# NLOS Error Mitigation in Hybrid RSS-TOA-Based Localization Through Semi-Definite Relaxation

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***Abstract*— This letter focuses on the accurate target localization in non-line of sight (NLOS) environment through effective hybridization of time of arrival (TOA) and received signal strength (RSS) measurements. A non-linear weighted least square (NLWLS) problem is formulated from general approximation of hybrid maximum-likelihood (ML) estimation problem using balancing parameter i.e., mean NLOS effect. The NLWLS problem is then solved using semi-definite relaxation (SDR) technique which employs a novel convex hull constraint. The proposed method requires neither prior NLOS statistics nor NLOS path identification. The numerical simulations validate that the proposed method significantly outperforms the existing methods and achieves the performance comparatively closer to that of Cramer-Rao lower bound (CRLB).**

***Index Terms*— Time of arrival (TOA), received signal strength (RSS), non-line of sight (NLOS), semi-definite relaxation (SDR).**

## I. Introduction

THE accuracy in target localization is one of the crucial aspects of indoor and outdoor wireless networks [1]–[9], especially in the applications where exact target positioning is required. Such applications include intelligent transportation systems, industrial automation, smart homes and cities and environmental monitoring [3]–[6]. The localization accuracy depends on line of sight (LOS) path and gets critically affected by the non-line of sight (NLOS) propagation errors [5]–[11]. Most of the localization methods trust on the extraction of range information through received signal strength (RSS) or time of arrival (TOA) measurements [1]–[11]. However, the RSS is more effective for short range applications whereas TOA is mostly preferred for considerably long range applications. The hybrid of RSS and TOA has been recently explored in [1]–[3], [10]–[15] to utilize their combined merits.

The maximum-likelihood (ML) relaxations and least squares methods were used in hybrid RSS-TOA localization procedures [1], [12]–[15] to provide range/target location estimation. However, these methods require prior knowledge of NLOS errors and path status to mitigate the NLOS effects. Both, prior statistics of NLOS errors and the path status can be unavailable due to dynamic communication environments [7]–[9]. Without any prior NLOS errors statistics, a hybrid closed-form weighted least square (CFWLS) method based on orientation information was formulated in [2]. But, its localization accuracy was highly susceptible to the noise levels due to perturbed angle estimation. The iterative generalized trust region sub-problem (GTRS) framework was proposed in [10] to approximate the non-convex hybrid ML estimation problem. The performance of GTRS model was further improved in [11] by considering the knowledge of upper bound on the NLOS errors. Even though, the existing works in [2], [10]–[13] employed low cost localization algorithms, the target estimation, however, was sub-optimal in ML sense. The sub-optimality of existing methods is the primary motivation for this work.

The ML approximation through convex-relaxation techniques can effectively mitigate NLOS and can perform significantly better than GTRS methods [8], [9]. However, the accuracy of the solution should be ensured in mixed LOS/NLOS conditions [7]–[9]. In this letter, we transform non-convex hybrid ML problem into non-linear weighted least square problem (NLWLS) by considering average effect of NLOS biases. Ultimately, the NLWLS problem is solved through semi-definite relaxation, and is further tightened using a convex hull constraint and regularization factor. The proposed method requires neither prior NLOS statistics nor NLOS path identification. Numerical simulations confirm that the proposed method significantly outperforms the existing methods and achieves the performance comparatively closer to that of Cramer-Rao lower bound (CRLB).

*Notations:* The scalar, vectors and matrices are represented by regular, bold lowercase and bold uppercase, respectively. $\mathbf{1}_N$ and $\boldsymbol{I}_N$ indicate column vector with all 1 and size $N$ and identity matrix of size $N$, respectively. $\mathbf{diag}(\boldsymbol{m})$ represents the diagonal matrix with elements of row/column vector $\boldsymbol{m}$ and $E\{\bullet\}$ represents expectation operator.

