered at the confluence of rivers and lakes because their impact is localized and the range of lakes is vast, which does not pose significant effects.

the water level of lakes may experience a slight increase in a short period of time, leading to an upward shift in the y-intercept. If the river is an inflow river, where the inflow volume equals the river flow rate plus precipitation volume, the slope will increase. If the river is an outflow river, where the river flow rate equals the outflow volume plus precipitation at the lake outlet, the slope will decrease.

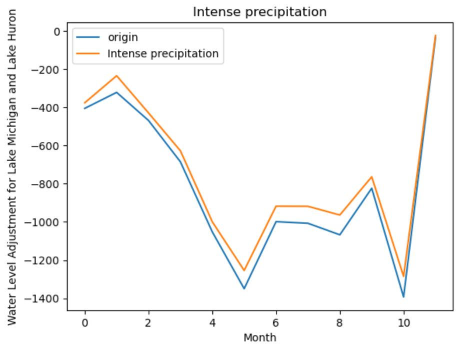


Figure 9: Water level adjustment for Lake H and M when there is intense precipitation

The curve represents the deviation between the calculated water levels and the optimal water levels. There is a certain deviation between the two curves during the months of August to October, while they are very close in other time periods. This indicates that heavy precipitation weather will have some impact on the water level results calculated by our algorithm, but the difference shows that the impact is not significant.

5.4.2 Lake surface freezing

the density of ice is lower, resulting in a larger volume of ice. Additionally, the ice cover reduces the evaporation of lake water, causing the water level of the lake to rise and thus increasing the y-intercept. However, the relationship between the change in water level and the change in flow rate remains relatively unchanged, resulting in the slope remaining constant. The curve represents the deviation between the calculated

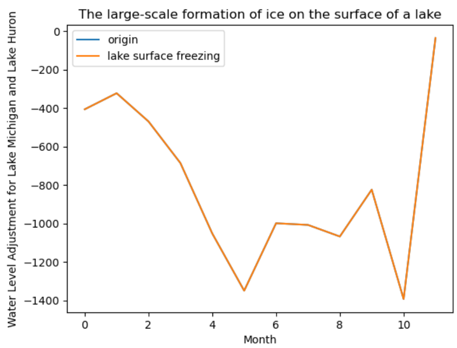


Figure 10: Water level adjustment for Lake H and M when large-scale ice formed on the lake

water levels and the optimal water levels. When the two curves coincide, it indicates that the algorithm's calculated water level results are hardly affected by the large-scale freezing of the lake.

5.4.3 Rapid melting of snow

Similar to intense precipitation, the melting of snow leads to an increase in the water level of lakes, resulting in an upward shift in the y-intercept. If the river is an inflow river, where the inflow volume equals the river flow rate plus the melting snow volume, the slope increases. If the river is an outflow river, where the river flow rate equals the outflow volume plus the melting snow volume, the slope decreases.

The curve represents the deviation in flow rate as the deviation curve. When the two

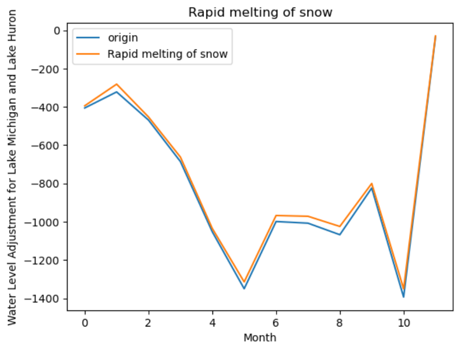


Figure 11: Water level adjustment for Lake H and M when there is rapid melting of snow

curves are very close, indicating a minimal difference in flow rate, it suggests that the algorithm's calculated water level results are hardly affected by melting snow in spring.

5.4.4 Conclusion

Our algorithm is not highly sensitive to extreme weather conditions such as freezing, snow melting, and heavy precipitation. It exhibits good robustness. Regarding the optimization of the algorithm, there is a deviation in the curve graph during the transition from summer to autumn, which lasts for several months in the case of heavy precipitation. To investigate the cause, it is likely that the constraints used in the optimization process were not precise enough. We can consider refining the constraints as a direction for improvement.

