

COSMOLOGICAL CONSTRAINTS FROM THE REDSHIFT DEPENDENCE OF THE VOLUME EFFECT USING THE GALAXY 2-POINT CORRELATION FUNCTION ACROSS THE LINE-OF-SIGHT

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ABSTRACT

We propose to use the redshift dependence of the galaxy 2-point correlation function across the line-of-sight (2pCF), $\xi(r_{\perp})$, to constrain cosmological parameters. When an incorrect set of parameters is assumed for the coordinate transformation from redshift space to comoving space, it produces *redshift-dependent mis-scaling* in the constructed galaxy distribution. As a result, we observe a redshift-dependent rescaling of the measured $\xi(r_{\perp})$. Using mock galaxy catalogues (containing hundreds of millions of galaxies at redshifts 0, 0.5, 1, 1.5, 2) from the Horizon Run 4 N-body simulation, we find that the redshift evolution of the $\xi(r_{\perp})$ shape, as a function of scale, is sensitive to the adopted cosmological parameters. However, it is relatively insensitive to the gravitational growth of structure, the galaxy bias, and the effect of redshift space distortions. Analyzing the 2pCF shape on scales $5h^{-1}\text{Mpc} \leq r_{\perp} \leq 40h^{-1}\text{Mpc}$, we derive tight constraints on Ω_m and w with 68.3% CL uncertainties of 0.015 and 0.05. The method could be applicable to future large scale structure surveys, especially photometric surveys such as DES, LSST, to derive tight cosmological constraints. This work is a continuation of our previous works (Li et al. 2014, 2015, 2016) as a strategy to constrain cosmological parameters using redshift-invariant physical quantities; they are all powerful in constrain cosmology and have a lot of merits, e.g., are less affected by the systematic effects introduced by non-linear clustering or RSD, does not require accurate modeling of galaxy clustering, and are applicable on relative small clustering scales.

Keywords: large-scale structure of Universe — dark energy — cosmological parameters

1. INTRODUCTION

The discovery of cosmic acceleration (Riess et al. 1998; Perlmutter et al. 1999) implies the existence of a “dark energy” component in the Universe or the breakdown of Einstein’s gravity theory on cosmological scales (see Yoo & Watanabe 2012, for a recent review). The theoretical explanation and observational probe of cosmic acceleration has attracted tremendous effort in the last two decades and are still far from well understood or accurately measured (Weinberg 1989; Li et al. 2011; Weinberg et al. 2013).

In an effort to probe the cosmic expansion history the large scale structure (LSS) surveys enable measurements of two geometrical quantities, the angular diameter distance D_A and the Hubble factor, H . If they were pre-

cisely measured as a function of redshift, tight constraints can be placed on cosmological parameters, e.g. the matter density Ω_m and the equation of state (EoS) of dark energy w .

When an incorrect cosmological model is assumed for the coordinate transformation from redshift space to comoving space, geometric distortions are produced in the constructed galaxy distribution. These distortions include the “volume effect” which is the mis-scaling of the size of structures, and the Alcock-Paczynski (AP) effect, the shape distortion induced by the fact that distances along and perpendicular to the line of sight are fundamentally different.

Various statistical methods have been proposed to measure these effects. The AP effect was first suggested to be measured through the anisotropic clustering of galaxies (Ballinger Peacock & Heavens 1996; Matsubara & Suto 1996). Later, with the advent of large obser-

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vational spectroscopic programs, this was applied to a series of LSS surveys (Outram et al. 2004; Blake et al. 2011; Chuang & Wang 2012; Reid et al. 2012; Beutler et al. 2013; Linder et al. 2014; Song et al. 2014; Jeong et al. 2014; Sutter et al. 2014; López-Corredoira 2014; Alam et al. 2016; Beutler et al. 2016; Sanchez et al. 2016), while considering the detailed effect of systematics for unbiased estimates of $D_A(z)$ and $H(z)$ (Vargas-Magaña et al. 2014; Ross et al. 2016; Sabiu & Song 2016).

Although there have also been alternative approaches using the symmetry properties of galaxy pairs (Marinoni & Buzzi 2010; Jennings et al. 2011; Bueno Belloso et al. 2012) and cosmic voids (Ryden 1995; Lavaux & Wandelt 2012; Sutter et al. 2014; Mao et al. 2016). Methods proposed to measure the volume effect include the number counting of galaxy clusters (Press & Schechter 1974; Viana & Liddle 1996), the topology of LSS (Park & Kim 2010), the BAO scale (Eisenstein et al. 1998; Blake & Glazebrook 2003; Seo & Eisenstein 2003), and the shape of 2pCF and power spectrum (Sánchez et al. 2006; Sánchez et al. 2009).

In Li et al. (2014, 2015, 2016) we developed a novel strategy to probe D_A and H from the LSS data. We found that the AP or volume effect in general introduces a geometric distortion that evolves significantly with redshift; this phenomenon can be utilized to distinguish the AP effect from the other effects, such as the redshift-space distortion (RSD). The radial distances of galaxies are inferred from the measured redshifts, which are distorted by the peculiar motion of galaxies, leading to apparent distortion in the redshift space galaxy distribution (Jackson 1972; Kaiser 1987; Ballinger Peacock & Heavens 1996). It is the major systematic effects limiting our ability to probe the geometry of LSS from the galaxy distribution. However, we found that the influence of RSD is significantly reduced when we consider only the redshift dependence of the distortion.

Li et al. (2014) attempted to use the *galaxy density gradient field* to characterize the redshift evolution of the LSS distortion. We tested the idea on the Horizon Run 3 (HR3) N-body simulations (Kim et al. 2011), showing that the redshift evolution of the gradient field can be used to infer unbiased constraints on cosmological parameters and that it is, largely, insensitive to RSD. The same topic was revisited in Li et al. (2015), using a slightly modified version of the usual anisotropic galaxy two-point correlation function (2pCF), $\xi(\mu)$, where $\mu \equiv \cos\theta$ and θ is the angle between the connection of the galaxy pairs and the line of sight direction. When an incorrect cosmological model is adopted, the shape of $\xi(\mu)$ is altered due to the anisotropic distortion of the LSS (the AP effect), and the amplitude is shifted due to the change in the comoving volume elements; both effects have significant redshift dependence. Although the RSD effect significantly distorts $\xi(\mu)$, it exhibits much less redshift evolution. The method was applied to BOSS DR12 galaxies (Li et al. 2016) to probe the redshift dependence of the AP effect, and we arrive at tight cosmological constraints competitive with the mainstream probes of SNIa, BAO, and CMB.

In this paper we continue our exploration of this strategy and present a new method using the redshift evolution of the shape of the 2-point correlation function across the line-of-sight as a function of radial scale, $\xi(r_\perp)$, to constrain cosmology. A wrongly adopted cosmology results in *redshift-dependent mis-scaling* of the galaxy distribution, which leads to redshift-evolution of the shape

of $\xi(r_\perp)$. The signal is sensitive to the cosmological parameters but relatively insensitive to the other effects such as the gravitational growth of structure, the galaxy bias, and the RSD effect. We apply our method to HR4 mock catalogues, using hundreds of millions of galaxies at redshifts $0 \leq z \leq 2$ and develop the statistical procedure to constrain cosmological parameters.

The outline of this paper proceeds as follows. In §2 we briefly review the nature and consequences of the LSS geometric distortion when performing coordinate transforms in a cosmological context. In §3 we describe the N-body mock galaxies used to test our method. The methodology is presented in §4. We conclude in §5.

2. VOLUME EFFECT IN A NUTSHELL

In this section we briefly introduce the scaling effect caused by incorrectly assumed cosmological parameters. A more detailed description has been provided in Li et al. (2014, 2015, 2016).