## II. Measurement Model and Problem Formulation

Consider a $k-$dimensional ($k = 2$ or 3) wireless network which consists of $N$ anchors and an unknown target node. Assuming that the anchors are positioned at $\boldsymbol{a}_i \in \mathbb{R}^k, i = 1, ..., N$ whereas the true position of target node which is to be estimated is defined as $\boldsymbol{u} \in \mathbb{R}^k$. The anchor nodes broadcast a radio signal and the target node extracts distance information from TOA and RSS observations on the received signal. The propagation delay, $\tau_i$ (in secs), and path loss, $L_i$ (in dB), of the signal between $i^{th}$ anchor and target can be given as

$$\begin{aligned} c\tau_i &= d_i + \alpha_i + n_{t_i} \\ L_i &= L_o + 10\gamma \log_{10} d_i + \beta_i + n_{r_i}, \end{aligned} \tag{1}$$

where $d_i = ||\boldsymbol{u} - \boldsymbol{a}_i||$ is true distance between $i^{th}$ anchor and target node, $L_o$ (in dB) is reference path loss of the signal

Manuscript received June 27, 2020; revised August 4, 2020; accepted August 27, 2020. Date of publication September 1, 2020; date of current version December 10, 2020. This work is supported in part by Centre of Excellence (CoE) on Commbedded Systems, funded by NPIU under TEQIP III. The associate editor coordinating the review of this letter and approving it for publication was W. Zhang. *(Corresponding author: Mayur Katwe.)*

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Digital Object Identifier 10.1109/LCOMM.2020.3020948

realized at distance 1m, $\gamma$ is path loss exponent (PLE) and $c = 3\times 10^8$ is propagation speed of the signal. The unavoidable measurements error in TOA and RSS observations, $n_{t_i}$ and $n_{r_i}$, are modeled as zero mean Gaussian random variable with variance of $\sigma_{t_i}^2$ and $\sigma_{r_i}^2$, respectively. $\alpha_i$ (in m) and $\beta_i$ (in dB) are unknown NLOS biases observed in TOA and RSS, respectively. The proposed analysis is valid for any arbitrary distribution of NLOS errors with general assumption of positive NLOS biases [8]–[11].

Let us collectively define all measurements as $\boldsymbol{T} = [\tau_1, ..., \tau_N]^T \in \mathbb{R}^N$, $\boldsymbol{L} = [L_1, ..., L_N]^T \in \mathbb{R}^N$ and the parameter of interest as $\boldsymbol{\xi} = [\boldsymbol{u}^T, \alpha_1, ..., \alpha_N, \beta_1, ..., \beta_N]^T \in \mathbb{R}^{2N+k}$. With the assumption[1] of independent and uncorrelated RSS and TOA measurements [1], [3], the joint probability density function of noisy measurements is given as

$$\begin{aligned}\mathcal{P}(\boldsymbol{T}, \boldsymbol{L}|\boldsymbol{\xi}) &= \frac{1}{(2\pi)^N}\prod_{i=1}^{N}\frac{1}{\sigma_{t_i}}\exp\left\{\frac{-(c\tau_i - d_i - \alpha_i)^2}{2\sigma_{t_i}^2}\right\}\\ &\times\prod_{i=1}^{N}\frac{1}{\sigma_{r_i}}\exp\left\{\frac{-(L_i - L_o - 10\gamma\log_{10}d_i - \beta_i)^2}{2\sigma_{r_i}^2}\right\}. \quad (2)\end{aligned}$$

The unbiased ML estimation of $\boldsymbol{\xi}$ is given as

$$\min_{\boldsymbol{\xi}}\sum_{i=1}^{N}\frac{(c\tau_i - ||\boldsymbol{u}-\boldsymbol{a}_i|| - \alpha_i)^2}{\sigma_{t_i}^2} + \sum_{i=1}^{N}\frac{(L_i - L_o - 10\gamma\log_{10}||\boldsymbol{u}-\boldsymbol{a}_i|| - \beta_i)^2}{\sigma_{r_i}^2}. \quad (3)$$

It should be noted that (3) is highly non-convex due to which estimation of $\boldsymbol{\xi}$ is nearly intractable. The non-convex optimization problem (3) can be relaxed into its equivalent convex form which will be discussed in the following section.