5.5 Question 5

We need to prioritize the consideration of natural factors and stakeholders' impact on the water level according to the requirements. Our algorithm has already taken Lake Ontario into separate consideration, so our focus should be on optimizing the constraints of G_j^2.

When considering the constraints, it is important to account for the strong regulating capacity of the Morse-Saunders Dam, which only takes into consideration the constraints of the St. Lawrence River. Additionally, we should also consider the constraints on Lake Ontario, and the constraints of the St. Lawrence River should vary with the season.

In spring (early February to late April), as the ice and snow melt, the water level rises, and the St. Lawrence River is not navigable due to freezing. There are fewer vessels in the Port of Montreal during this season. Therefore, the constraint interval is up to about 500.

In early summer (early May to early June), the lake level reaches its maximum. Therefore, the constraint interval should continually increase by about 500.

In peak summer (mid-June to late July) The lake level remains relatively stable. The shipping and port activities are busy. Hence, the constraint interval should return to its original position.

In autumn (early August to late October), the lake level decreases, but The shipping and port activities are still busy. Therefore, the constraint should decrease by about 500.

In winter (early November to late January of the following year), Lake Ontario partially freezes, and the river is not navigable. To prevent ice blockage at the lake's entrance, the constraint conditions should continuously decrease by about 500.

6 Memo

When determining the optimal water level for each lake throughout the year, we divide the entire year into two distinct periods: one during lake ice cover and the other during lake thawing. During the former, we do not consider the interests of shipping companies, as they are unable to navigate on frozen lakes and must rely on unfrozen tributaries. However, due to the limited carrying capacity of these tributaries, profits during this period can be considered negligible compared to when the lakes are unfrozen.

In calculating the importance of each stakeholder during these two periods, we utilize their annual profit data from Canada Statistics. After aggregating profit data from the past five years and calculating the average, we obtain an approximate profit distribution, establishing the reliability of our stakeholder importance analysis.

When adjusting the Morse-Saunders Dam to control the water level of Lake Ontario, we establish a linear relationship between the dam's water release and the water level of Lake Ontario. Consequently, we minimize the square difference sum between the water level of all lakes adjusted by the dam each month and the optimal water level to determine the necessary water adjustment for that month. Subsequently, we identify the constraint G2, representing the approximate linear relationship between the water level of Lake Ontario and the flow rate of the Lawrence River. This relationship serves as a constraint on the dam's flood control capacity. Additionally, we consider the interests of stakeholders along the Lawrence River. For instance, shipping companies upstream on the Lawrence River require higher water levels, while downstream at Montreal Port, we must ensure that the flow rate of the Lawrence River, combined with that of the Ottawa River, does not exceed certain limits to meet the interests of Montreal Port. It's essential to note that our approach is an online algorithm, distinct from the offline algorithm previously presented in the Appendix. In the online setting, real-time monitoring of the Lawrence River's water flow allows us to determine the current water level of Lake Ontario, and by applying this level to the optimization equation, we can calculate the required dam adjustment for the month without needing extensive historical data.

In summary, firstly, our algorithm employs the principles of linear regression and since our target function is convex, the global optimum is easy to solve. Secondly, we leveraged reliable data from official sources. Thirdly, We conducted detailed discussions and considerations for each month to meet the specific interests of stakeholders, rendering our model reasonably dependable.

7 Our first model – Offline model

Another algorithm that can control the water level through two dams.

7.1 Question 2 Analysis

First of all, we make an assumption, due to Lake Michigan, Lake Huron, Lake St. Clair, and Lake Erie, the four lakes' water level difference is relatively small, the four lakes can be regarded as lakes, notation for Lake MHSE, and the average water level of the four lakes as Lake MHSE's water level.

The main objective of the first optimization problem is to maintain optimal water levels in Lake Superior and Lake MHSE by controlling the Soo Locks Dam. The second optimization problem is to maintain optimal water levels in Lake Ontario by controlling Moses-Saunders Dam.