Suppose that we are probing the size of some objects in the Universe. We measure their redshift span Δz and angular size $\Delta\theta$, then compute their sizes in the radial and transverse directions using the following formulas

$$\Delta r_\parallel = \frac{c}{H(z)} \Delta z, \quad \Delta r_\perp = (1+z) D_A(z) \Delta\theta, \quad (1)$$

where H is the Hubble parameter and D_A is the angular diameter distance. In the particular case of a flat universe composed of a cold dark matter component and a constant EoS dark energy, they take the forms

$$H(z) = H_0 \sqrt{\Omega_m a^{-3} + (1 - \Omega_m) a^{-3(1+w)}},$$

$$D_A(z) = \frac{1}{1+z} r(z) = \frac{1}{1+z} \int_0^z \frac{dz'}{H(z')}, \quad (2)$$

where $a = 1/(1+z)$ is the cosmic scale factor, H_0 is the present value of Hubble parameter and $r(z)$ is the comoving distance.

When incorrect values of Ω_m and w are adopted, the inferred Δr_\parallel and Δr_\perp are also incorrect, resulting in erroneous estimation of the object's shape (AP effect) and size (volume effect). These effects and their cosmological consequences have been studied in Li et al. (2014, 2015, 2016).

In this paper we focus on the mis-estimation of the angular size Δr_\perp . The ratio of mis-estimation is

$$\alpha_\perp \equiv \frac{\Delta r_{\perp, \text{wrong}}}{\Delta r_{\perp, \text{true}}} = \frac{D_{A, \text{wrong}}}{D_{A, \text{true}}}, \quad (3)$$

where “true” and “wrong” denote the values in the true cosmology and wrongly assumed cosmologies respectively.

An illustration is provided in the left panels of Figure 1. Suppose that the true cosmology is a flat Λ CDM with present matter ratio $\Omega_m = 0.26$ and standard dark energy EoS $w = -1$. If we were to distribute a series of perfect squares at distances ranging from 500 Mpc/h to 3000 Mpc/h, and an observer located at the origin were to measure their redshifts and infer the sizes of the squares using the distance-redshift relations of four incorrect cosmologies

- (i). $\Omega_m = 0.40$, $w = -1.0$,
- (ii). $\Omega_m = 0.15$, $w = -1.0$,
- (iii). $\Omega_m = 0.26$, $w = -0.5$,

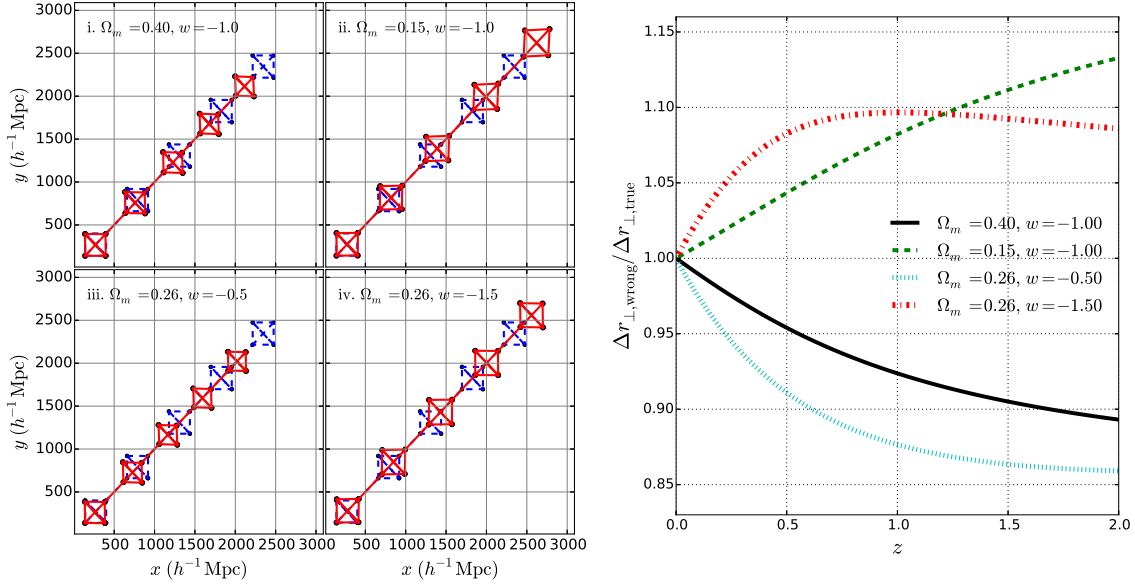


Figure 1. A description of how incorrectly assumed cosmologies can distort the interpretation of observation, assuming $\Omega_m = 0.26, w = -1$ is the true cosmology. Left panel shows a series of five regular squares, measured by an observer at the origin. Their true positions and shapes are plotted in blue dashed lines. When the observer adopts incorrect cosmologies when computing distances from redshifts and inferring the positions and shapes of the squares, they obtain distorted shapes (red solid lines). Right panel shows the redshift evolution of the wrongly estimated angular diameter distance (divided by the correct value).

(iv). $\Omega_m = 0.26, w = -1.5$,

then the shapes of the squares would appear distorted (AP effect), and their sizes would be wrongly estimated (volume effect). Cosmological models (i,iii) result in compressed volume elements (in both angular and LOS directions), and the degree of compression increases with increasing distance; the situation is opposite for the other two cosmologies.

The mis-estimation of angular size, $\Delta r_{\perp, \text{wrong}} / \Delta r_{\perp, \text{true}}$, is displayed in the right panel of Figure 1. In all cosmologies, $\Delta r_{\perp, \text{wrong}} / \Delta r_{\perp, \text{true}}$ evolves significantly in the redshift range $0 \leq z \leq 2$. As an example, when adopting the quintessence cosmology $\Omega_m = 0.26, w = -0.5$, the angular size is underestimated by 8.9%, 12.3%, 13.6%, 14.1% at $z = 0.5, 1, 1.5, 2$.

In sum, as a consequence of incorrect cosmologies the size of objects is mis-estimated and the magnitude of mis-estimation depends on the redshift. We use the galaxy 2pCF across the line-of-sight to probe the mis-estimation of angular size, $\Delta r_{\perp, \text{wrong}} / \Delta r_{\perp, \text{true}}$.

3. THE SIMULATION DATA

We test the method using the mock galaxy samples produced by the Horizon Run 4 (HR4) N-body simulation (Kim et al. 2015; Hong et al. 2016).

HR4 was made within a cube of volume $(3.15 h^{-1} \text{Gpc})^3$ using 6300^3 particles with mass $m_p \simeq 9 \times 10^9 h^{-1} \text{M}_{\odot}$. The simulation adopted the second order Lagrangian perturbation theory (2LPT) initial conditions at $z_i = 100$ and a WMAP5 cosmology $(\Omega_b, \Omega_m, \Omega_{\Lambda}, h, \sigma_8, n_s) = (0.044, 0.26, 0.74, 0.72, 0.79, 0.96)$ (Komatsu et al. 2011).

Mock galaxies are produced from the simulation based on a modified one-to-one correspondence scheme (Hong et al. 2016). The most bound member particles (MBPs) of simulated halos are adopted as tracers of galaxies. The merger timescale is computed to obtain the lifetime of merged halos. Merger trees of halos are constructed by

tracking their MBPs from $z = 12$ to 0; when a merger event occurs, the merger timescale is computed using the formula of Jiang et al. (2008) to determine when the satellite galaxy is completely disrupted.

The resulting mock galaxies were found to reproduce the 2pCF of the SDSS DR7 volume-limited galaxy sample (Zehavi et al. 2011) very well. The mock galaxies shows a similar finger of god (FOG) feature (Jackson 1972) as the observation. The projected 2pCF of the mock and observational samples agree within 1σ CL on scales greater than $1 h^{-1} \text{Mpc}$.

We take the snapshot data of the mock galaxies at $z = 0, 0.5, 1, 1.5, 2$. Setting a minimal halo mass of $3 \times 10^{11} h^{-1} \text{M}_{\odot}$, we select 457, 406, 352, 306 and 228 million mock galaxies at the five redshifts, corresponding to a number density of 1.46, 1.30, 1.13, 0.98 and 0.73 in units of $10^{-2} h^3 \text{Mpc}^{-3}$ respectively. Applying a uniform mass cut, at higher redshift we obtained a smaller number of galaxies with *larger bias*.