## III. NLOS Mitigation for Hybrid Localization Through Semi-Definite Relaxation

The problem of maximizing likelihood function in (3) has $2N + k$ unknowns and $2N$ measurements due to which ML problem is under-determined and its solution does not converge. To deal with this, we consider the average effect of NLOS errors and introduce balancing parameter in all the measurements [9], [10]. Though, this approximation partially alleviates NLOS effects, it simplifies ML problem from $2N+k$ to only $k+2$ optimization variables. The balancing parameters, $\alpha$ and $\beta$ in TOA and RSS measurements, respectively, are included as

$$\begin{aligned}c\tau_i &= ||\boldsymbol{u}-\boldsymbol{a}_i|| + \alpha + n_{t_i}\\ L_i &= L_o + 10\gamma\log_{10}||\boldsymbol{u}-\boldsymbol{a}_i|| + \beta + n_{r_i}. \quad (4)\end{aligned}$$

Now, the RSS expression in (4) can be rearranged as

$$10^{-\frac{L_i - L_o}{10\gamma}}||\boldsymbol{u}-\boldsymbol{a}_i|| = 10^{-\frac{\beta + n_{r_i}}{10\gamma}}. \quad (5)$$

For sufficiently low noise ($\beta_i + n_{r_i} < 10\gamma$), the right side expression of (5) can be approximated[2] into $1 - \frac{(\beta + n_{r_i})\ln 10}{10\gamma}$ using first order Taylor series expansion. So, (4) can be rewritten as

$$\begin{aligned}\mu_i - ||\boldsymbol{u}-\boldsymbol{a}_i|| - \alpha &= n_{t_i}\\ \eta - \eta\lambda_i||\boldsymbol{u}-\boldsymbol{a}_i|| - \beta &\approx n_{r_i}, \quad (6)\end{aligned}$$

where $\mu_i = c\tau_i, \lambda_i = 10^{\frac{-(Li - Lo)}{10\gamma}}$ and $\eta = \frac{10\gamma}{\ln 10}$.

Using (6), the ML problem in (3) can be approximated as

$$\min_{\boldsymbol{u},\alpha,\beta}\sum_{i=1}^{N}\frac{(\mu_i - ||\boldsymbol{u}-\boldsymbol{a}_i|| - \alpha)^2}{\sigma_{t_i}^2} + \sum_{i=1}^{N}\frac{(\eta - \eta\lambda_i||\boldsymbol{u}-\boldsymbol{a}_i|| - \beta)^2}{\sigma_{r_i}^2}. \quad (7)$$

To exploit range-based characteristics of RSS and TOA measurements, a hybrid weighing method with weighted linear combination of range measurements is used. For weighing coefficients, the average weighing concentrated to adjacent links were used in [2], [10], [11]. In this letter, however, we use the exponential weights which are given as,

$$w_{r_i} = \exp\left(\frac{-0.347\nu_i}{d_c}\right),\ w_{t_i} = \sqrt{1 - w_{r_i}^2},\quad i = 1, \ldots, N, \quad (8)$$

where $\nu_i = 0.5(\mu_i + \lambda_i^{-1})$ is approximate distance obtained from measurements by neglecting effect of NLOS paths. $d_c = \sum_{i=1}^{N}\sigma_{t_i}\left(\exp(\sigma_{r_i}^2/\eta^2) - 1\right)^{-0.5}$ is critical distance obtained as [1], above and below which TOA and RSS are assigned with higher preference, respectively. At critical distance, RSS and TOA gain approximately equal weights.

Considering the above weights, (7) is rewritten as

$$\begin{aligned}\min_{\boldsymbol{u},\alpha,\beta}&\sum_{i=1}^{N}\frac{w_{t_i}(\mu_i - ||\boldsymbol{u}-\boldsymbol{a}_i|| - \alpha)^2}{\sigma_{t_i}^2} + \sum_{i=1}^{N}\frac{w_{r_i}(\eta - \eta\lambda_i||\boldsymbol{u}-\boldsymbol{a}_i|| - \beta)^2}{\sigma_{r_i}^2}\\ \text{s.t.}\quad &\alpha \geq 0, \beta \geq 0. \quad (9)\end{aligned}$$