we suppose F_j^l as Lake l's change in lake water volume other than surface runoff in month j. $l = 1$ represents Lake Superior; $l = 2$ represents Lake MHSE; $l = 3$ represents Lake Ontario. We suppose $\overrightarrow{F^l} = [F_1^l\ F_2^l\ ...\ F_{12}^l]$ as a constant vector of Lake l. Since the lake level, as well as the river runoff in previous years are known in the Excel sheet, F_j^l can be expressed as the difference between the amount of change in the lake level and the amount of river runoff into and out of the lake in month j of Lake l. So we have

$$F_j^1 = c_1(\hat{h_j^1} - \hat{h_{j-1}^1})S_1 - c_2X_j^1 \tag{25}$$

$$F_j^2 = c_1(\hat{h_j^2} - \hat{h_{j-1}^2})S_2 + c_2(X_j^1 - X_j^2) \tag{26}$$

$$F_j^3 = c_1(\hat{h_j^3} - \hat{h_{j-1}^3})S_3 + c_2(x_j^2 - X_j^3) \tag{27}$$

$c_1 = 10^6$ and $c_2 = 2.592 * 10^6$ are constant numbers for maintaining unit consistency. $\hat{h_j^l}$ represents the water level of Lake l in month j which we predicted for this year. $\hat{h_0^l}$ represents the water level of Lake l in December last year. X_j^l represents the river runoff of l River in month j which we predicted this year. $l = 1$ represents St. Mary's River; $l = 2$ represents Niagara River; $l = 3$ represents St. Lawrence River. S_l represents the area of Lake l. We suppose $\overrightarrow{X^l} = [X_1^l\ X_2^l\ ...\ X_{12}^l]$ as a vector of l River.

We suppose G_j^1 as Soo Locks Dam's flow in month j and $\overrightarrow{G^1} = [G_1^1\ G_2^1\ ...\ G_{12}^1]$ as a variable of the first optimization problem. We suppose G_j^2 as Morse Saunders Dam's flow in moth j and $\overrightarrow{G^2} = [G_1^2\ G_2^2\ ...\ G_{12}^2]$ as a variable of the second optimization problem.

We suppose $\bar{h_j^l}$ as the optimal water level of Lake l in month j from Question 1 and h_j^l as the water level of Lake l in month j after algorithmic regulation. We hope that $|h_j^l - \bar{h_j^l}|$ is small enough for each $j \in [1, 12]$.

7.2 The solution of the first optimization problem

Since the first problem involves two lakes, we will first analyze the water levels of each lake separately. For Lake Superior, the St. Mary's River is the dominant surface runoff, and because the water level of Lake Superior is higher than that of Lakes Michigan and Huron, it is dominated by outflow. Whereas Soo Locks Dam controls the flow of the St. Mary's River, so h_j^l can be expressed as:

$$h_j^1 = \hat{h_0^1} + \frac{(\overrightarrow{F^1} - \overrightarrow{G^1})A_j^T}{c_1S_1} \tag{28}$$

A_j is a $1 * 12$ row vector which the first j terms are 1 while the rest are 0.

For the four lakes in the middle, surface runoff is divided between the St. Mary's River, which enters the lakes, and the Niagara River, which exits the lakes, so only the surface runoff into the lakes is controlled by Soo Locks Dam, so h_j^2 can express as

$$h_j^2 = \hat{h_0^2} + \frac{(\overrightarrow{F^2} + \overrightarrow{G^1} - c_2\overrightarrow{X^2})A_j^T}{c_1 S_2} \tag{29}$$

Since our goal is to keep the lakes at their optimal levels, the water levels of the two lakes each month need to be as close as possible to their respective optimal water levels after algorithmic regulation. We can construct a function f_j^1

$$f_j^1 = (h_j^1 - \bar{h_j^1})^2 + (h_j^2 - \bar{h_j^2})^2 \tag{30}$$

We construct $f^1 = [f_1^1 \ f_2^1 \ ... \ f_{12}^1]$ as an objective function with the goal of solving for the minimum of their components.

Next, we will discuss the constraints of $\overrightarrow{G^1}$. Since Soo Locks Dam was built on the St. Mary's River, there is a limit to the flow of the river due to its course and other influences. Based on the data in the Excel sheet, we suppose x_{ij} is the flow of St Mary's River in year i month j ($i \in [2009, 2022]$). We expressed its average as

$$x_j = \frac{1}{14}\sum_{i=2009}^{2022} x_{ij} \tag{31}$$

Therefore, we specify his constraint interval to be roughly $G_j^1 \in [0, x_j]$.

