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The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: Examining the observational evidence for dynamical dark energy

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We examine the ability of the ΛCDM model to simultaneously fit different types of cosmological observations and apply a recently proposed test, based on the Kullback-Leibler divergence, to quantify the tension between different subsets of data. We find a tension between the distance indicators derived from a ΛCDM model using a combined dataset, and the recent H_0 measurements, as well as the high redshift Baryon Acoustic Oscillations (BAO) obtained from the Lyman-α forest spectra. We then allow for a dynamical dark energy (DE) and perform a Bayesian non-parametric reconstruction of the DE equation of state as a function of redshift. We find that the tension with H_0 and Lyman-α forest BAO is effectively relieved by a dynamical DE. Although a comparison of the Bayesian evidence for dynamical DE with that of ΛCDM shows that the tension between the datasets is not sufficiently strong to support a model with more degrees of freedom, we find that an evolving DE is preferred at a 3.5σ significance level based solely on the improvement in the fit. We also perform a forecast for the upcoming DESI survey and demonstrate that, if the current best fit DE happens to be the true model, it will be decisively supported by the Bayesian evidence.

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A flat Friedman-Roberson-Walker (FRW) universe with a content dominated by a positive cosmological constant (Λ) and cold dark matter (CDM), has been the working model preferred by cosmologists since the discovery of cosmic acceleration [1]. It is capable of describing the background expansion and the growth of cosmic structure as measured by a diverse array of observations. Still, the nature of CDM remains unknown, as well as the way in which the vacuum energy is supposed to gravitate, meaning that about 95% of the content of the ΛCDM universe is effectively postulated. Hence, while ΛCDM may well turn out to be the correct model, the lack of basic understanding of its constituents warrants an agnostic approach to interpreting data as broader ranges of scales and redshifts are surveyed with a high accuracy.

Within the ΛCDM model, several tensions of various degrees of significance are known to be present among existing datasets [2–8]. In particular, the high redshift Lyman-α (Lyα) forest measurement of the Baryon Acoustic Oscillations (BAO) by the Baryon Oscillation Spectroscopic Survey (BOSS) [2, 9] prefers a smaller value of $\Omega_{\rm M}$ compared to the value preferred by cosmic microwave background (CMB). Also, the recently measured value of H_0 [10], 73.24 ± 1.74 km s^{-1} Mpc^{-1}, is 3.4σ higher than 66.93 ± 0.62 km s^{-1} Mpc^{-1} inferred from *Planck* data [6].

The Kullback-Leibler (KL) divergence [11], also known as relative entropy, has been extensively utilized as a way of quantifying the degree of tension between different datasets within the ΛCDM model [12–19]. Rather than focusing on particular model parameters, it is designed to compare the overall concordance of datasets within a given model (see also [20]). The difference between the actual and the expected KL divergence, called "Surprise" [12], was applied in [14] to quantify the agreement between datasets. It was shown, for instance, that while the Surprise between WMAP9 [21] and *Planck* 2013 [22] was large, it disappeared with *Planck* 2015 [6].

Various ways to "relieve" the tension have been proposed, including neutrinos and dynamical dark energy (DE) [23], interacting vacuum [24, 25] etc. As always, when comparing performance of models with different numbers of degrees of freedom, it is pertinent to ask if the improvement in the fit is sufficiently large to warrant a more complicated model. In what follows, we will use the KL divergence to quantify the degree of tension between different datasets within the ΛCDM model. We will then allow for a dynamical DE, reconstruct the DE equation of state, w, as a function of redshift, recalculate the tension, and compare the Bayesian evidence for the $w(z)$CDM model to that of ΛCDM.

The KL divergence quantifies the proximity of two probability density functions (PDFs), P_1 and P_2, of a multi-dimensional random variable θ. If both P_1 and P_2 are assumed to be Gaussian [12], and data are assumed to be more informative than the priors, we can quantify the tension between them in terms of,

$$T \equiv \frac{S}{\Sigma} = \frac{(\theta_1-\theta_2)^T \mathcal{C}_1^{-1}(\theta_1-\theta_2) - \mathbf{Tr}\left(\mathcal{C}_2\,\mathcal{C}_1^{-1}+\mathbb{I}\right)}{\sqrt{\mathbf{Tr}\left(\mathcal{C}_2\,\mathcal{C}_1^{-1}+\mathbb{I}\right)^2}}\,, \tag{1}$$

