Constructing three-way decision with fuzzy granular-ball rough sets based on uncertainty invariance (Supplementary materials)

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This is the supplementary file for the paper entitled Constructing three-way decision of fuzzy granular-ball rough sets from the perspective of uncertainty in IEEE Transactions on Fuzzy Systems.

S1 THE RELATED ALGORITHMS

Algorithm S1: Generate-GB_list [1]

Input: $\mathbb{D}$: the dataset
Output: GB_list:the set of granular balls

```
Initializing: n = |D|, gb = D, GB_list = ∅; Add gb to an empty queue Q;
while Q is not empty do
    Get the first element gb from Q and delete it from Q; if the size of gb is larger than √n then
        Employ 2-means algorithm to divide gb into two sub-balls Sub1 and Sub2;
        Add Sub1 and Sub2 to the tail of Q;
    end
    if the size of gb is less than or equal to √n then
        Compute the center c and radius r of gb according to Eq. 2-3;
        GB_list = GB_list ∪ gb;
    end
end
Return GB_list;
```

S2 THE PROOFS OF RELATED THEOREMS

Theorem 1. *Let $S = (U, AT \cup D, V, f)$ be a decision system, Seq_GBS $= (U/GB(B_1), U/GB(B_2), \ldots, U/GB(B_N))$ and X be a target fuzzy subset on U. Then, $F_{(X^J_{GB_{j+1}})} \le F_{(X^J_{GB_j})}$ holds.*

Proof. Suppose $U/GB(B_j) = \{gb^j_1, gb^j_2, \ldots, gb^j_m\}$ and $U/GB(B_{j+1}) = \{gb^{j+1}_1, gb^{j+1}_2, \ldots, gb^{j+1}_l\}$ are two granular-ball spaces in Seq_GBS, respectively. Because $U/GB(B_{j+1}) \subseteq U/GB(B_j)$, to simplify the proof, suppose that only a GB gb^j_1 is subdivided into two GBs gb^{j+1}_1 and gb^{j+1}_2 from $U/GB(B_j)$ to $U/GB(B_{j+1})$, while no change in other granular balls (more complex situations can be translated into this case, so no more repeat here). Based on the above assumptions, we have $gb^j_1 = gb^{j+1}_1 \cup gb^{j+1}_2, gb^j_2 = gb^{j+1}_3, gb^j_3 = gb^{j+1}_4, \ldots, gb^j_m = gb^{j+1}_l (l = m + 1)$. That is, $[x]_{GB_j} = \{gb^{j+1}_1, gb^{j+1}_2, gb^j_2, gb^j_3, \ldots, gb^j_m\}$.

$$
\begin{aligned}
&F_{(X^J_{GB_{j+1}})} - F_{(X^J_{GB_j})} \\
&= \frac{4}{|U|}\Big(h(gb^j_1) - h(gb^{j+1}_1) - h(gb^{j+1}_2)\Big) \\
&= \frac{4}{|U|}\Big(\bar{\mu}(gb^j_1)(1 - \bar{\mu}(gb^j_1)) - \bar{\mu}(gb^{j+1}_1)(1 - \bar{\mu}(gb^{j+1}_1)) \\
&\quad - \bar{\mu}(gb^{j+1}_2)(1 - \bar{\mu}(gb^{j+1}_2))\Big)
\end{aligned}
$$

Because $gb^j_1 = gb^{j+1}_1 \cup gb^{j+1}_2$, then $\bar{\mu}(gb^j_1) = \frac{|gb^{j+1}_1|}{|gb^j_1|}\bar{\mu}(gb^{j+1}_1) + \frac{|gb^{j+1}_2|}{|gb^j_1|}\bar{\mu}(gb^{j+1}_2)$, we have

$$
\begin{aligned}
&F_{(X^J_{GB_{j+1}})} - F_{(X^J_{GB_j})} \\
&= 4\frac{|gb^{j+1}_1||gb^{j+1}_2|}{|U||gb^j_1|}(\bar{\mu}(gb^{j+1}_1) - \bar{\mu}(gb^{j+1}_2))^2 \ge 0
\end{aligned}
$$