7.3 The solution of the second optimization problem

The second issue considers the control of the Morse-Saunders Dam on the St. Lawrence River, which is a non-landlocked river that ultimately flows into the ocean, so only Lake Ontario needs to be considered. Surface runoff from Lake Ontario can be roughly divided into the Niagara River, which enters the lake, and the St. Lawrence River, which exits the lake. The Morse-Saunders Dam primarily controls the flow of the St. Lawrence River, so h_j^3 can be expressed as

$$h_j^3 = \hat{h_0^3} + \frac{(\overrightarrow{F^3} + \overrightarrow{G^2} - c_2\overrightarrow{X^2})A_j^T}{c_1 S_3} \tag{32}$$

Since our goal is to keep the lake at its optimal levels, the water level of Ontario Lake each month needs to be as close as possible to its respective optimal water level after algorithmic regulation. We can construct a function f_j^2

$$f_j^2 = (h_j^3 - \bar{h_j^3})^2 \tag{33}$$

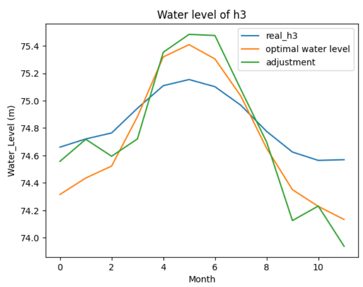
We can construct $f^2 = [f_1^2 \ f_2^2 \ ... \ f_{12}^2]$ as an objective function with the goal of solving for the minimum of their components.

The constraint interval of G_j^2 is $B2_j \cap B3_j$. $B2_j$ and $B3_j$ are intervals constructed in Question 2.

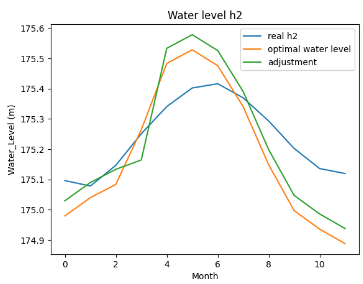
7.4 Method Feasibility

The components of the objective function of both multi-objective optimization problems are convex functions, so the local optimal solution of the components is the global optimal solution, as long as the local optimal solution can be found within the constraints. We can use a genetic algorithm to achieve it.

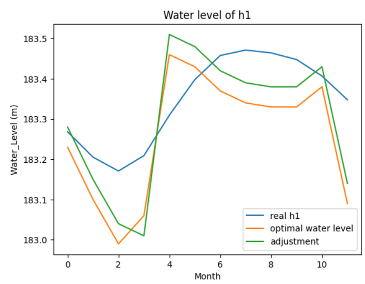
The outcomes demonstrate the effectiveness of our algorithm for Lake Ontario, as well as the intermediate lakes, and Lake Superior. As depicted in the figures below, integrating the annual water average of Lake Ontario into our algorithm resulted in upward adjustments of water levels during spring and summer, followed by downward adjustments in other seasons. Specifically, when applying the water level data from Lake Ontario in 2017, our algorithm efficiently lowered the unexpectedly high water levels to align with the optimal values. This pattern of explanation also holds for the intermediate four lakes and Lake Superior.



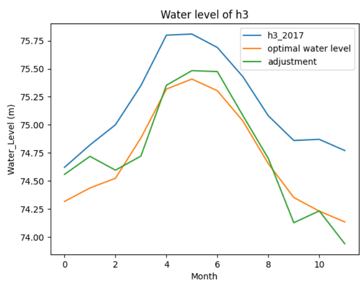
(a) G2 applies to the average water level of Lake Ontario



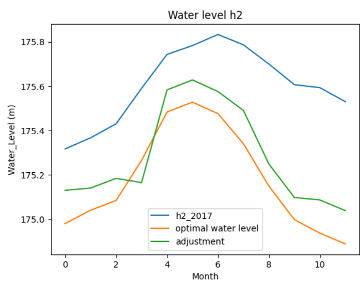
(b) G1 applies to the average water level of middle four lakes



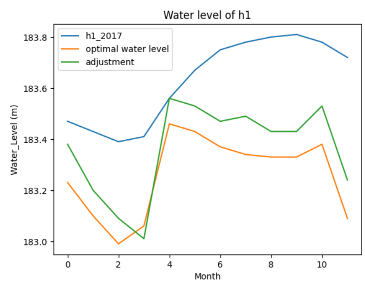
(c) G1 applies to the average water level of Lake Superior