The NLWLS problem obtained in (9) is difficult to solve due to its non-convexity. Its equivalent matrix-vector form can be realized as

$$\begin{aligned}\min_{\boldsymbol{b},\boldsymbol{u}}\ &(\boldsymbol{A}\boldsymbol{b} - \boldsymbol{c})^T\boldsymbol{W}(\boldsymbol{A}\boldsymbol{b} - \boldsymbol{c})\\ \text{s.t.}\ \ &\boldsymbol{b} = [||\boldsymbol{u}-\boldsymbol{a}_1||, \ldots, ||\boldsymbol{u}-\boldsymbol{a}_N||, \alpha, \beta]^T, \boldsymbol{b} \geq 0, \quad (10)\end{aligned}$$

where

$$\begin{aligned}&\boldsymbol{W} = \mathbf{diag}\left([w_{t_1}/\sigma_{t_1}^2, ..., w_{t_N}/\sigma_{t_N}^2, w_{r_1}/\sigma_{r_1}^2, ..., w_{r_N}/\sigma_{r_N}^2]\right),\\ &\boldsymbol{A} = \begin{bmatrix}\mathbf{diag}(\boldsymbol{\lambda}) & 1 & 0\\ \boldsymbol{I}_N & 0 & 1\end{bmatrix},\quad \boldsymbol{\lambda} = [\eta\lambda_1, \ldots, \eta\lambda_N]^T,\\ &\boldsymbol{c} = [\mu_1, \ldots, \mu_N, \eta\boldsymbol{1}_N^T]^T.\end{aligned}$$

Before solving (10), we analyze the mixed condition of LOS and NLOS propagation paths. As the NLOS errors in the measurements contributes positive bias in range information, the distance between anchor and the target follows that

$$\mu_i \geq ||\boldsymbol{u}-\boldsymbol{a}_i|| \leq \lambda_i^{-1},\quad i = 1, \ldots, N. \quad (11)$$

This restricts the target estimation within the region (i.e., circle for $k = 2$ and sphere for $k = 3$) from an anchor.

[1]The experiments conducted in [3], [15] validate that the RSS and TOA measurements when extracted from the same signal exhibit weak correlation. So, the assumption of uncorrelated measurements is not detrimental to generality.

[2]The strict mathematical formulations are relaxed to some extent due to simplicity of presentation.

However, (11) will not be satisfied for the case when LOS connection is established between the anchor and target or the magnitude of negative effect of measurement error is greater than NLOS biases in measurements i.e., $n_{t_i} < 0, |n_{t_i}| > \alpha_i$ or $n_{r_i} < 0, |n_{r_i}| > \beta_i$. To avoid this issue of convex-hull [5], we introduce a soft variable, $r_i$ such that

$$\nu_i + r_i \geq ||\boldsymbol{u} - \boldsymbol{a}_i||, r_i \geq 0, \quad i = 1, \ldots, N. \tag{12}$$

The variable $r_i$ should be sufficiently large to avoid the convex hull issue. However, its very large value loosens the constraint in (12) and thus, degrades the localization accuracy. So, a regularization term, $\rho_c \sum_{i=1}^{N} r_i^2$, is introduced in the main objective function of (10), where $\rho_c > 0$ is the penalization factor which is required to regularize the variable $r_i$ [4]. The optimization problem in (10) is now written as,

$$\begin{aligned} \min_{\boldsymbol{b},\boldsymbol{u},\boldsymbol{r}} \;\; & (\boldsymbol{A}\boldsymbol{b} - \boldsymbol{c})^T \boldsymbol{W} (\boldsymbol{A}\boldsymbol{b} - \boldsymbol{c}) + \rho_c \sum_{i=1}^{N} r_i^2 \\ \text{s.t. } & \boldsymbol{b} = \left[||\boldsymbol{u} - \boldsymbol{a}_1||, \ldots, ||\boldsymbol{u} - \boldsymbol{a}_N||, \alpha, \beta\right]^T, \boldsymbol{b} \geq 0 \\ & \nu_i + r_i \geq ||\boldsymbol{u} - \boldsymbol{a}_i||, r_i \geq 0, \quad i = 1, \ldots, N. \end{aligned} \tag{13}$$

The problem (13) is non-convex for the optimization variables. Using the semi-definite constraints such that $\boldsymbol{B} \succeq \boldsymbol{b}\boldsymbol{b}^T$ and $v \succeq \boldsymbol{u}^T\boldsymbol{u}$, (13) can be relaxed into semi-definite programming (SDP) as