where θ_1 and θ_2 are the best-fit parameter vectors, $\mathcal{C}_1$ and $\mathcal{C}_2$ are the covariance matrices for P_1 and P_2, and $\mathbb{I}$ is the unity matrix. The numerator S is the Surprise, which is the difference between the actual and the expected KL divergence, while the denominator Σ is the standard deviation of the expected KL divergence. If $T \lesssim 1$, then P_1 and P_2 are consistent with each other, while a negative tension T (*i.e.*, a negative Surprise) indicates a perfect agreement between the two PDFs [14].

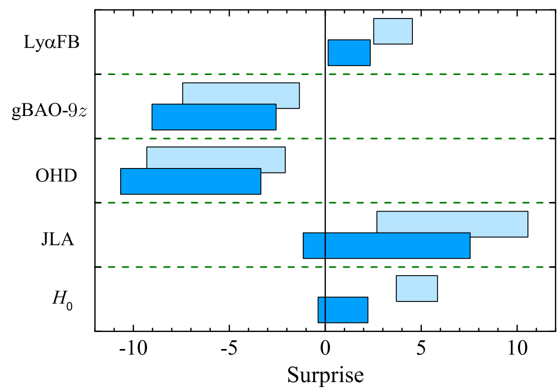

FIG. 1. The Surprise between the PDFs for $D_A(z)$ and $H(z)$ derived from the best fit model using the combined dataset of ALL16, and the directly observed $D_A(z)$ and $H(z)$ from H_0, JLA, OHD, gBAO-9z and LyαFB respectively. The light blue horizontal bars indicate the 68% CL range of Surprise in ΛCDM, while the dark blue bars correspond to $w(z)$CDM.

Acronym	Meaning	References
P15	The *Planck* 2015 CMB power spectra	[6]
JLA	The JLA supernovae	[28]
6dF	The 6dFRS (6dF) BAO	[29]
MGS	The SDSS main galaxy sample BAO	[30]
$P(k)$	The WiggleZ galaxy power spectra	[31]
WL	The CFHTLenS weak lensing	[32]
H_0	The Hubble constant measurement	[10]
OHD	$H(z)$ from galaxy age measurements	[33]
gBAO-3z	3-bin BAO from BOSS DR12 galaxies	[34]
gBAO-9z	9-bin BAO from BOSS DR12 galaxies	[35, 36]
LyαFB	The Lyα forest BAO measurements	[2, 9]
B	P15 + JLA + 6dF + MGS	
ALL12	The combined dataset used in [27]	
ALL16-3z	B+$P(k)$+WL+H_0+OHD+gBAO-3z+LyαFB	
ALL16	B+$P(k)$+WL+H_0+OHD+gBAO-9z+LyαFB	
DESI++	P15 + mock DESI BAO [49] + mock SN [50]	

TABLE I. The datasets used in this work.

In what follows, we consider the datasets summarized in Table I. Rather than comparing the PDFs for the ΛCDM parameters θ for every pair of datasets, we take the combined dataset ALL16 and find the *derived* PDFs for the angular diameter distance $D_A(z)$ and the Hubble parameter $H(z)$ at redshifts corresponding to the available data. We then compute the KL divergence between