As an illustration, Figure 2 displays the 2D distribution of a subsample of mock galaxies in five redshifts, with x, y coordinates computed using the “correct” cosmological parameters $\Omega_m = 0.26, w = -1$ (left panels) and a set of wrong cosmological parameters $\Omega_m = 0.05, w = -1.5$ (right panels), respectively.

When the correct cosmology is adopted the cosmic scale is correctly estimated, the main factor leads to redshift evolution of galaxy distribution is the gravitational growth of structure. With decreasing redshift, we clearly see the clusters and filaments form and grow. On the other hand, when the wrong cosmology is adopted, there is an artificial scaling of distances in the constructed map. The separations among galaxies are over-estimated by 25.6%, 47.3%, 62.2%, 71.7% at redshifts of 0.5, 1.0, 1.5, 2.5, leading to a clear redshift evolution of sizes of structures.

The growth of structure increases the clustering and re-

sults in more compact structures, while the volume effect maintains the clustering pattern and uniformly re-scales structures on all scales. The imprint of each of these effects on the large scale structure is different and should be distinguishable. In the following section we show that they affect $\xi(r_\perp)$ measurements in different ways, and thus can be easily separated.

4. METHODOLOGY

We use the 2pCF across the line-of-sight as a statistical tool to probe the volume effect. The galaxy 2pCF as a function of galaxy separation perpendicular to the LOS, $\xi(r_\perp)$, is computed for snapshot data of mock galaxies at redshifts 0.5, 1, 1.5 and 2. We adopt the Landy-Szalay estimator (Landy & Szalay 1993),

$$\xi(r_\perp) = \frac{DD - 2DR + RR}{RR}, \quad (4)$$

where DD is the number of galaxy-galaxy pairs, DR the number of galaxy-random pairs, and RR is the number of random-random pairs, all separated by a distance defined by $r_\perp \pm \Delta_{r_\perp}$ where we choose $\Delta_{r_\perp} = 1h^{-1}\text{Mpc}$. The random catalogue consists of unclustered points uniformly distributed in the same space as the simulated data. In an effort to reduce the statistical variance of the estimator, we use 50 times as many random points as we have galaxies.

Considering the large number of galaxies and random points the 2pCF is computed part by part in subsamples with size of $1575 h^{-1}\text{Mpc} \times 1575 h^{-1}\text{Mpc} \times 105 h^{-1}\text{Mpc}$. The ‘sheet’-like shape of the subsample is similar to the shape of redshift shells in the real observational case. The Z direction with thickness $105 h^{-1}\text{Mpc}$ is treated as the radial direction (the r direction) and the X-Y directions are the angular plane. For our simulation box size we have 120 such subsamples. The average of the measurements in all subsamples is taken as the 2pCF of the whole sample, while the covariances of them are taken to be the covariance matrix (after multiplying by a factor of $1/\sqrt{119}$).

4.1. Galaxy 2pCF across the line-of-sight: at different redshifts

The upper-left panel of Figure 3 displays the 2pCF across the line-of-sight measured from HR4 mock galaxies. We multiply ξ by the separation r_\perp to obtain a similar statistical uncertainty on all scales. For convenience we will use the denotation

$$\xi_{r_\perp} \equiv r_\perp \xi. \quad (5)$$

Among the different redshift bins there is large variation in the amplitude of $\xi(r_\perp)$. The amplitude is proportional to the clustering strength, which is affected by the gravitational growth of structures and the galaxy bias. The amplitude is highest at $z = 0$ where the structures experienced the most growth. $\xi(r_\perp)$ increases at $z > 1$ with increasing redshift due to the increasing galaxy bias.

Although there is large variation in amplitude between the different redshifts bins, the shape of ξ_{r_\perp} remains similar at all redshifts; in general it peaks at $r \sim 9h^{-1}\text{Mpc}$ and monotonically drops or increases at larger or smaller scales. The only exception is the small enhancement at $r \lesssim 2h^{-1}\text{Mpc}$, which is caused by the non-linear growth of structures and is much more significant at lower redshift.

In order to directly compare the shape of the 2pCF at different redshifts, in the middle panel of Figure 3 we show the $r_\perp \xi$ normalized by the overall amplitude within $5h^{-1}\text{Mpc} < r < 40h^{-1}\text{Mpc}$ (here after $\hat{\xi}_{r_\perp}$):

$$\hat{\xi}_{r_\perp} \equiv \frac{r_\perp \xi(r_\perp)}{\int_{r_{\perp,\min}}^{r_{\perp,\max}} r_\perp \xi(r_\perp) dr_\perp / (r_{\perp,\max} - r_{\perp,\min})}, \quad (6)$$

where we choose $r_{\perp,\min} = 5h^{-1}\text{Mpc}$, $r_{\perp,\max} = 40h^{-1}\text{Mpc}$ in this analysis. Below $5h^{-1}\text{Mpc}$ the non-linear growth of structure leads to systematic redshift evolution which could be difficult to be reliably accounted; On scales larger than $40h^{-1}\text{Mpc}$, the analytical modeling of the shape of ξ_{r_\perp} is relatively well understood; one can just fit the 2pCF with the theoretical predictions (Bernardeau et al. 2002; Salvador et al. 2014, 2016) instead of using our method (although our method should also be applicable).

In the upper-middle panel, the overlapping of $\hat{\xi}_{r_\perp}$ clearly show the minimal redshift evolution of the shape. In the upper-right panel, we further show the residual evolution at high redshifts with respect to $z = 0$. There is only 1-4% enhancement at $r < 10h^{-1}\text{Mpc}$, and $< 1.5\%$ suppression at $r > 25h^{-1}\text{Mpc}$. The trend is monotonic with redshift.

There is one detail in the analysis that should be emphasized here. In real observations we may be selecting *different* types of galaxies at different redshifts. Considering this fact in the comparison of $\hat{\xi}_{r_\perp}$ we compare subsamples of galaxies at *different* locations. As an example, if at $z = 0$ we take $\hat{\xi}_{r_\perp}$ measured within $0h^{-1}\text{Mpc} < Z < 105h^{-1}\text{Mpc}$, at higher redshifts we then adopt measurement within $105h^{-1}\text{Mpc} < Z < 210h^{-1}\text{Mpc}$ for a comparison. If we simply compare the 2pCF of the *same* subsample of galaxies at different redshifts, one would significantly underestimate the statistical uncertainty of $\delta\hat{\xi}_{r_\perp}$ by ignoring cosmic variance.

4.2. Galaxy 2pCF across the line-of-sight: cosmological effect

The 2pCF across the line-of-sight for galaxy positions constructed in a wrongly assumed cosmology $\Omega_m = 0.05$, $w = -1.5$ is displayed in the lower panels of Figure 3. The mis-scaling uniformly shifts the clustering pattern on all scales, leading to a biased $\hat{\xi}_{r_\perp}$, which is related with the true $\hat{\xi}$ as

$$\hat{\xi}_{r_\perp, \text{wrong}}(r) = \hat{\xi}_{r_\perp, \text{correct}}(\alpha_\perp r), \quad (7)$$

a simple consequence of the fact that the clustering pattern at scale r is rescaled to $\alpha_\perp r$.

The redshift evolution of α_\perp leads to redshift evolution of $\hat{\xi}_{r_\perp, \text{correct}}$, which is displayed in the lower-middle panel. The upscaling of comoving distance leads to a stretched shape. When increasing redshift, the peak location is shifted from $\sim 9h^{-1}\text{Mpc}$ to $\sim 15h^{-1}\text{Mpc}$ at $z = 2$, as a result of the fact that the $\hat{\xi}_{r_\perp}$ separation is upscaled by 71.7%. Correspondingly, the right panel shows that, compared with $\hat{\xi}_{r_\perp}$ at $z = 0$ there is a 20-40% change at higher redshifts.