$$\begin{aligned} \min_{\substack{\boldsymbol{u},v,\\ \boldsymbol{b},\boldsymbol{B},\boldsymbol{r}}} \;\; & \text{trace}\left(\boldsymbol{W}\left(\boldsymbol{A}\boldsymbol{B}\boldsymbol{A}^T - 2\boldsymbol{A}\boldsymbol{b}\boldsymbol{c} + \boldsymbol{c}\boldsymbol{c}^T\right)\right) + \rho_c \sum_{i=1}^{N} r_i^2 \\ & + \rho_n \left(b_{N+1}^2 + b_{N+2}^2\right) \\ \text{s.t. } & \begin{bmatrix} \boldsymbol{B} & \boldsymbol{b} \\ \boldsymbol{b}^T & 1 \end{bmatrix} \succeq \boldsymbol{0}_{N+3}, \begin{bmatrix} \boldsymbol{I}_k & \boldsymbol{u} \\ \boldsymbol{u}^T & v \end{bmatrix} \succeq \boldsymbol{0}_{k+1} \\ & B_{i,i} = \begin{bmatrix} \boldsymbol{a}_i \\ -1 \end{bmatrix}^T \begin{bmatrix} \boldsymbol{I}_k & \boldsymbol{u} \\ \boldsymbol{u}^T & v \end{bmatrix} \begin{bmatrix} \boldsymbol{a}_i \\ -1 \end{bmatrix}, \quad b_{i,} \geq 0, \\ & \nu_i + r_i \geq ||\boldsymbol{u} - \boldsymbol{a}_i||, r_i \geq 0, \quad i = 1, \ldots, N, \end{aligned} \tag{14}$$

where $\rho_n > 0$ is penalization factor which is used to avoid ill-posed conditions [8]. It regularizes $\alpha$ and $\beta$ with a reasonable size. The penalization factors $\rho_c$ and $\rho_n$ can be considered as optimization variables and need to be optimally selected. Without any prior information and perfect knowledge of NLOS errors, it is difficult to obtain optimal values of $\rho_c$ and $\rho_n$ prior to SDP solution [4]. To obtain their optimal values, a heuristic approach involving various numerical simulations is used. The optimality of penalization factors is discussed later in this letter.

The solution for (14) can be obtained by standard interior point methods [16]. According to [16], the worst-case computational complexity of the proposed SDR based optimization method is approximately given as

$$\mathcal{O}\left\{\sqrt{N}\log\left(\frac{1}{\epsilon}\right)\left(N(N+3)^2 + N^2(N+3)^2 + N^3\right)\right\} \tag{15}$$

where $\epsilon$ is the solution precision. On an average, the worst case complexity of proposed algorithm is in the order of $\mathcal{O}(N)^{4.5}$. Though, it is considerably higher than the least squares [2], [12]–[14] and bisection methods [10], [11], it performs exceptionally well in terms of accuracy level which is validated in the next section.

The CRLB for target location which states the lower threshold on the variance of unbiased estimation [18] under LOS condition is obtained from likelihood function (2) as

$$I(\boldsymbol{u})_{i,j} = -E\left[\frac{\partial^2(\log_e \mathcal{P})}{\partial u_i \partial u_j}\right], \quad i,j = 1, \ldots, k \tag{16}$$

where $\boldsymbol{I}(\boldsymbol{u}) \in \mathbb{R}^{k \times k}$ is Fisher information matrix (FIM). So, the variance of location estimation, $\sigma_{\boldsymbol{u}}^2$, follows that

$$\sigma_{\boldsymbol{u}}^2 \geq \text{trace}(\boldsymbol{I}^{-1}(\boldsymbol{u})) = C_{\boldsymbol{u}} \tag{17}$$

where $C_{\boldsymbol{u}}$ is the CRLB for the target location estimation.

## IV. Simulation Results

In this section, the performance of the proposed algorithm is examined through Monte-Carlo ($M_c$) simulations with 5000 samples. In the simulation environment, we consider a square region of $R \times R$ m$^2$. The target is randomly placed in the region at every run of simulation whereas N anchors are uniformly placed on the edges of the square. Without loss of generality, the noise in RSS and TOA measurements are assumed to have common variance $\sigma^2$ i.e., $\sigma_{t_i}^2 = \sigma_{r_i}^2 = \sigma^2$. The NLOS errors in the radio measurements are assumed to follow the uniform distribution i.e., $\alpha_i, \beta_i \in \mathcal{U}(0, \zeta_m)$. For sake of expression, we represent both $\sigma$ and $\Lambda$ in terms of m and dB for TOA and RSS, respectively as (m,dB). The penalization factors are heuristically set as $\rho_c = 0.1$ and $\rho_n = 0.1$ which is justified shortly in the paper. To account for realistic measurement model in dynamic environment and to validate the robustness of the proposed method to imperfect knowledge of the network parameters i.e., noise variances and PLE, we consider that $\boldsymbol{v} = \boldsymbol{v}^\circ + \Delta\boldsymbol{v}$ where $\boldsymbol{v} = [\sigma, \gamma]$ is the estimated value, $\boldsymbol{v}^\circ = [\sigma^\circ, \gamma^\circ]$ is the true value and $\Delta\boldsymbol{v}$ is uniformly (or normally) distributed random estimation error in the network parameters. The true RSS parameters are set as $L_o = 40$ dB and $\gamma^\circ = 3$. The RSS and TOA measurements are generated from (1) using true values.