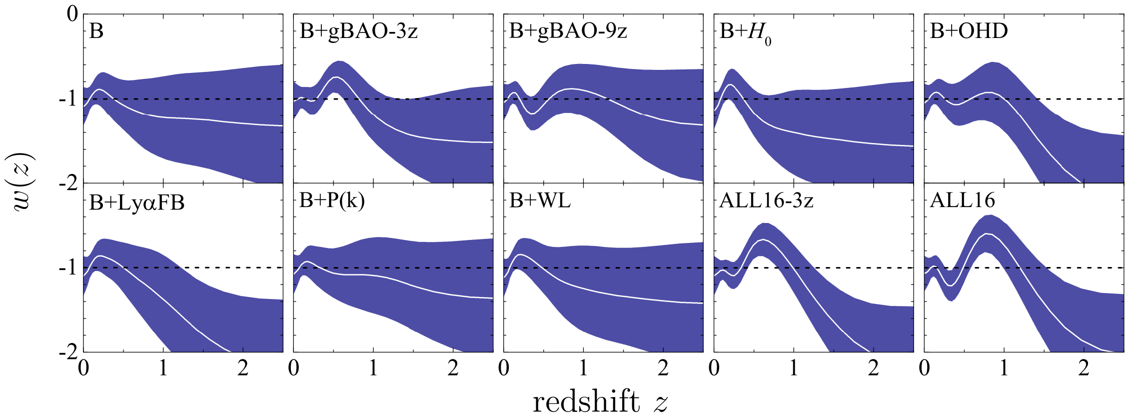

FIG. 2. The reconstructed $w(z)$ (white solid line) and the 68% CL uncertainty (dark blue shading) from different data combinations shown in the legend. The correlated prior parameters are $a_c = 0.06$ and $\sigma_{\mathrm{m}} = 0.04$.

the derived PDFs and the directly observed $D_A(z)$ and $H(z)$ from H_0, JLA, OHD, gBAO-9z and LyαFB, and evaluate the corresponding Surprise and the standard deviation Σ. The results are shown with light blue bars in Fig. 1. They indicate that the H_0, LyαFB and JLA measurements are in tension with the combined dataset, with tension values of $T = 4.4$, 3.5, and 1.7, respectively.

Next, we check if the tension within the ΛCDM model can be interpreted as evidence for a dynamical DE. To address this, we allow for a general evolution of the DE equation of state and use the correlated prior method[26] to reconstruct $w(z)$. To start, $w(z)$ is parameterized in terms of its values at discrete steps in z, or the scale factor a. Fitting a large number of uncorrelated bins would result in extremely large uncertainties and, in fact, would prevent the Monte Carlo Markov Chains (MCMC) from converging because of the many degenerate directions in the parameter space. On the other hand, fitting only a few bins could significantly bias the result. Our approach is to introduce a prior covariance between the bins based on a specified two-point function that correlates values of w at different a, $\xi_w(|a-a'|) \equiv \langle [w(a) - w^{\mathrm{fid}}(a)][w(a') - w^{\mathrm{fid}}(a')] \rangle$, which can be taken to be of the form proposed in [37], $\xi_{\mathrm{CPZ}}(\delta a) = \xi_w(0)/[1 + (\delta a/a_c)^2]$, where a_c describes the typical smoothing scale, and $\xi_w(0)$ is a normalization factor set by the expected variance in the mean w, $\sigma^2_{\bar{w}}$. As shown in [26], results are largely independent of the choice of the correlation function. The prior covariance matrix $\mathbf{C}$ is obtained by projecting $\xi_w(|a-a'|)$ onto the discrete w bins [26, 37], and the prior PDF is taken to be of Gaussian form: $\mathcal{P}_{\mathrm{prior}}(\mathbf{w}) \propto \exp[-(\mathbf{w}-\mathbf{w}^{\mathrm{fid}})^T \mathbf{C}^{-1} (\mathbf{w}-\mathbf{w}^{\mathrm{fid}})/2]$, where $\mathbf{w}^{\mathrm{fid}}$ is the fiducial model. The reconstructed model is that which maximizes the posterior probability, which by Bayes' theorem is proportional to the likelihood of the data times the prior probability, $\mathcal{P}(\mathbf{w}|\mathbf{D}) \propto \mathcal{P}(\mathbf{D}|\mathbf{w}) \times \mathcal{P}_{\mathrm{prior}}(\mathbf{w})$. Effectively, the prior results in a new contribution to the total χ^2 of a model, which penalizes models that are less smooth.

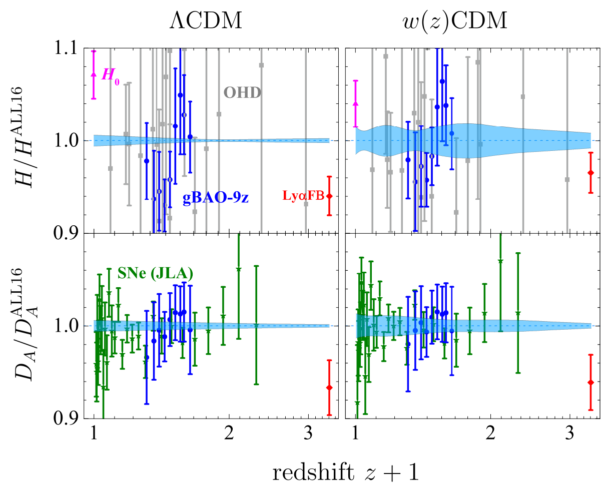

FIG. 3. The $H(z)$ (top) and the $D_A(z)$ (bottom) data rescaled by the values derived from the ALL16 best fit ΛCDM (left) and $w(z)$CDM (right) models. Datasets are labeled by accordingly coloured text, and the shaded bands indicate the 1σ uncertainty in the rescaled $H(z)$ and D_A.