Therefore, $F_{(X^J_{GB_{j+1}})} \le F_{(X^J_{GB_j})}$. □

Theorem 2. *Let $S = (U, AT \cup D, V, f)$ be a decision system, Seq_GBS $= (U/GB(B_1), U/GB(B_2), \ldots, U/GB(B_N))$ and X be a target fuzzy subset on U. Then, $|F_{X^J_{GB_{j+1}}} - F_{X^J_{GB_N}}| \le |F_{X^J_{GB_j}} - F_{X^J_{GB_N}}|$ holds.*

Proof. From Corollary 1, $F_{X^J_{GB_j}} \ge F_{X^J_{GB_N}}$ holds. Then, from Definition 7, we have

$$
\begin{aligned}
&|F_{X^J_{GB_{j+1}}} - F_{X^J_{GB_N}}| - |F_{X^J_{GB_j}} - F_{X^J_{GB_N}}| \\
&= F_{X^J_{GB_{j+1}}} - F_{X^J_{GB_j}}
\end{aligned}
$$

Obviously, according to Theorem 1, $|F_{X^J_{GB_{j+1}}} - F_{X^J_{GB_N}}| \le |F_{X^J_{GB_j}} - F_{X^J_{GB_N}}|$ holds. □

Theorem 3. *Let $S = (U, AT \cup D, V, f)$ be a decision system, Seq_GBS $= (U/GB(B_1), U/GB(B_2), \ldots, U/GB(B_N))$ and X be a target fuzzy subset on U. If only the GBs contained in $POS(X^J_{GB_j})$ are subdivided into finer GBs from $U/GB(B_j)$ to $U/GB(B_{j+1})$, then $F(BND(X^J_{GB_{j+1}})) \ge F(BND(X^J_{GB_j}))$ holds.*

Proof. Based on the assumption in the proof of Theorem 1, the theorem can be proved by the following three cases:

Case 1. When $\bar{\mu}(gb_1^{j+1}) \geq \alpha$ and $\bar{\mu}(gb_2^{j+1}) \leq \beta$, that is, $gb_1^{j+1} \subseteq POS(X^J_{GB_{j+1}})$ and $gb_2^{j+1} \subseteq NEG(X^J_{GB_{j+1}})$, because $gb_2^j = gb_3^{j+1}, gb_3^j = gb_4^{j+1}, \ldots, gb_m^j = gb_l^{j+1} (l = m+1)$, $BND(X^J_{GB_j}) = BND(X^J_{GB_{j+1}})$, then $F(BND(X^J_{GB_{j+1}})) = F(BND(X^J_{GB_j}))$.

Case 2. When $\bar{\mu}(gb_1^{j+1}) \geq \alpha$ and $\bar{\mu}(gb_2^{j+1}) \geq \alpha$, that is, $gb_1^{j+1} \subseteq POS(X^J_{GB_{j+1}})$ and $gb_2^{j+1} \subseteq POS(X^J_{GB_{j+1}})$, because $gb_2^j = gb_3^{j+1}, gb_3^j = gb_4^{j+1}, \ldots, gb_m^j = gb_l^{j+1} (l = m+1)$, $BND(X^J_{GB_j}) = BND(X^J_{GB_{j+1}})$, then $F(BND(X^J_{GB_{j+1}})) = F(BND(X^J_{GB_j}))$.

Case 3. When $\bar{\mu}(gb_1^{j+1}) \geq \alpha$ and $\beta \leq \bar{\mu}(gb_2^{j+1}) \leq \alpha$, that is, $gb_1^{j+1} \subseteq POS(X^J_{GB_{j+1}})$ and $gb_2^{j+1} \subseteq BND(X^J_{GB_{j+1}})$, $BND(X^J_{GB_{j+1}}) = BND(X^J_{GB_j}) \cup gb_2^{j+1}$, then $F(BND(X^J_{GB_{j+1}})) = F(BND(X^J_{GB_j})) + F(gb_2^{j+1}) > F(BND(X^J_{GB_j}))$.