(d) G2 applies to the water level of Lake Ontario in 2017



(e) G1 applies to the water level of middle four lakes in 2017



(f) G1 applies to the water level of Lake Superior in 2017

Figure 12: Final adjusted water level vs Average water level vs Optimal water level vs water level in 2017

However, it's important to note that the algorithm discussed thus far operates offline, requiring the availability of historical data to inform adjustment decisions. To address this limitation, our main focus in this article is on introducing an online algorithm. With this approach, we can determine the required water level adjustments solely based on the current water levels of lakes surrounding the target lake. This is achieved through the implementation of a linear relationship, providing a more real-time and adaptive mechanism for making adjustment decisions.

References

[1] Canada Statistics: https://www.statcan.gc.ca/en/start

[2] International Lake Ontario-St. Lawrence River Board: http://ijc.org/en/loslrb/library/modules

[3] Sivanandam, S. N., et al. "Genetic algorithm optimization problems." Introduction to genetic algorithms (2008): 165-209.

[4] https://glisa.umich.edu/sustained-assessment/ontario-climatology/

[5] https://www.glerl.noaa.gov/data/ice/historical

Online Model

```
baseLakeLevel = []
sheetBase = LakeData['base']
for row in sheetBase.iter_rows(min_row=1,max_row=4,min_col=1,max_col=12,values_only=True):
        baseLakeLevel.append(np.array(row))
baseLakeLevel = np.array(baseLakeLevel)
for row in sheetBase.iter_rows(min_row=5,max_row=5,min_col=1,max_col=12,values_only=True):
        BaseAn = np.array(row)
        BasicAn = np.array(row)
baseLakeLevel = []
updata_Sup = []
updata_HM = []
updateC = []
updateE = []
updateAn = []
AN = [74.73284031423275, 74.80576048702197, 74.85018186237909, 75.02625664481361, 75.16625730478503,
            75.21141945045423, 75.15193998083, 75.00948967719775, 74.8107324398323, 74.6565617967145,
  74.60277098375674, 74.63988185386764]
Task2 = ["Niagara River","Lake Ontario","St. Lawrence River"]
sheetBase = LakeData['base']
for row in sheetBase.iter_rows(min_row=1,max_row=4,min_col=1,max_col=12,values_only=True):
        baseLakeLevel.append(np.array(row))
baseLakeLevel = np.array(baseLakeLevel)
optimalWaterLevel = np.array(optimalWaterLevel)
for month in range(1,13):
        base = baseLakeLevel[:,month-1]
        sheet = LakeData[LakeName[0]]
        k = [1,1]
        b = []
        std_dev = []
        Jan = []
        column_number = month + 1
        Jan.append([])
        plt.figure()
        for row in sheet.iter_rows(min_row=8, max_row=30, min_col=column_number, max_col=column_number, values_only=True):
                Jan[0].append(row[0])
        for i in range(1,3):
                Jan.append([])
                sheet = LakeData[LakeName[i]]
                for row in sheet.iter_rows(min_row=8, max_row=30, min_col=column_number, max_col=column_number, values_only=True):
                        Jan[i].append(row[0])
                m, n = np.polyfit(Jan[0],Jan[i], 1)
                fit_x = np.linspace(175,178, 100)
                fit_y = m * fit_x + n
                st = np.std(np.array(Jan[i])-(m * np.array(Jan[0]) + n))
                k.append(m)
                b.append(n)
        k = np.array(k) * np.array(1)
        delta = 0
        sum1 = 0
        sum2 = 0
        for i in range(0,4):
                sum1 = sum1 + (optimalWaterLevel[i][month-1] - base[i]) * k[i]
                sum2 = sum2 + k[i] * k[i]
        for i in range(0,4):
                base[i] = base[i] + (sum1/sum2) * k[i]
        updata_Sup.append(base[0])
        updata_HM.append(base[1])
        updateC.append(base[2])
        updateE.append(base[3])
        sheet = LakeData[Task2[0]]
        k = []
        b = []
        std_dev = []
        Jan = []
        column_number = month + 1
        Jan.append([])
        plt.figure()
        Nia = []
        Ontar = []
        Law = []
        for row in sheet.iter_rows(min_row=20, max_row=28, min_col=column_number, max_col=column_number, values_only=True):
                Nia.append(row[0])
```

```
        Nia.append(row[0])
        sheet = LakeData[Task2[1]]
        for row in sheet.iter_rows(min_row=20, max_row=28, min_col=column_number, max_col=column_number, values_only=True):
                Ontar.append(row[0])
        sheet = LakeData[Task2[2]]
        for row in sheet.iter_rows(min_row=20, max_row=28, min_col=column_number, max_col=column_number, values_only=True):
                Law.append(row[0])
        Nia = np.array(Nia)
        Ontar = np.array(Ontar)
        Law = np.array(Law)
        minus = Law
        m,n = np.polyfit(minus,Ontar,1)
        fit_x = np.linspace(5000,10000)
        fit_y = m * fit_x + n
        if (abs(BaseAn[month - 1] - AN[month - 1]) <= 0.2):
                BaseAn[month - 1] = AN[month - 1]
        else:
                if ((BaseAn[month - 1] - AN[month - 1]) <= -0.2):
                        BaseAn[month - 1] = BaseAn[month - 1] + 0.2
                else:
                        BaseAn[month - 1] = BaseAn[month - 1] - 0.2
        updateAn.append(BaseAn[month - 1])
for row in sheetBase.iter_rows(min_row=2,max_row=2,min_col=1,max_col=12,values_only=True):
        a = np.array(row)
plt.figure()
plt.plot(np.array(updateAn),label='Our method')
plt.plot(np.array(BasicAn),label='origin')
plt.plot(AN,label='optimal')
plt.title('Water Level for Lake Ontario')
plt.xlabel('Month')
plt.ylabel('Lake Depth')
plt.legend()
plt.figure()
plt.plot(np.array(updata_HM),label="update")
plt.plot(np.array(a),label = "origin")
plt.plot(np.array(optimalWaterLevel[1]))
plt.legend()
```