The mis-scaling not only changes the shape of the 2pCF but also changes the value of ξ_{r_\perp} . As is shown in the left panel, due to the stretch of scale the amplitude

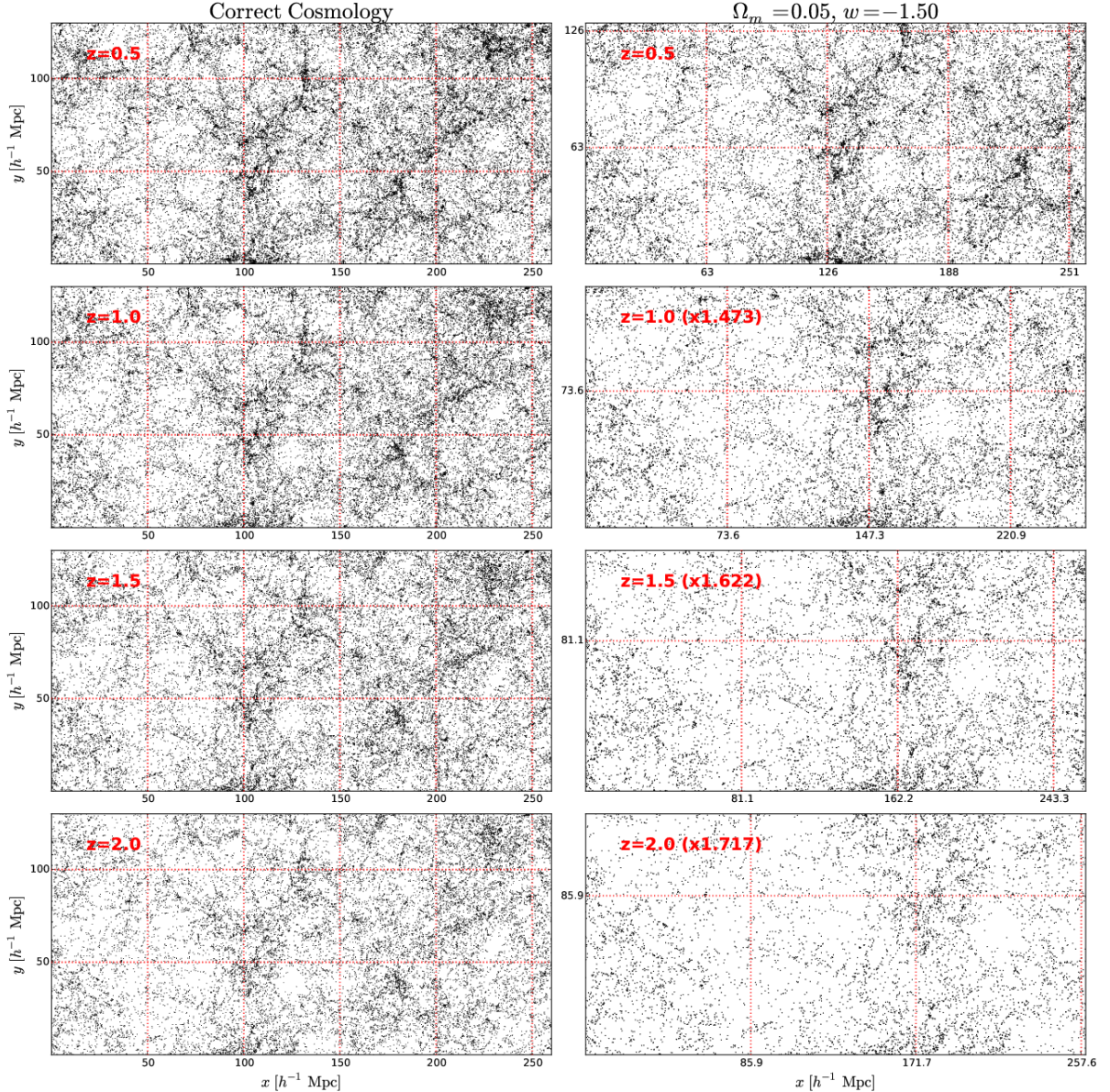


Figure 2. The 2D distribution of a $260 \times 130 \times 105 (h^{-1} \text{Mpc})^3$ subsample of HR4 mock galaxies, at redshifts 0.5, 1, 1.5 and 2. Left and right panels show the x, y coordinates computed using the “correct” parameters $\Omega_m = 0.26$, $w = -1$ and a set of wrong cosmological parameters $\Omega_m = 0.05$, $w = -1.5$, respectively. The growth of structure strengthens the clustering and make structures more compact at lower redshift. When the wrong cosmology is adopted, the comoving distances are mis-scaled and over-estimated by 25.6%, 47.3%, 62.2%, 71.7% at redshifts of 0.5, 1.0, 1.5, 2.5, respectively, leading to a clear redshift evolution of the sizes of structures.

is enhanced at higher redshifts; the higher the redshift, the greater the enhancement².

Although the change of amplitude could be a more significant cosmological consequence than the alteration of shape, it is mixed with the other effects; the growth of structure and a larger galaxy bias can also lead to a stronger clustering and thus an enhanced amplitude. In order to reliably extract the cosmological information, we just utilize the redshift evolution of the 2pCF shape, which is less affected by other factors.

4.3. Systematic effects

² The value of ξ is not affected by the mis-scaling. But since we are using $r_{\perp}\xi$ rather than ξ , the y-axis values are affected. For a 20% up-rescaling, the peak value of $r_{\perp}\xi$ is also increased by 20%.

In Sec. 4.1 we showed that $\hat{\xi}_{r\perp}$ measured from the constant mass cut samples at different redshifts show good agreement. The gravitational growth of structure will not have a large impact on $\hat{\xi}_{r\perp}$ on scales $\gtrsim 5h^{-1} \text{Mpc}$, yet there are many other factors that may induce redshift evolution including galaxy bias, RSD, redshift error, and the redshift evolution of galaxies properties such as mass, morphology, color, concentration. Here we test two “major” systematical effects; the RSD effect and galaxy bias.

4.3.1. Redshift Space Distortion

The galaxy peculiar velocity contaminates the observed redshift and distorts the inferred galaxy radial position. It is the major systematic in our previous works (Li et al. 2014, 2015, 2016) of statistical analysis on the 3D galaxy distribution.