In our reconstruction of $w(z)$, we set $a_c = 0.06$ and $\sigma_{\bar{w}} = 0.04$, which is the "weak prior" used in [27]. To calculate the observables, we use a version of `CAMB` [40] modified to include DE perturbations for an arbitrary $\mathbf{w}$ [41]. We use `PolyChord` [38], a nested sampling plug-in for `CosmoMC` [39], to sample the parameter space $\mathbf{P} \equiv (\omega_b, \omega_c, \Theta_s, \tau, n_s, A_s, w_1, ..., w_{30}, \mathcal{N})$ where ω_b and ω_c are the baryon and CDM densities, Θ_s is the angular size of the sound horizon at decoupling, τ is the optical depth, n_s and A_s are the spectral index and the amplitude of the primordial power spectrum, and $w_1, ..., w_{30}$ denote

	P15	JLA	gBAO-9z	$P(k)$	WL	H_0	LyαFB	OHD
$\Delta\chi^2$	-0.7	-1.6	-2.8	$+1.1$	-0.1	-2.9	-3.7	-2.3

	ALL12	ALL16	DESI++
S/N	2.5σ	3.5σ	6.4σ
ΔAIC	-0.3	-4.3	-24.6
$\Delta\ln E$	-6.7 ± 0.3	-3.3 ± 0.3	11.3 ± 0.3

TABLE II. Top: the change in χ^2 for individual datasets between the best-fit $w(z)$CDM model and the best fit ΛCDM model. Bottom: the signal-to-noise in $w(z)$ deviating from -1, and ΔAIC and the Bayes factor $\Delta\ln E$ between the ΛCDM and $w(z)$CDM models for the ALL12 and ALL16 datasets along with the forecast for DESI++.

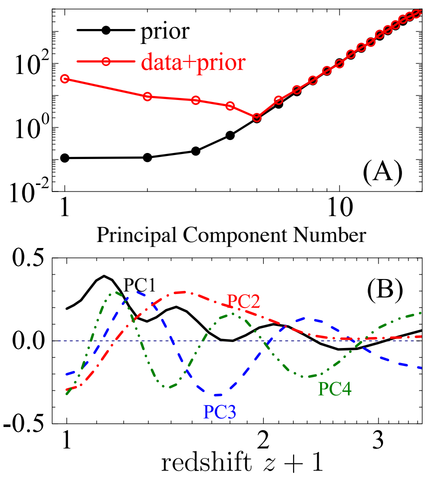

FIG. 4. Panel (A): the inverse eigenvalues of the prior covariance matrix (black line with filled dots) and of the posterior covariance (red line with unfilled dots); Panel (B): the first four posterior eigenmodes of $w(z)$ for the ALL16 dataset combined with the correlated prior.

the 30 w-bin parameters. The first 29 w bins are uniform in $a \in [0.286, 1]$, corresponding to $z \in [0, 2.5]$, and the last wide bin covers $z \in [2.5, 1100]$. We marginalize over nuisance parameters such as the intrinsic SN luminosity.

Fig. 2 presents the reconstructed $w(z)$, along with the 68% CL uncertainty, derived from ten different data combinations. One can note that the dip in $w(z)$ at $z \sim 0.4$ is more pronounced for ALL16 compared to ALL16-3z, thanks to BAO-9z being more informative than BAO-3z. As we will see shortly, this makes the ALL16 result more consistent with the $w(z)$ reconstructed in [27] using a different combination of data (ALL 12).