The existing NLOS mitigation based hybrid methods i.e., CFWLS [2], R-GTRS [11], GTRS [10] and HWLS [12] are considered as the strong competitor for proposed SDP method. The iterative GTRS and R-GTRS algorithms are solved with 30 iterations [10], [11]. The joint adhoc (JAH) estimator in [13] based on LOS condition is considered as a performance benchmark. Moreover, the derived hybrid CRLB in (17) is also included in the results as a benchmark. The proposed SDP method is implemented in MATLAB by `CVX` toolbox with `sedumi` as the solver [17]. The performance comparison of the proposed SDP method with existing methods is carried out for various networks scenarios. Moreover, the performance results of the proposed method when using RSS-only (SDP$_{\text{RSS}}$) and TOA-only (SDP$_{\text{TOA}}$) are also shown. The root mean square error (RMSE) is considered as main performance metric which is calculated as $\text{RMSE}_{\boldsymbol{u}} = \sqrt{\frac{1}{M_c}\sum_{k=1}^{M_c}\|\boldsymbol{u}_k - \hat{\boldsymbol{u}}_k\|^2}$ where, $\boldsymbol{u}_k$ and $\hat{\boldsymbol{u}}_k$ are true value and estimated value of parameter, respectively in the $k^{th}$ run.

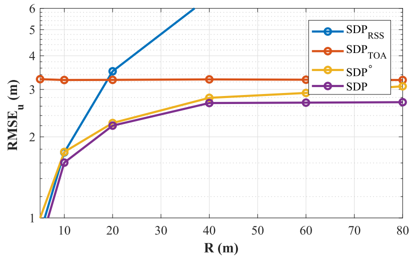


Fig. 1. Localization error versus deployment area when $N = 8, \sigma^{\circ} = 3$ (m, dB), $\sigma \in \mathcal{U}[2, 4]$ (m, dB), $\zeta_m = 5$ (m, dB), $\gamma \in \mathcal{U}[2.5, 3.5]$.

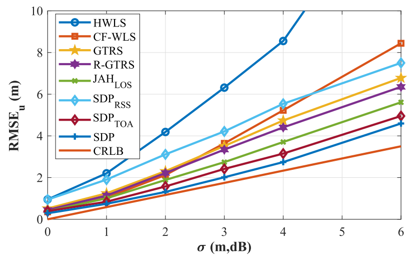


Fig. 2. Localization error versus measurement noise when $N = 8$, $R = 40$m, $\sigma = \sigma^{\circ}$, $\zeta_m = 1$ (m, dB), $\gamma \in \mathcal{U}[2.5, 3.5]$.

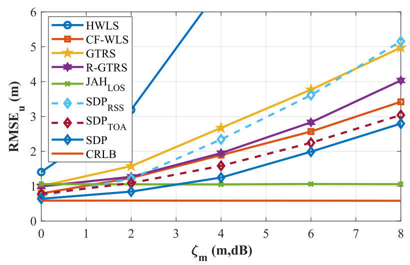


Fig. 3. Localization error versus upper bound on NLOS errors when $N = 8, R = 40$m, $\sigma^{\circ} = 1$ (m, dB), $\sigma \in \mathcal{U}[0, 2]$ (m, dB), $\gamma \in \mathcal{U}[2.5, 3.5]$.