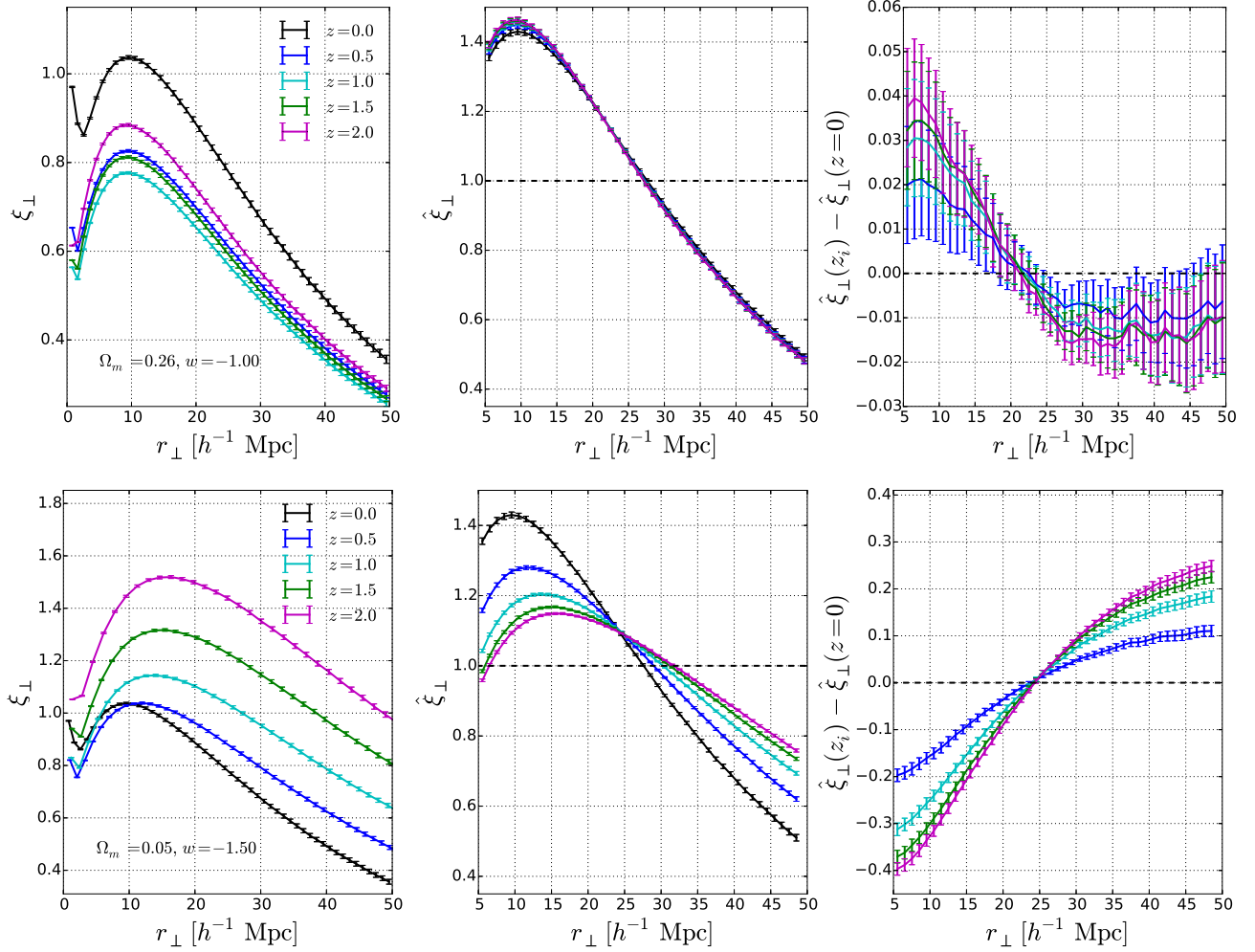


Figure 3. The 2pCF across the line-of-sight, ξ_{r_\perp} (left panels), the shape of $\hat{\xi}_{r_\perp}$ (middle panels), and the evolution with respect to the $z = 0$ value, $\hat{\xi}_{r_\perp}(z = z_i) - \hat{\xi}_{r_\perp}(z = 0)$ (right panels). Upper panels show that in the correct cosmology the redshift evolution of the shape of 2pCF is rather small regardless of the large redshift evolution of the amplitude, whereas the lower panels show that, in a wrong cosmology $\Omega_m = 0.05, w = -1.5$, the redshift evolution of $\hat{\xi}_{r_\perp}$ is very significant.

The effect of RSD is much milder in this work. The angular positions of galaxies are not shifted by RSD at all; the only effect of RSD enter in the procedure of splitting galaxies into redshift shells. The galaxies observed in a survey are split into shells of subsamples, with different redshift ranges. This allows us to obtain 2pCF measurement at various redshifts. RSD distorts the galaxy redshift and as a result some galaxies (especially those close to the boundaries of shells) are assigned to the wrong redshift shells.

We shift the radial coordinates of galaxies according to the relation

$$\Delta z = (1 + z) \frac{v_Z}{c}, \quad (8)$$

where v_Z (Z is the third coordinate of the galaxy in the box, treated as the radial direction in this analysis) is the galaxy peculiar velocity in the LOS direction. This lead to *misclassification* of some galaxies when we split the box into slices.

The upper panel of Figure 4 shows how it affects on the 2pCF. Comparing this plot with Figure 3 one can clearly see the effect of RSD. The amplitude of the measured 2pCF is enhanced by $\sim 10\%$ when considering the RSD

effect and the slop of $\hat{\xi}_{r_\perp}$ is suppressed. However, the redshift evolution of $\hat{\xi}_{r_\perp}$ remains small, $\lesssim 4\%$ at $0.5 \leq z \leq 2$. Therefore RSD should not significantly affect the application of our method, as long as its redshift evolution is small; the small effect of RSD can be precisely modelled by simulations and corrected.

Similar to RSD, redshift errors of galaxies also results in fuzzy boundaries of redshift shells. Thankfully, there are some methods for mitigating this effect in photometric surveys using pair-center binning (Nock et al. 2010).

4.3.2. Galaxy Bias

Galaxies are biased tracers of the dark matter field; more massive galaxies reside in regions with higher density contrast and exhibit stronger clustering (i.e. larger bias).

We vary the mass cut and check the effect of galaxy bias on $\hat{\xi}_{r_\perp}$. The second row of Figure 4 displays the 2pCF measured from galaxies within a $3150 \times 3150 \times 105 (h^{-1} \text{ Mpc})^3$ volume, taken from the $z = 0$ snapshot. Four values of minimal mass limits, $3 \times 10^{11} M_\odot$, $1 \times 10^{12} M_\odot$, $4 \times 10^{12} M_\odot$ and $1 \times 10^{13} M_\odot$, are imposed to create samples with different galaxy biases. The measured 2pCF