Therefore, $F(BND(X^J_{GB_{j+1}})) \geq F(BND(X^J_{GB_j}))$ holds. □

Theorem 4. *Let $S = (U, AT \cup D, V, f)$ be a decision system, Seq_GBS $= (U/GB(B_1), U/GB(B_2), \ldots, U/GB(B_N))$, and X be a target fuzzy subset on U. If only the GBs contained in $NEG(X^J_{GB_j})$ are subdivided into finer GBs from $U/GB(B_j)$ to $U/GB(B_{j+1})$, then $F(BND(X^J_{GB_{j+1}})) \geq F(BND(X^J_{GB_j}))$ holds.*

Proof. From the proof of Theorem 3, Theorem 4 is obviously easy to prove. □

Theorem 5. *Let $S = (U, AT \cup D, V, f)$ be a decision system, Seq_GBS $= (U/GB(B_1), U/GB(B_2), \ldots, U/GB(B_N))$, and X be a target fuzzy subset on U. If only the GBs contained in $BND(X^J_{GB_j})$ are subdivided into finer GBs from $U/GB(B_j)$ to $U/GB(B_{j+1})$, then $F(BND(X^J_{GB_{j+1}})) \leq F(BND(X^J_{GB_j}))$ holds.*

Proof. Based on the assumption in proof of Theorem 1, the theorem can be proved by the following four cases:

Case 1. When $\beta \leq \bar{\mu}(gb_1^{j+1}) \leq \alpha$ and $\beta \leq \bar{\mu}(gb_2^{j+1}) \leq \alpha$, that is, $gb_1^{j+1} \subseteq BND(X^J_{GB_{j+1}})$ and $gb_2^{j+1} \subseteq BND(X^J_{GB_{j+1}})$, because $gb_2^j = gb_3^{j+1}, gb_3^j = gb_4^{j+1}, \ldots, gb_m^j = gb_l^{j+1}$ $(l = m+1)$, $BND(X^J_{GB_j}) = BND(X^J_{GB_{j+1}})$,$\bar{\mu}(gb_1^j) = \frac{|gb_1^{j+1}|}{|gb_1^j|}\bar{\mu}(gb_1^{j+1}) + \frac{|gb_2^{j+1}|}{|gb_1^j|}\bar{\mu}(gb_2^{j+1})$, we have

$$\begin{aligned}&F(BND(X^J_{GB_{j+1}})) - F(BND(X^J_{GB_j}))\\&= F(gb_1^j) - (F(gb_1^{j+1}) + F(gb_2^{j+1}))\\&= 4\frac{|gb_1^{j+1}||gb_2^{j+1}|}{|U||gb_1^j|}(\bar{\mu}(gb_1^{j+1}) - \bar{\mu}(gb_2^{j+1}))^2 \geq 0.\end{aligned}$$

Therefore, $F(BND(X^J_{GB_{j+1}})) \leq F(BND(X^J_{GB_j}))$.

Case 2. When $\beta \leq \bar{\mu}(gb_1^{j+1}) \leq \alpha$ and $\bar{\mu}(gb_2^{j+1}) \geq \alpha$, that is, $gb_1^{j+1} \subseteq BND(X^J_{GB_{j+1}})$ and $gb_2^{j+1} \subseteq POS(X^J_{GB_{j+1}})$, $BND(X^J_{GB_{j+1}}) = BND(X^J_{GB_j}) \cup gb_2^{j+1}$, then $F(BND(X^J_{GB_j})) = F(BND(X^J_{GB_{j+1}})) + F(gb_2^{j+1}) > F(BND(X^J_{GB_{j+1}}))$. Therefore, $F(BND(X^J_{GB_{j+1}})) < F(BND(X^J_{GB_j}))$.