Offline Model

```
# -*- coding: utf-8 -*-
"""该案例展示了一个带约束连续决策变量的最小化目标的双目标优化问题的求解。详见MyProblem.py."""
from MyProblem3 import MyProblem  # 导入自定义问题接口
import numpy as np

import geatpy as ea  # import geatpy

if __name__ == '__main__':
        # 实例化问题对象
        problem = MyProblem()
        # 构建算法
        algorithm = ea.moea_NSGA2_templet(
                problem,
                ea.Population(Encoding='RI', NIND=50),
                MAXGEN=1000,  # 最大进化代数
                logTras=0)  # 表示每隔多少代记录一次日志信息，0表示不记录。
        # 求解
        res = ea.optimize(algorithm,verbose=False,drawing=1,outputMsg=True,drawLog=False,saveFlag=False)
        std = []
        for i in range(50):
                std.append(np.std(res["Vars"][i]))
        print(np.argmin(std))
        print(res["Vars"][np.argmin(std)])
```

```
class MyProblem(ea.Problem):  # 继承Problem父类

        def __init__(self, M=12):  # 一共有12个优化目标：分别是每个月水位与理想水位的差值
                name = 'MyProblem3'  # 初始化name（函数名称，可以随意设置）
                Dim = 12  # 初始化Dim（决策变量维数）共12个决策变量，即G2十二个月的取值
                # 初始化maxormins（目标最小最大化标记列表，1：最小化该目标；-1：最大化该目标）这里要最小化目标，目标是一个max函数
                maxormins = [1] * M
                # 初始化varTypes（决策变量的类型，0：实数；1：整数）这里决策变量由于是G1放水流量->实数
                varTypes = [0] * Dim
                # 存放G2的上界
```

```
        g2_down = [7256.834807, 7251.68629, 7890.102407, 7635.250812,
                   7655.84488, 8193.864914, 8440.993733, 8471.884835, 8235.05305,
                   7779.40929, 7558.023056, 7637.82507]
        g2_up = [7571.25529340956, 7588.45963438245, 8819.17399834767,
                 9138.02379537105, 10241.3428280286, 9477.2403949754,
                 8886.37217493604, 8714.79455469297, 8627.17134277923,
                 8426.65155083204, 8283.25214478705, 7917.3877125051]

        for i in range(6, 12):
            g2_down[i] -= 6000
            g2_up[i] -= 6000
        # 修改！ 存放G2各个月的下界！
        lb = g2_down  # 决策变量下界 存放G1各个月的下界
        # 修改！存放G2各个月的上界
        ub = g2_up  # 决策变量上界 存放G1各个月的上界
        lbin = [1] * Dim  # 决策变量下边界（0表示不包含该变量的下边界，1表示包含）
        ubin = [1] * Dim  # 决策变量上边界（0表示不包含该变量的上边界，1表示包含）
        self.S1 = 82414
        self.S2 = 143344
        self.S3 = 19000
        # 修改！存lake每个月的optimal水位 修改！！得找出optimal_h1!!!
        self.opt_h1 = [183.268260869565, 183.205217391304, 183.170869565217,
                       183.209130434783, 183.309565217391, 183.397826086957,
                       183.457826086957, 183.471304347826, 183.464347826087,
                       183.447826086957, 183.406956521739, 183.347826086957]
        self.opt_h2 = [174.97950038453766, 175.03975692512606, 175.08342108409943, 175.26419070586454, 175.48312374735477, 175.5280978282853
                       175.34034194269134, 175.14865628118287, 174.99714164171195, 174.93601181814495, 174.88780565169787]