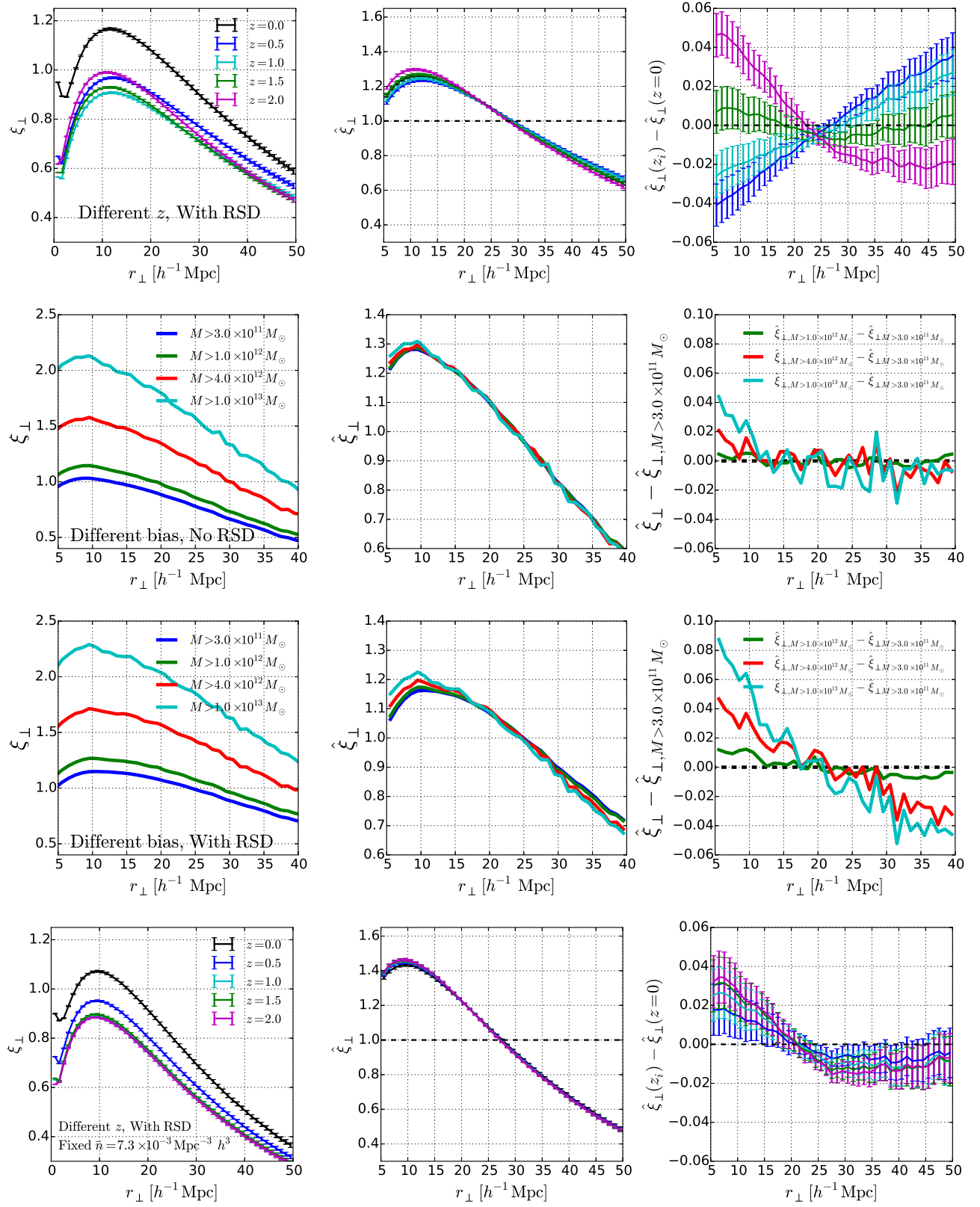


Figure 4. First row: ξ_{r_\perp} (left), $\hat{\xi}_{r_\perp}$ (middle), and $\hat{\xi}_{r_\perp}(z = z_i) - \hat{\xi}_{r_\perp}(z = 0)$ (right) calculated with the RSD effect included. Second row: ξ_{r_\perp} (left) and $\hat{\xi}_{r_\perp}$ (middle) are shown for four different halo-mass-cuts, 3×10^{11} , 1×10^{12} , 4×10^{12} , & $1 \times 10^{13} M_\odot$, below which we remove from 2pCF calculation. The right panel shows the difference in $\hat{\xi}_{r_\perp}$ between the $3 \times 10^{11} M_\odot$ mass-cut case and the other mass-cut cases. Third row: The same as the second row panels, except that the RSD effect is considered. Fourth row: The redshift evolution in case of using samples with constant number density $\bar{n} = 7.3 \times 10^{-3} \text{Mpc}^{-3} h^3$, and the RSD effect is considered.

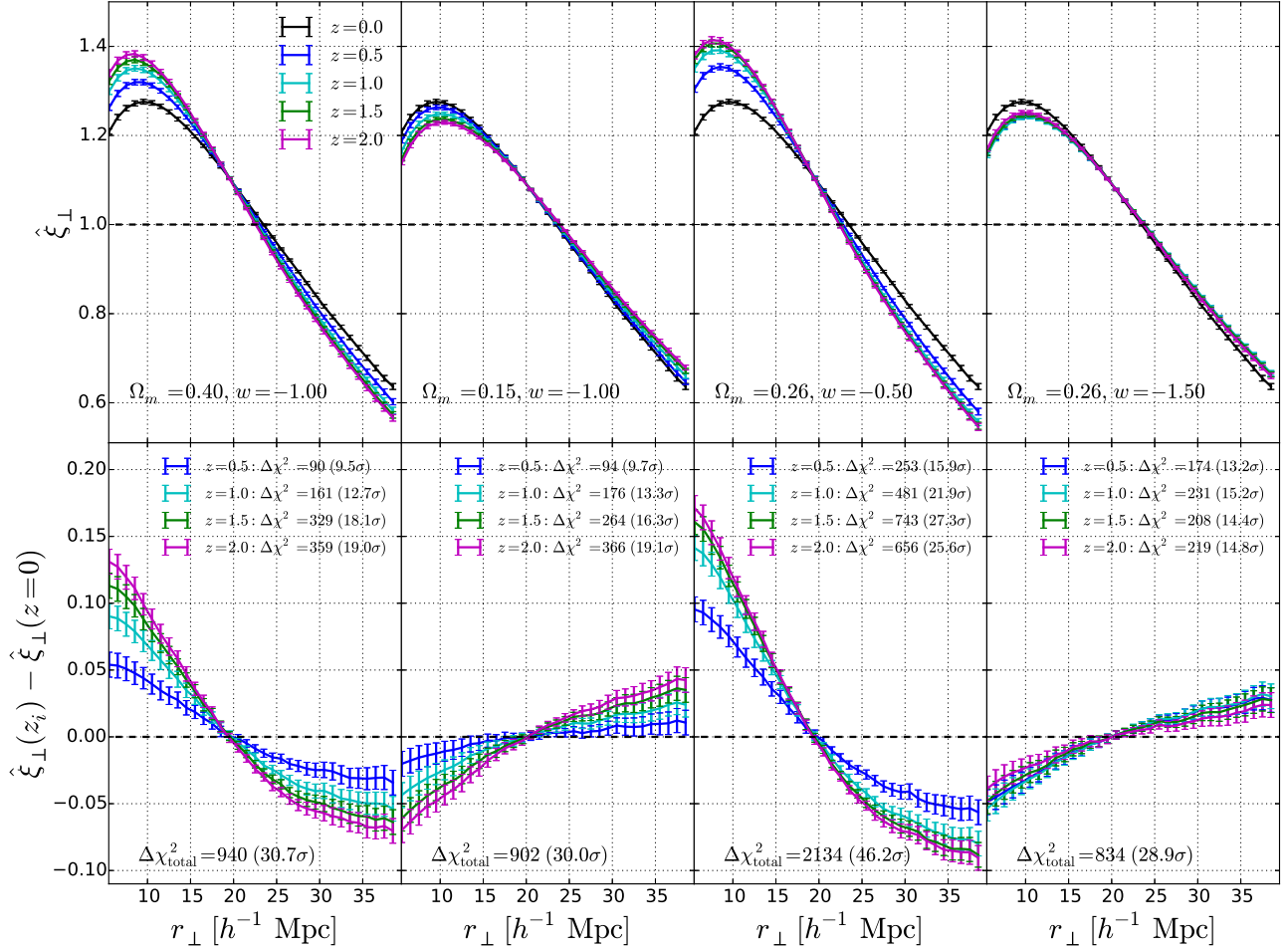


Figure 5. $\hat{\xi}_{r_\perp}$ (upper panels) and $\hat{\xi}_{r_\perp}(z = z_i) - \hat{\xi}_{r_\perp}(z = 0)$ (lower panels) at several redshifts, for four wrong cosmologies. The redshift evolution of the $\hat{\xi}_{r_\perp}$ shape in these wrong cosmologies are detected at high CL.