Case 3. When $\bar{\mu}(gb_1^{j+1}) \leq \beta$ and $\beta \leq \bar{\mu}(gb_2^{j+1}) \leq \alpha$, that is, $gb_1^{j+1} \subseteq NEG(X^J_{GB_{j+1}})$ and $gb_2^{j+1} \subseteq BND(X^J_{GB_{j+1}})$, $BND(X^J_{GB_{j+1}}) = BND(X^J_{GB_j}) \cup gb_2^{j+1}$, $F(BND(X^J_{GB_j})) = F(BND(X^J_{GB_{j+1}})) + F(gb_2^{j+1}) > F(BND(X^J_{GB_{j+1}}))$. Therefore, $F(BND(X^J_{GB_{j+1}})) < F(BND(X^J_{GB_j}))$ holds.

Case 4. When $\bar{\mu}(gb_1^{j+1}) \leq \beta$ and $\bar{\mu}(gb_2^{j+1}) \geq \alpha$, that is, $gb_1^{j+1} \subseteq NEG(X^J_{GB_{j+1}})$ and $gb_2^{j+1} \subseteq POS(X^J_{GB_{j+1}})$, $BND(X^J_{GB_{j+1}}) < BND(X^J_{GB_j})$.Because $gb_2^j = gb_3^{j+1}, gb_3^j = gb_4^{j+1}, \ldots, gb_m^j = gb_l^{j+1} (l = m+1)$

Therefore, $F(BND(X^J_{GB_{j+1}})) \leq F(BND(X^J_{GB_j}))$. □

Theorem 6. *Let $S = (U, AT \cup D, V, f)$ be a decision system, X be a target fuzzy subset on U. $U/GB(B_1)$, $U/GB(B_2)$, and $U/GB(B_3)$ are three granular-ball spaces in Seq_GBS. If $U/GB(B_3) \preceq U/GB(B_2) \preceq U/GB(B_1)$, then $GBSD(U/GB(B_1), U/GB(B_2)) \leq GBSD(U/GB(B_1), U/GB(B_3))$ holds.*

Proof. Suppose $U/GB(B_1) = \{gb_1^1, gb_2^1, \ldots, gb_m^1\}$, $U/GB(B_2) = \{gb_1^2, gb_2^2, \ldots, gb_l^2\}$, and $U/GB(B_3) = \{gb_1^3, gb_2^3, \ldots, gb_p^3\}$ are three granule-ball spaces in Seq_GBS, respectively. Because $U/GB(B_3) \subseteq U/GB(B_2) \subseteq U/GB(B_1)$, to simplify the proof, suppose that only a GB gb_1^1 is subdivided into two GBs gb_1^2 and gb_2^2 from $U/GB(B_1)$ to $U/GB(B_2)$, and only a granular ball gb_1^2 is subdivided into gb_1^3 and gb_2^3 from $U/GB(B_2)$ to $U/GB(B_3)$. Based on the above assumptions, we have $gb_1^1 = gb_1^2 \cup gb_2^2$, $gb_2^1 = gb_3^2$, $gb_3^1 = gb_4^2$, $\ldots$, $gb_m^1 = gb_l^2$ $(l = m+1)$ and $gb_1^2 = gb_1^3 \cup gb_2^3$, $gb_2^2 = gb_3^3$, $gb_3^2 = gb_4^3$, $\ldots$, $gb_l^2 = gb_p^3$ $(p = l+1)$. That is, $U/GB(B_2) = \{gb_1^2, gb_2^2, gb_2^1, gb_3^1, \ldots, gb_m^1\}$ and $U/GB(B_3) = \{gb_1^3, gb_2^3, gb_2^2, gb_3^2, \ldots, gb_l^2\}$. We have

$$\begin{aligned}&GBSD(U/GB(B_1), U/GB(B_2))\\&= \frac{(\bar{\mu}(gb_1^1) - \bar{\mu}(gb_1^2))|gb_1^2|}{|U|} + \frac{(\bar{\mu}(gb_1^1) - \bar{\mu}(gb_2^2))|gb_2^2|}{|U|},\\&GBSD(U/GB(B_1), U/GB(B_3))\\&= \frac{(\bar{\mu}(gb_1^1) - \bar{\mu}(gb_1^3))|gb_1^3|}{|U|} + \frac{(\bar{\mu}(gb_1^1) - \bar{\mu}(gb_2^3))|gb_2^3|}{|U|}\\&+ \frac{(\bar{\mu}(gb_1^1) - \bar{\mu}(gb_3^3))|gb_3^3|}{|U|}.\end{aligned}$$