The results for tension between datasets, re-evaluated for the ALL 16 best fit $w(z)$CDM model, are shown with dark blue bars in Fig. 1. We find that $T = 0.7$, 1.1 and 0.7 for H_0, LyαFB and JLA, respectively, indicating that tensions that existed in the ΛCDM model are fully released within $w(z)$CDM. Table II shows the change in χ^2 relative to ΛCDM for each individual dataset for the best fit $w(z)$CDM model derived from ALL16. Overall, the χ^2 is improved by -12.3, which can be interpreted as the dynamical DE being preferred at 3.5σ.

Fig. 3 shows the Hubble parameter and the angular diameter distance data rescaled by the values derived from the ALL16 best fit ΛCDM and $w(z)$CDM models. The shaded bands indicate the uncertainty in the rescaled $H(z)$ and D_A. One can see that, in the case of $w(z)$CDM, the data points are much more consistent with the corresponding values derived from ALL16, demonstrating the significant reduction in tension.

With 30 additional w-bin parameters, it appears that the improvement in the fit is achieved by $w(z)$CDM at the cost of a huge increase of the parameter space. However, correlations between the w-bins induced by the prior constrain most of that freedom. One way to estimate the effective number of additional degrees of freedom is to perform a principal component analysis (PCA) [42] of the posterior covariance matrix of the w-bin parameters and compare it to that of the prior. First, we diagonalize the posterior covariance of w-bins to find their uncorrelated linear combinations (eigenmodes), along with the eigenvalues, which quantify how well a given eigenmode is constrained. We plot the inverse eigenvalues of the posterior covariance, ordered according to the number of nodes in the eignemodes, in Panel A of Fig. 4. The number of nodes is representative of the smoothness in the evolution of eigenmodes, with the first four posterior eigenmodes shown in Panel B of Fig. 4. Next, we perform a PCA of the prior covariance matrix and plot its inverse eigenvalues alongside those of the posterior. We see that the fifth and higher number eigenvalues of the two matrices coincide, which means that they are fully determined by the prior. However, the first four inverse eigenvalues of the posterior are significantly larger than that of the prior, indicating that they are constrained primarily by the data. This is precisely the intent of the correlated prior method: the smooth features in $w(z)$ are constrained by the data, with no bias induced by the prior, while the high frequency features are constrained by the prior. Thus, our $w(z)$CDM model effectively has only four additional degrees of freedom compared to ΛCDM.

	ALL12	ALL16
SNe	SNLS3 (472) [44]	JLA (740)
gBAO	DR9; $z_{\rm eff} = 0.57$ [45]	DR12; 9 $z_{\rm eff} \in [0.2, 0.75]$
LyαFB	none	DR11; $z_{\rm eff} = 2.34$
$100h_0$	74.2 ± 3.6 [46]	73.24 ± 1.74
CMB	WMAP7 [47]	*Planck* 2015

TABLE III. Comparing ALL12 and ALL16 datasets.

It is interesting to compare $w(z)$ reconstructed from ALL16 to that obtained in [27] using the same prior

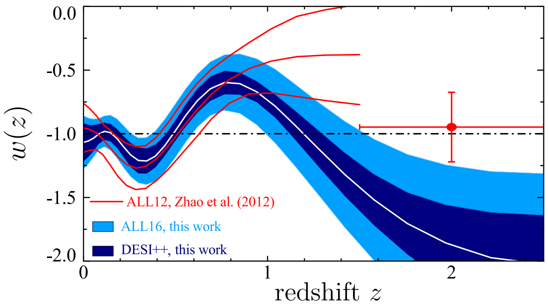

FIG. 5. The mean and the 68% CL uncertainty of the $w(z)$ reconstructed from ALL16 (light blue) compared to the ALL12 $w(z)$ reconstructed in [27] (red lines showing the mean and the 68% CL band). The red point with error bars is the value of $w(z)$ at $z = 2$ "predicted" by the ALL12 reconstruction. The dark blue band around the ALL16 reconstruction is the forecasted 68% CL uncertainty from DESI++.