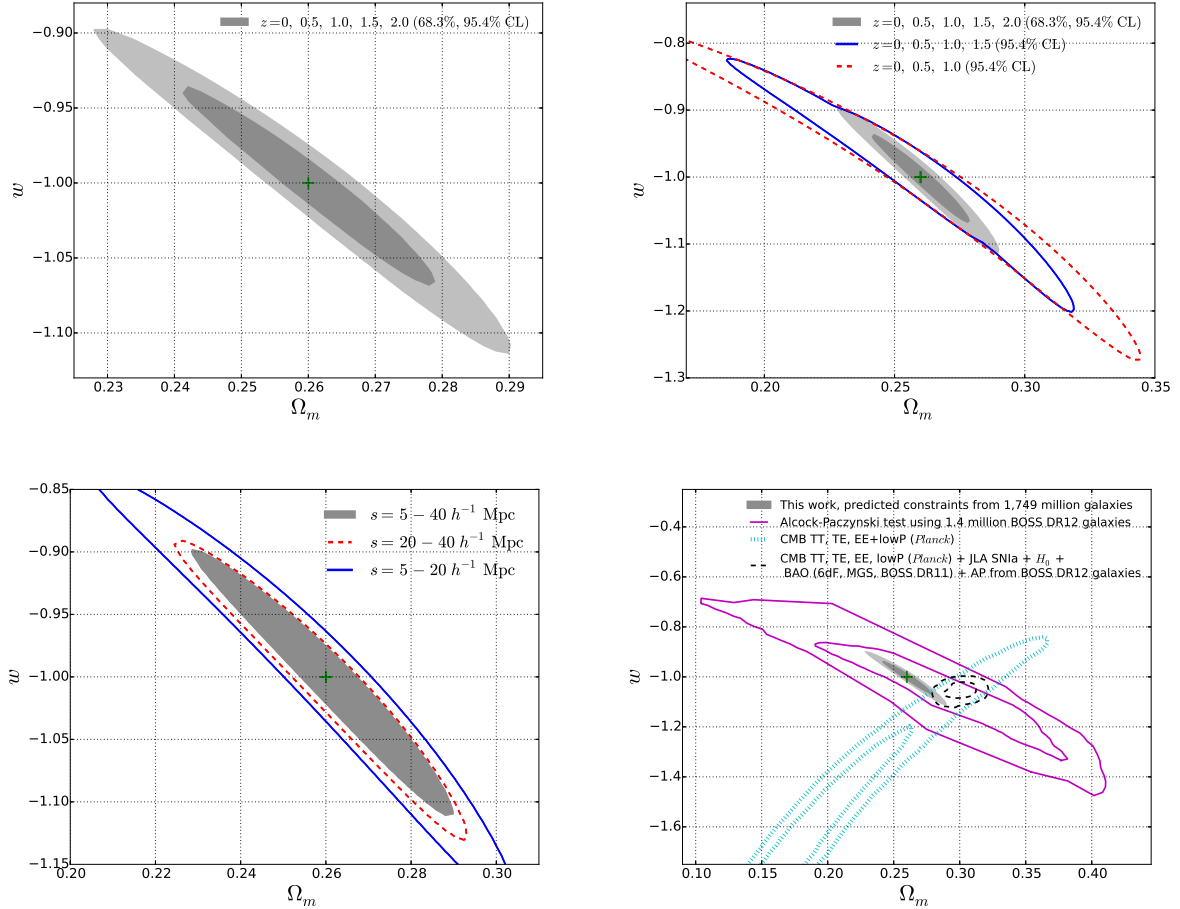


Figure 6. Upper left: Likelihood contours (68.3%, 95.4%) in the $\Omega_m - w$ plane from our method, based on the redshift evolution of the angular 2pCF shape $\hat{\omega}_{r\perp}$. The green point denotes our fiducial cosmological model and the contours denote the statistical error. Utilizing the HR4 snapshots data (containing hundreds of millions of galaxies at redshifts 0, 0.5, 1, 1.5, 2), the method lead to very tight cosmological constraint. Upper right: The constraints in case that one or two high redshift bins are not used. Lower left: The 95.4% likelihood contours using different clustering scales. Lower right: A comparison of the constraint with the cosmological constraints from the current observational data.

across the line-of-sight are displayed.

Different mass cuts result in large variation in the amplitude of the 2pCF (left panel); but the shape of the normalised 2pCF remains less affected (middle panel). Compared with the sample of $M > 3 \times 10^{11} M_\odot$, samples with mass limits of $1 \times 10^{12} M_\odot$, $4 \times 10^{12} M_\odot$ and $1 \times 10^{13} M_\odot$ have amplitude of 2pCFs enhanced by 10%, 50% and 100%, while the change in the shape is only 0.5%, 2% and 4%, respectively. The effect of galaxy bias becomes much less significant by utilizing the shape of the 2pCF rather than the amplitude.

The combined effect of galaxy bias and RSD results in larger systematics. Galaxies that are more biased predominantly reside within nonlinear, over-dense regions and thus have larger larger peculiar velocities. The third row of Figure 4 shows that, the difference of $\hat{\xi}_{r\perp}$ among different mass cut samples reaches 1.5%, 4% and 8% in the case when including the RSD effect.

Finally, the fourth row of the Figure displays the redshift evolution in case of using samples with constant number density $\bar{n} = 7.3 \times 10^{-3} \text{ Mpc}^{-3} h^3$, with the RSD effect considered. We see the redshift evolution of $\hat{\xi}_{r\perp}$ is as small as $\lesssim 3\%$.

4.3.3. More discussion

In summary, we aim to minimise the redshift evolution of the shape of the normalised angular 2pcf; the shape change is dramatic when considering a cosmology much deviated from the fiducial model. However even in the case of the correct cosmology there remains a small signal, due to growth of structures, RSD and galaxy bias. These systematics are removed with the aid of simulations. The procedure of the correction of systematics is similar to what was implemented in our previous papers Li et al. (2014, 2015, 2016).

The systematics are estimated from a simulation conducted using a particular cosmology. There could be a bias in the estimation if the simulation cosmology is different from the truth. The bias would not be serious if the simulation cosmology turns out to be consistent with the best-fit cosmological parameters obtained from the analysis Li et al. (2016). In the case that there is a large discrepancy between the simulation cosmology and the best-fit cosmology, one needs to adjust the simulation cosmology and re-do the analysis.

A more precise approach would involve estimating the systematic effect from a set of simulations covering the relevant volume of the parameter space. This

would remove the remaining uncertainty associated with the cosmological dependence. This approach would also enable the estimation of the *systematic error*, which could be important when the sample size is large enough and the systematic error becomes as small as the *statistical error*.

The third row of Figure 4 shows that, if having a large difference in bias between the samples, there is a large systematic variation on small scales. This means that an accurate modelling of systematics from simulations is required. To avoid the possible difficulty, one can conduct the analysis using a volume-limited sample, where the redshift evolution of galaxy properties is small and the systematic effect from RSD and galaxy bias is also small, as shown by the fourth row of Figure 4.

To avoid the large systematics in the nonlinear regime, another solution would be to increase the scale of analysis to larger separations where the non-linear effect is becoming subdominant. Figure 4 shows that, on scales of $s = 20 - 50h^{-1}\text{Mpc}$ the systematic effect is 2-3 times smaller than that of $s = 5 - 20h^{-1}\text{Mpc}$. Actually, we tested and found that, using $s = 5 - 40h^{-1}\text{Mpc}$ and $20 - 40h^{-1}\text{Mpc}$, yields cosmological constraints that are pretty close to each other (see §4).

We do not consider the systematic effects in case of non-standard cosmological scenarios such as modified gravity and warm dark energy. These topics go beyond the scope of this work, and they are worthy future investigations.

4.4. Likelihood Analysis

Adopting the correct cosmological model will result in minimal evolution of the normalised 2pCF. Thus we proceed to construct a quantitative likelihood estimator that reflects this property. The procedure here is very similar to Li et al. (2014, 2015, 2016). We compare the high redshift $\hat{\xi}_{r\perp}$ and the lowest redshift measurement,

$$\delta\xi_{r\perp}(z_i, z_1) \equiv \hat{\xi}_{r\perp}(z_i) - \hat{\xi}_{r\perp}(z_1), \quad (9)$$

and design the likelihood function to quantify the redshift evolution of $\hat{\xi}_{r\perp}$:

$$\chi^2 \equiv \sum_{i=2}^{n_z} \sum_{j=1}^{n_r} \sum_{j_2=1}^{n_r} \mathbf{p}(z_i, r_{j_1}) (\mathbf{Cov}_i^{-1})_{j_1, j_2} \mathbf{p}(z_i, r_{j_2}), \quad (10)$$

where n_z is the number of redshifts which is 5 in this analysis, n_r is number of binning in $\hat{\xi}_{r\perp}(r_\perp)$, which is 35 since we have r_\perp bins with width $1h^{-1}\text{Mpc}$ in a range of $5h^{-1}\text{Mpc} \leq r_\perp \leq 40h^{-1}\text{Mpc}$. $\mathbf{p}(z_i, r_j)$ is the redshift evolution of the correlation function shape with systematic effects subtracted

$$\mathbf{p}(z_i, r_j) \equiv \delta\hat{\xi}_\perp(z_i, z_1, r_j) - \delta\hat{\xi}_{r\perp, \text{sys}}(z_i, z_1, r_j) \quad (11)$$

\mathbf{Cov}_i is the covariance matrix estimated from the $\mathbf{p}(z_i, r_j)$ measured from 120 subsamples. As mentioned previously, for a robust estimation of the covariance matrix, we always compare slices at different locations to include the cosmic variance.