Because $gb_1^1 = gb_1^2 \cup gb_2^2$ and $gb_1^2 = gb_1^3 \cup gb_2^3$, $\bar{\mu}(gb_1^1) = \bar{\mu}(gb_1^2) + \bar{\mu}(gb_2^2)$, $\bar{\mu}(gb_1^2) = \bar{\mu}(gb_1^3) + \bar{\mu}(gb_2^3)$, and $|gb_1^1| = |gb_1^2| + |gb_2^2|$, $|gb_1^2| = |gb_1^3| + |gb_2^3|$.

$$\begin{aligned}&GBSD(U/GB(B_1), U/GB(B_3)) - GBSD(U/GB(B_1), U/GB(B_2))\\&= \frac{\bar{\mu}(gb_1^3)|gb_2^3| + \bar{\mu}(gb_2^3)|gb_1^3|)}{|U|} \geq 0.\end{aligned}$$

Therefore, $GBSD(U/GB(B_1), U/GB(B_2)) \leq GBSD(U/GB(B_1), U/GB(B_3))$. Similarly, we can draw $GBSD(U/GB(B_2), U/GB(B_3)) \leq GBSD(U/GB(B_1), U/GB(B_3)$. □

Theorem 7. *Let $S = (U, AT \cup D, V, f)$ be a decision system, $Seq\_GBS = (U/GB(B_1), U/GB(B_2), \ldots, U/GB(B_N))$, and X be a target fuzzy subset on U. Then, $GBSD(U/GB(B_j), U/GB(B_N))$ is a granularity measure.*

Proof. (1) Obviously, $GBSD(U/GB(B_j), U/GB(B_N)) \geq 0$;
(2) When $U/GB(B_j) = U/GB(B_k)$ and $U/GB(B_j), U/GB(B_k) \in Seq\_GBS$, we have $GBSD(U/GB(B_j), U/GB(B_N)) = GBSD(U/GB(B_k), U/GB(B_N))$;
(3) From Theorem 6, if $U/GB(B_k) \prec U/GB(B_j)$, then $GBSD(U/GB(B_j), U/GB(B_N)) > GBSD(U/GB(B_k), U/GB(B_N))$. □

S3 THE REAL-TIME APPLICATIONS OF 3WC- FGBRS

We have added a case study in the paper.Our proposed method is a versatile classifier that can be applied in various real-time applications. In our selected UCI datasets, the Rice dataset includes 7 attributes extracted from images to identify different rice varieties. These attributes are derived from images of rice samples, each labeled to indicate the rice variety. As shown in the Table S1S1, A_1 to A_7 represent continuous attributes, while D is the label column, where 0 and 1 indicate the rice varieties Cammeo or Osmancik.

TABLE S1: Rice's data

	A_1	A_2	A_3	…	A_7	D
X_1	15231	525.579	229.7499	…	0.572896	1
X_2	14656	494.311	206.0201	…	0.615436	1
⋮	⋮	⋮	⋮	…	⋮	⋮
X_3810	11434	404.71	161.0793	…	0.802949	0

In our experiments, we demonstrated that 3WC-FGBRS outperforms other classifiers. We selected several well-performing, representative comparison algorithms, and the comparison results are shown in the Figure.S1. As analyzed in the paper, the combination with GB gives 3WC-FGBRS an advantage over traditional three-way classifiers. The classification process, which incorporates the shadow set and our proposed classification rules, provides a further edge over conventional GB classifiers. Additionally, comparing 3WC-FGBRS with CART algorithms offers a more comprehensive illustration of its strengths.