        self.opt_h3 = [74.31583592, 74.43583592, 74.52279244, 74.88279244, 75.31879432, 75.40835953,
                       75.30401171, 75.03444649, 74.65270736, 74.35096822, 74.22922909, 74.13322723]
        # 存放每个月平均的水位 real_h1 real_h2 real_h3
        self.real_h1 = [183.268260869565, 183.205217391304, 183.170869565217,
                        183.209130434783, 183.309565217391, 183.397826086957,
                        183.457826086957, 183.471304347826, 183.464347826087,
                        183.447826086957, 183.406956521739, 183.347826086957]
        self.real_h2 = [175.096086956522, 175.078260869565, 175.14652173913,
                        175.250869565217, 175.340579710145, 175.402028985507,
                        175.416086956522, 175.369565217391, 175.293043478261,
                        175.202463768116, 175.135942028985, 175.119420289855]
        self.real_h3 = [74.6608695652174, 74.7208695652174, 74.764347826087,
                        74.944347826087, 75.1091304347826, 75.1539130434783,
                        75.1017391304348, 74.9669565217391, 74.7760869565217,
                        74.6252173913043, 74.5643478260869, 74.5695652173913]
        # 存放Mary river, Niagara river,law river的月流量
        self.X_1 = [1994.67893508928, 1944.74015828652, 1939.98697377916,
                    1916.1401459742, 2031.04585314979, 2256.08385667656,
                    2480.35326015534, 2507.59810922513, 2356.30525767199,
                    2308.591375745, 2242.12769791029, 2057.34940054137]
        self.X_2 = [5851.42857142857, 5788.57142857143, 5894.28571428571,
                    6115.71428571429, 6318.09523809524, 6253.33333333333, 6220,
                    6084.28571428571, 5857.61904761905, 5839.04761904762,
                    5876.19047619048, 5963.80952380952]
        self.X_3 = [7256.834807,  7251.68629, 7890.102407, 7635.250812, 7655.84488,
                    8193.864914, 8440.993733, 8471.884835, 8235.05305, 7779.40929, 7558.023056, 7637.82507]
        # 调用父类构造方法完成实例化
        ea.Problem.__init__(self,name,M,maxormins,Dim,varTypes,lb,ub,lbin,ubin)

    def evalVars(self, Vars):
        coff_1 = 3600*24*30
        coff_2 = 10**6
        F = []
        h3 = []
        for j in range(12):
            F3 = (self.real_h3[j]-self.real_h3[j-1]) * \
                self.S3*coff_2 + (self.X_2[j]-self.X_3[j])*coff_1
            tmp_h3 = self.real_h3[-1]
            for k in range(j+1):
                tmp_h3 += (F3-(Vars[:, [j]]-self.X_2[j])
                                     * coff_1)/(self.S3*coff_2)
            h3.append(tmp_h3)
        for j in range(12):
            f = abs(h3[j]-self.opt_h3[j])**2
            F.append(f)
        # 无可行性检验
        # CV = 0
        ObjV = np.hstack(F)
        return ObjV
```