but a different dataset (ALL12). Namely (see Table III), ALL16 contains about 40% more supernovae than ALL12, primarily coming from the SDSS-II survey [43]. In ALL12, the BAO measurement derived from the BOSS DR9 sample was at a single effective redshift, while in ALL16 it is tomographic at nine redshifts from BOSS DR12, which contains four times more galaxies than DR9. In addition, ALL16 includes a high-redshift BAO measurement from Lyα forest. This helped to constrain $w(z)$ at redshifts where constraints from supernovae are weak. The new 2016 H_0 measurement is consistent with that in 2009, with the error bar halved. The Planck 2015 CMB data is also much more informative than the WMAP 7-year release, thanks to a higher angular resolution of the temperature and polarization maps, and lower levels of statistical uncertainties. Overall, ALL16 is more constraining: the effective number of $w(z)$ degrees of freedom constrained by ALL12 was three, compared to four constrained by ALL16. Fig. 5 compares the two results and shows that they are highly consistent. We quantify the agreement in two ways: by evaluating the dot-product of the $\hat{\mathbf{w}}$ vectors from the two reconstructions (the vectors are normalized so that a dot-product is unity if they are identical), and by calculating the tension using Eq. (1). We find

$$\hat{\mathbf{w}}_{\rm ALL12} \cdot \hat{\mathbf{w}}_{\rm ALL16} = 0.94 \pm 0.02; \quad T = -1.1 \ , \tag{2}$$

indicating an excellent agreement.

To check if the improvement in the fit warrants introducing four additional effective degrees of freedom, we evaluate the difference in the Akaike Information Criterion (AIC) [48] defined as $\Delta{\rm AIC} \equiv \Delta\chi^2 + 2\Delta N_{\rm p}$, where $\Delta\chi^2$ is the difference in χ^2 and $\Delta N_{\rm p}$ is the difference in the number of parameters. Table II shows ΔAIC between the best-fit $w(z)$CDM model and ΛCDM for ALL16 ($\Delta N_{\rm p} = 4$) and ALL12 ($\Delta N_{\rm p} = 3$). The ΔAIC values are negative, indicating a preference for the $w(z)$CDM, with the ALL16 dataset having a stronger preference for a dynamical DE model than ALL12.

Another way of performing model selection is to compare the Bayesian evidence, $E \equiv \int d\theta\, \mathcal{L}(\mathbf{D}|\theta)\, P(\theta)$, which tends to penalize having additional parameters more than the AIC. We calculate E for both ΛCDM and $w(z)$CDM using `PolyChord` [38]. The Bayes' factors, which are the differences in $\ln E$ between the two models, are shown at the bottom of Table II. As shown, the Bayes' factor for both ALL12 and ALL16 are negative, indicating that ΛCDM is favoured by this criterion. Our forecast for a future dataset, DESI++, comprised of BAO measurements from DESI [49], supernovae measurements from future surveys [50] and CMB (assuming the Planck sensitivity), shows that $w(z)$CDM would be *decisively* (according to Jefferys' scale) supported if it happened to be the true model.

One may ask how much the evidence for DE depends on the particular choice of the prior parameters. In principle, the choice of the smoothing scale should be guided by theory. The value used in [27] and this work, $a_c = 0.06$, was based on variations in $w(z)$ seen in quintessence models. It is contrary to the principles of Bayesian inference to attempt to optimize the prior according to the best outcome for the $\Delta\chi^2$ or the Bayesian evidence. Also, for the inference to be conclusive, the evidence for a dynamical DE should be strong over a wide range of the prior parameters. We found that neither ALL12 nor ALL16 dataset provided evidence for a dynamical DE at all prior strengths, although the Bayes factor for ALL16 is generally much less negative than that of ALL12 for all priors, *e.g.*, it grows from -6.7 ± 0.3 to -3.3 ± 0.3 for the prior used in this work. On the other hand, our forecast for DESI++ shows that, if the $w(z)$CDM model was true, it would be decisively supported by Bayesian evidence over a wide range of prior strengths.

In summary, there is tension between different types of cosmological observations within the context of the ΛCDM model, and a dynamical DE model can effectively reduce the tension. Although the Bayesian evidence for the dynamical DE from current observations is not sufficiently strong to prefer it over Λ, we have observed a hint of DE dynamics, at a significance level of 3.5σ, with the DE equation of state crossing the -1 boundary. Such an evolution of $w(z)$ is prohibited in single field minimally coupled quintessence models [51, 52], but is possible in models with more degrees of freedom, *e.g.*, the quintom model [53, 54], or in the case of a non-minimal coupling between DE and matter [55]. Future data will be able to decisively confirm such DE dynamics if it happens to be true.

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