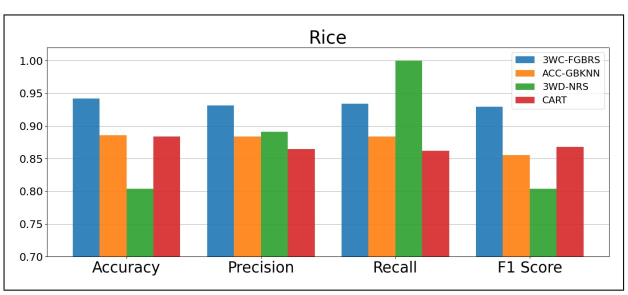


Fig. S1: Metrics of each algorithm.

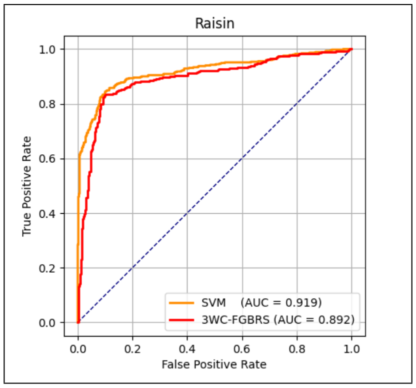
Overall, 3WC-FGBRS can efficiently and accurately classify these instances. When combined with an appropriate image recognition algorithm, it can serve as a simple yet effective rice varieties detector. We believe this capability highlights its potential for real-time applications in fields that require immediate classification decisions.

S4 THE CLASSIFICATION CAPABILITY ASSESSMENT OF 3WC- FGBRS

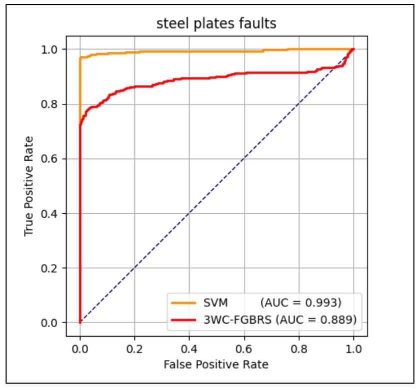
Regarding the ROC curves, since 3WC- FGBRS is not a probabilistic classifier, we had to use an alternative method to generate pseudo-probabilities. We followed the approach used in k-NN, utilizing the membership degree of the closest k spheres as the probability output. We compared the results with SVM, one of the best-performing traditional classifiers. The ROC curves and AUC values results are shown in Figure S1. It is evident that in datasets Mushroom where SVM performs poorly, 3WC- FGBRS still maintains strong classification ability. However, in some datasets, 3WC- FGBRS's performance is not as strong as SVM. This does not imply that 3WC- FGBRS has inferior classification ability. The main reason is that the simple membership degree used as a pseudo-probability output does not fully represent 3WC- FGBRS's classification capability. The main limitations of this pseudo-probability approach are as follows:

(1) Neglecting the role of the shadow set. The mapping of the shadow set plays a crucial role in classification, but the pseudo-probability approach does not account for the shadow set's mapping relationship. Currently, there is no research on how to convert this mapping into a probabilistic representation.
(2) Overlooking the classification rules of 3WC- FGBRS. The pseudo-probability only uses the sample's membership degree to the GB as the output, without incorporating the classification rules proposed in our method. In our classification rules, both the membership degree and the sample's position within the GB region must be considered. The latter cannot be directly converted into a probabilistic output.
(3) Ignoring the importance of three-way classification. Three-way classification is a significant innovation over binary classification and effectively reduces classification risk. However, in calculating the ROC curve, the traditional binary classification metrics (FP and TP) are used, without considering the impact of the uncertain class.