The merits of hybridization over individual utilization of RSS and TOA measurements can be observed in Fig. 1. We fix the postion of source node and vary the deployment area, R×R m$^2$. The proposed hybrid SDP method outperforms the SDP$_{\text{RSS}}$ and SDP$_{\text{TOA}}$ for entire span of R as shown in Fig. 1. The weighing in proposed SDP method provides improved accuracy at shorter and longer ranges when compared with the SDP method without weighing (labeled as SDP$^{\circ}$). The performance results hold the essence of critical distance analysis mentioned in [1]. Although, the improvement due to hybrid measurements is minor in small (R≤10m) and large deployment areas (R≥40m), the results are more profound for moderate areas. The recent advances in modern receivers [6] make them capable to extract both the measurements simultaneously without any extra hardware overhead [1], [3].

Fig. 2 and Fig. 3 respectively depicts the impact of noise levels on the considered algorithms. The performance of CFWLS degrades as the standard deviation of LOS errors increases due to highly perturbed orientation estimation. The R-GTRS and GTRS methods undergoes significant degradation due to the presence of outliers at high noise levels. Intuitively, the performance of localization algorithms degrades with increase in the measurement noise. But, the proposed method performs exceptionally well over entire noise levels. The convex-hull constraint in SDP provides optimal performance at higher noise levels. Moreover, it closely attains CRLB at low noise power and and its performance is comparatively closer to CRLB at high noise powers.

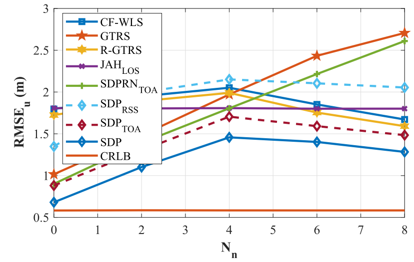


Fig. 4. Localization error versus NLOS paths, when $N = 8$, $R = 40$m, $\sigma^{\circ} = 2$ (m, dB), $\sigma \in \mathcal{U}[1, 3]$ (m, dB), $\zeta_m = 2$ (m, dB), $\gamma \in \mathcal{U}[2.5, 3.5]$.

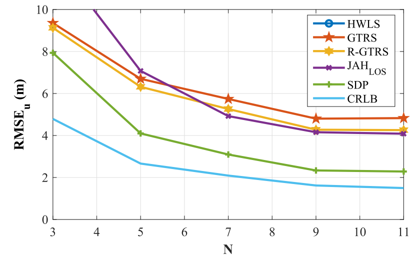


Fig. 5. Localization error versus number of anchors, when $N_{\text{nlos}} = N, R = 40$m, $\sigma^{\circ} = 4$ (m, dB), $\sigma \in \mathcal{U}[3, 5]$ (m, dB), $\zeta_m = 5$ (m, dB), $\gamma \in \mathcal{U}[2.5, 3.5]$.

Next, the performance of the proposed algorithm is evaluated under mixed LOS-NLOS conditions. The performance results for localization erorr w.r.t. number of NLOS paths, N$_{\text{n}}$ are shown in Fig. 4. The CF-WLS and R-GTRS algorithms utilize the known upper bound on the NLOS errors which do not fit for all scenarios. In LOS or mixed LOS-NLOS conditions, their accuracy becomes poorer as the considered upper bound does not suit for all paths. As the number of NLOS paths increases, the accuracy of GTRS decreases which is commonly observed in balancing parameter estimation methods [9]. The TOA based SDP method in [8] (labeled as SDPRN$_{\text{TOA}}$) employed the regularization of NLOS biases, but, its accuracy degrades with increase in $N_n$ due to increase in inaccurate measurements. However, our proposed method performs equally well at LOS, NLOS and mixed LOS-NLOS conditions due to the convex-hull constraint and regularization involved.

Fig. 5 shows the behavior of proposed algorithm in contrast with randomly deployed anchor nodes. The CF-WLS [2] method is not compared as it is topology dependent. The proposed algorithm outperforms existing algorithms for the entire span. However, the accuracy of proposed algorithm with reference to CRLB is poorer with few anchors as compared to the situation when there are large number of anchors. It is possibly due to the placement of target outside the convex hulls formed by few anchors. As anchors increases, the probability of the target inside the convex-hull increases, hence the accuracy improves [5].