The covariance matrix inferred from a finite number of samples is always a biased estimate of the true matrix (Hartlap et al. 2006). This can be corrected by rescaling the inverse covariance matrix as

$$\mathbf{Cov}_{ij, \text{Hartlap}}^{-1} = \frac{N_s - n_r - 2}{N_s - 1} \mathbf{Cov}_{ij}^{-1}, \quad (12)$$

where $N_s = 120$ is the number of mocks used in covariance estimation. For this analysis the rescaling is as large as 1.43. In the case that one has 2000 mocks the rescaling is less than 1.02.

Figure 5 displays the 2pCF and the likelihood values when adopting the four incorrect cosmologies used in Figure 1. For the cosmologies $\Omega_m = 0.4, w = -1$ and $\Omega_m = 0.26, w = -0.5$, the compression shifts the clustering patterns to smaller scales at higher redshift and make the 2pCFs steeper. For the cosmologies $\Omega_m = 0.15, w = -1$ and $\Omega_m = 0.26, w = -1.5$, the stretch of structure leads to a shallower slope of $\hat{\xi}_{r\perp}$ at higher redshift. In all cosmologies there is significant detection of redshift evolution. We compute the χ^2 values according to Equation 10, and find that these cosmologies are disfavored at $\gtrsim 30\sigma$ CL.

5. COSMOLOGICAL CONSTRAINT

We constrain Ω_m and w through a Bayesian analysis (Christensen et al. (2001); also see Lewis & Bridle (2002); Li et al. (2016)). We assume that the likelihood takes the form

$$\mathcal{L} \propto \exp \left[-\frac{\chi^2}{2} \right], \quad (13)$$

and scan the parameter space in the $\Omega_m - w$ plane to obtain the 68.3% and 95.4% CL regions. The result is displayed in the left panel Figure 6.

We obtain tight constraints on the two parameters. The 2σ contour lies within the region $0.23 < \Omega_m < 0.29$, $-1.1 < w < -0.9$. The two parameters are very strongly degenerated with each other, and the area of constrained region is quite small. The thin shape of the contour implies that, when combining with other observational data with a different direction of degeneracy (e.g. CMB, BAO), a very tight combined constraint can be obtained.

In the case of fixing one parameter at its best-fit value and constraining the other, one obtains very small 1σ uncertainties of $\delta\Omega_m \approx 0.003, \delta w \approx 0.01$.

The simulation sample used in this analysis reaches $z = 2$. The constraint becomes weaker if we limit the highest redshift of the samples. The upper right panel displays that, in case that we exclude the $z = 2$ galaxies, the 2σ uncertainties of $\delta\Omega_m \approx 0.01$, $\delta w \approx 0.1$ (the other parameter fixed at its best-fit value). The area of contour is further enlarged by $\approx 50\%$ if we only use galaxies at $z = 0, 0.5$ and 1.

So far we simply follow the convention of Li et al. (2016) and adopts $5h^{-1}\text{Mpc} < s < 40h^{-1}\text{Mpc}$. As discussed in the §4.3.3, one may want to avoid large systematics in the non-linear regime. In this case one can adopt $20h^{-1}\text{Mpc} < s < 40h^{-1}\text{Mpc}$, which significantly reduces the systematic effects while not much weaken the power of the final constraints, as shown by the lower left panel of Figure 6.

The choice of the redshift width $105h^{-1}\text{Mpc}$ is also slightly little arbitrary. It should be larger than the distance distortion from the RSD and redshift errors. The optimistic scheme of clustering scales and redshift width should depend on the specific galaxy sample used in the analysis. One could adjust them to have a good balance between the power of cosmological constraint and the size of systematic effect.

Finally, to be connected with the real observations, the lower right panel displays the constraints from Planck CMB (dotted blue, Ade et al. (2015)), the

AP analysis using BOSS DR12 galaxies (solid magenta, Li et al. (2016)), and the joint constraint $\Omega_m = 0.301 \pm 0.006$, $w = -1.054 \pm 0.025$ obtained using the CMB+SNIa+BAO+AP (dashed black, Beutler et al. (2011); Ross et al. (2015); Riess et al. (2011); Betoule et al. (2014); Anderson et al. (2013); Li et al. (2016)).

The predicted constraints from this method clearly wins due to the large sample size. The total number of galaxies used in this work is 1,749 million, which is 1,000 times of the BOSS DR12 sample size. So the constrained area is also ~ 30 times smaller. Such a large sample size will be enabled by LSST (the Large Synoptic Survey Telescope)³, which probe tens of billions of galaxies at $z \lesssim 2$.

6. CONCLUDING REMARKS

We have developed a method for constraining certain cosmological parameters by measuring the redshift evolution of the shape of the galaxy correlation function across the line-of-sight, $\hat{\xi}_{r\perp}$. We found that a wrongly adopted cosmology results in a redshift-dependent mis-scaling in the constructed galaxy distribution, which in turn leads to a redshift-dependent rescaling of $\hat{\xi}_{r\perp}(r_\perp)$. The redshift dependent effect is sensitive to cosmology while being relatively insensitive to the gravitational growth of structure, the galaxy bias, and the RSD effect. We tested our method on the HR4 mock galaxies having 457, 406, 352, 306 and 228 million galaxies at redshifts 0, 0.5, 1, 1.5, 2. Analyzing the redshift evolution of $\hat{\xi}_{r\perp}(r_\perp)$ on scales $5h^{-1}\text{Mpc} \leq r_\perp \leq 40h^{-1}\text{Mpc}$, we derive tight constraints on Ω_m and w .

In this analysis we restrict our focus to the rescaling in the direction perpendicular to line-of-sight. One can also use the 2D 2pCF $\xi(s, \mu)$ as a function of both angular and radial scale to fully probe the scaling effect in the 3D galaxy distribution.

In our early work of Li et al. (2015, 2016), we proposed to use the angular-dependence of the galaxy clustering to apply an AP test and derive cosmological constraints. In contrast, in this work we focus on the scale-dependence of the clustering. The two methods focus on different parts of the clustering information, and the constraints derived from the them should be fairly independent. But it is still suggested to have an estimation of the covariance between them in case one is using both methods to do an analysis. An alternative way is to investigate the redshift evolution of the correlation map $\xi(s, \mu)$ as a function of both scale and direction, which captures all the information.

There are already works using the angular 2pCF to constrain cosmology by directly comparing the theoretical 2pCF and the observed one (Salvador et al. 2014, 2016). Our method is simpler and just uses the fact that in the correct cosmology the shape of the 2pCF across the line-of-sight does not exhibit significant redshift evolution. Our method is complementary to these works in that it does not require accurate analytic modeling, and could be applicable on smaller scales.

Li et al. (2015, 2016) shows that using the redshift dependence of the AP effect, one can derive very tight cosmological constraints. The method presented in this paper studies the redshift evolution of the volume effect. These two methods probe different information encoded in the 2pCF and can be combined to give a complete

geometric probe of the LSS galaxy distribution.

This method is designed to be applied to current and future LSS surveys, with a particular emphasis on *photometric surveys*. The galaxy 2pCF across the line-of-sight in redshift shells is much less affected by the large redshift errors compared with the 3D 2pCF. The Dark Energy Survey will probe hundreds of millions of galaxies at $z \lesssim 1.5$ ⁴, and LSST will probe tens of billions of galaxies at $z \lesssim 2$. We expect very tight cosmological constraints from these surveys.

Our series of works Li et al. (2014, 2015, 2016), together with this analysis, and also Park & Kim (2010); Morandi & Sun (2016), are introducing a new strategy to constrain cosmology from galaxy clustering. We utilize a statistical pattern which remains constant at all redshifts; this invariance is broken when incorrect cosmologies are adopted to construct the galaxy distribution and introduces redshift evolution of geometric distortion. The systematic effects can be corrected by performing an N-body simulation in a cosmology with values of parameters close enough to the underlying correct one. This avoids the difficulty in the analytical modeling the galaxy clustering, galaxy bias and RSD, and enables reliable cosmological constraint on relatively small clustering scales. We hope these methods play an important role in deriving cosmological constraints from future LSS surveys.

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³ <https://www.lsst.org/>

⁴ <https://www.darkenergysurvey.org/>

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