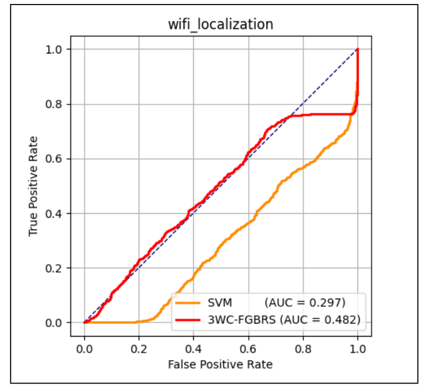
To date, there is no probabilistic classifier that fully captures the characteristics of GBs and shadow sets. Using the pseudo-probability outlined above to plot ROC curves only demonstrates the classification ability of GBs under traditional binary classification scenarios. Thus, we believe that the pseudo-probabilities generated by existing methods only partially represent 3WC- FGBRS 's true capabilities, and we have decided not to include these results in the main paper. Further research is needed to refine this aspect, and this will be one of the directions for our future work.



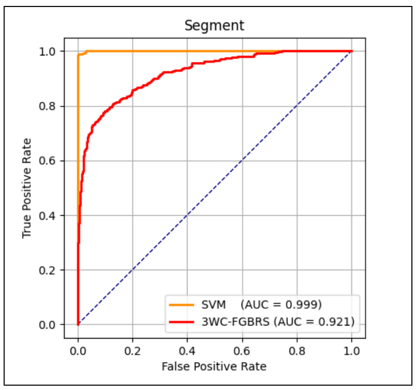
(a) Raisin



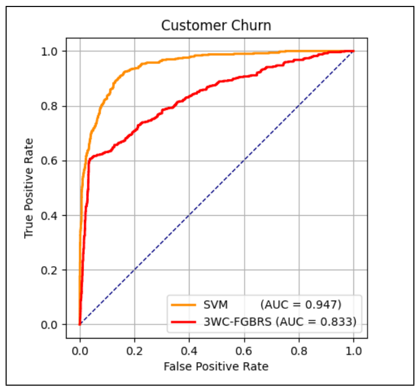
(b) SPF



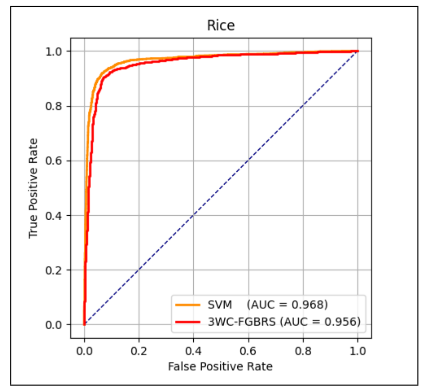
(c) Wifi_Loc



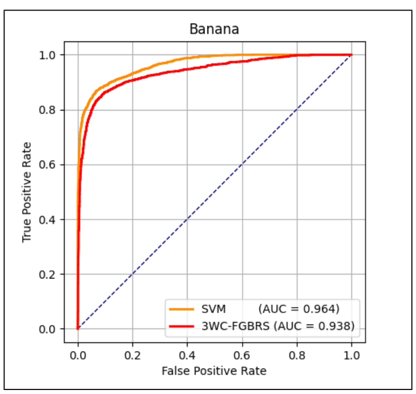
(d) Segment



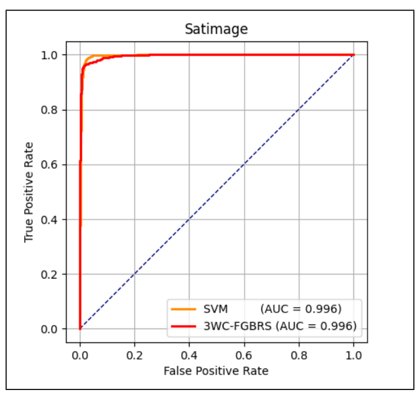
(e) Iranian Churn



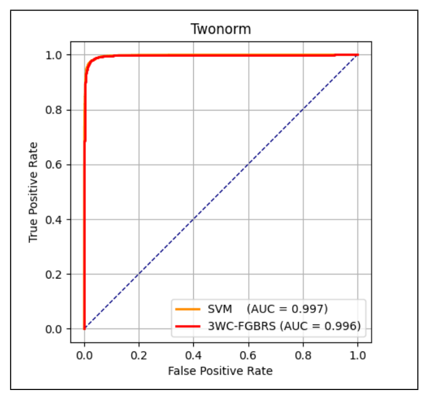
(f) Rice



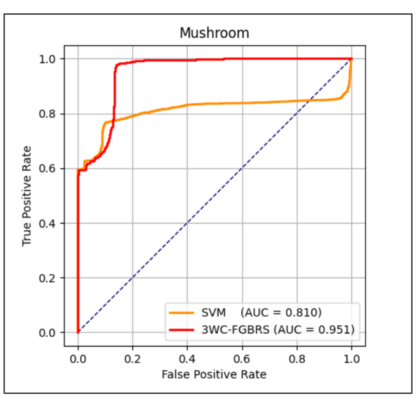
(g) Banana