Next, the optimality of the penalization factors is examined under different LOS-NLOS conditions. The performance analysis of the proposed method with different values $\rho_c$ and $\rho_n$ under different conditions i.e., severe NLOS (C1), mild NLOS (C2), LOS with low measurement noise (C3), LOS with high measurement noise (C4) and mixed LOS-NLOS (C5) is described in Fig. 6. Intuitively, the optimal values of

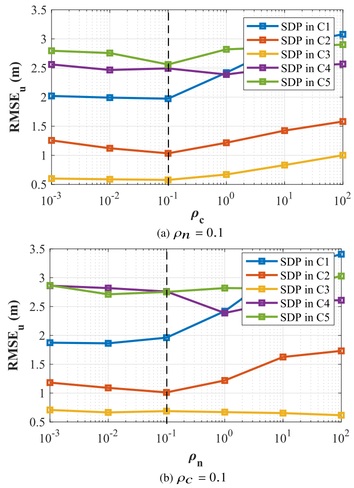


Fig. 6. Localization error versus penalization factors under conditions C1 ($\sigma^\circ=1, \zeta_m=8, N_n=8$), C2 ($\sigma^\circ=1, \zeta_m=4, N_n=8$), C3 ($\sigma^\circ=1, \zeta_m=0, N_n=8$) C4 ($\sigma^\circ=4, \zeta_m=0, N_n=8$) and C5 ($\sigma^\circ=4, \zeta_m=4, N_n=4$) when $N=8, R=40\text{m}, \sigma=\sigma^\circ, \gamma \in \mathcal{U}[2.5, 3.5]$.

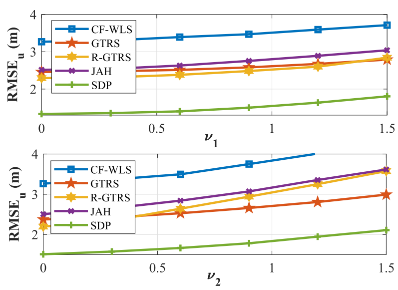


Fig. 7. Localization error versus imperfection in network parameters, when $N_n=N=8, R=40\text{m}, \sigma^\circ=3$ (m, dB), $\zeta_m=4$ (m,dB), $\gamma^\circ=3$.

$\rho_c$ and $\rho_n$ depend on the noise levels (measurement noises and NLOS biases). For low noise levels, the large values of $\rho_c$ and $\rho_n$ provide low estimation of the variable $\boldsymbol{r}$ and the NLOS parameters $\alpha$ and $\beta$ and thus, attain better localization accuracy. The similar argument holds for low values of $\rho_c$ and $\rho_n$ at high noise levels. However, the proposed method is robust to the unavailable information of NLOS biases, hence, it is quite reasonable to assume the $\rho_c$ and $\rho_n$ as constant [4], [8]. Under most of the conditions, $\rho_c=0.1$ and $\rho_n=0.1$ with proposed method provides optimal performance as these value considerably regularizes the $\boldsymbol{r}, \alpha$ and $\beta$ at moderate and high noise levels.

The robustness of proposed method against estimation errors in the network parameters is validated as shown in Fig. 7. We consider two different cases in which the estimation errors $\Delta\boldsymbol{\nu}$ in the network parameters are uniform and normal distributed i.e., $\Delta\boldsymbol{\nu} \in \mathcal{U}[-\nu_1, +\nu_1]$ and $\Delta\boldsymbol{\nu} \sim \mathcal{N}(0, \nu_2^2)$, respectively. For both the cases, the imperfection in network parameters does not significantly alter the performance of proposed method and it still outperforms the state of art algorithms.

Among all the considered scenarios, the proposed SDP method performs better than existing hybrid methods at the expense of slightly higher computational cost. On an average, the proposed method is found to be $20-40\%$ more accurate but 5-10 times computationally slower than existing algorithms. The issue of trade-off between accuracy level and latency in localization methods is commonly encountered and the preferences to these algorithms are usually application dependent [5]–[7].

## V. Conclusion

In this letter the problem of target localization was addressed through effective utilization of RSS and TOA measurements and NLOS mitigation technique. A NLWLS problem was formulated by approximating the original non-convex hybrid ML solution. The NLWLS was efficiently solved using SDR relaxation and was further tightened with soft constraint to avoid convex-hull problem. The proposed hybrid method earned the combined advantage of RSS and TOA measurements at shorter and longer ranges, respectively. The simulations results validated the excellence of the proposed method over existing hybrid algorithms in terms of accuracy level.

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