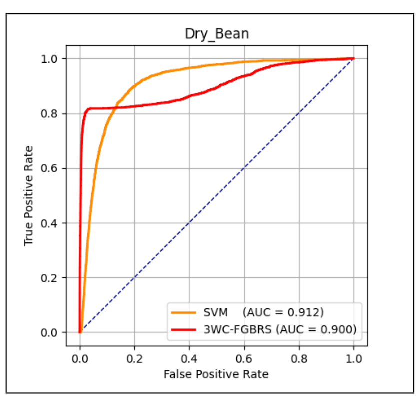
(h) Satimage



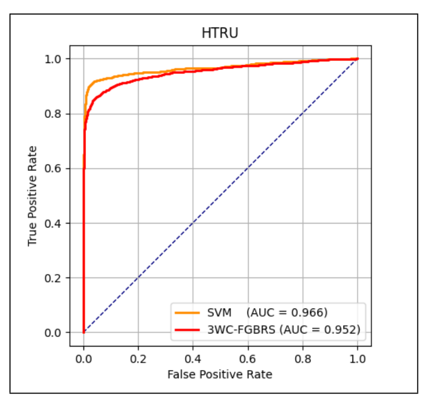
(i) Twonorm



(j) Mushroom



(k) Dry Bean

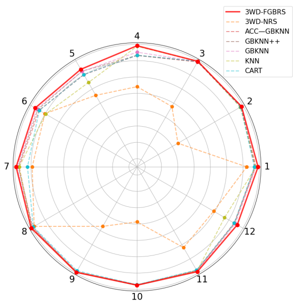


(l) HTRU

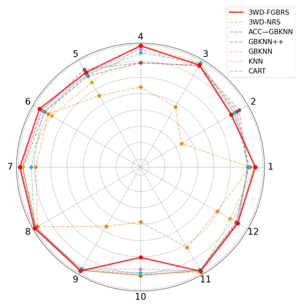
Fig. S2: The ROC curve and AUC value of 3WC- FGBRS.

S5 PERFORMANCE ANALYSIS FIGURE ABOUT THE 3WC- FGBRS WITH ALL COMPARISON CLASSIFIERS.

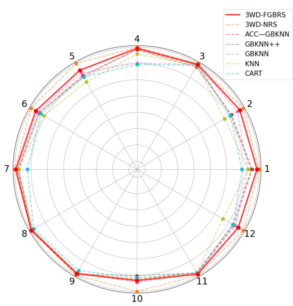
3WC-FGBRS has better performance than all the compared classifiers in various indicators. In order to better show the performance of 3WC-FGBRS on various metrics, we use the radar chart to visually show the excellent performance of 3WC-FGBRS, as shown in FIG.S3.



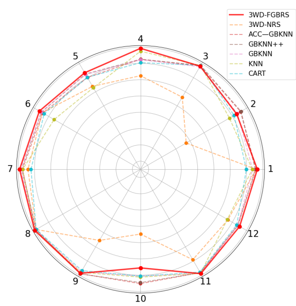
(a) Accuracy



(b) Precision



(c) Recall



(d) F1 score

Fig. S3: Performance analysis figure about the 3WC- FGBRS with all comparison classifiers.

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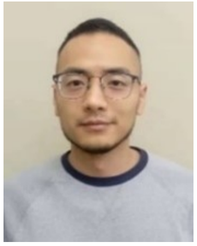

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