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Accelerated expansion from cosmological holography

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ABSTRACT
It is shown that holographic cosmology implies an evolving Hubble radius $c^{-1}\dot{R}_H = -1+3\Omega_m$ in the presence of a dimensionless matter density Ω_m scaled to the closure density $3H^2/8\pi G$, where c denotes the velocity of light and H and G denote the Hubble parameter and Newton's constant. It reveals a dynamical dark energy and a sixfold increase in gravitational attraction to matter on the scale of the Hubble acceleration. It reproduces the transition redshift $z_t \simeq 0.4$ to the present epoch of accelerated expansion and is consistent with $(q_0,(dq/dz)_0)$ of the deceleration parameter $q(z) = q_0 + (dq/dz)_0 z$ observed in Type Ia supernovae.

Key words: cosmology: theory — cosmology: dark energy

1 INTRODUCTION

General relativity describes a four-covariant geometric theory of gravitation with an exception record in describing linearized gravity in systems much smaller than the Universe, notably in our solar system (e.g. Binney & Tremaine (1987)) and compact binaries evolving by gravitational radiation losses (Hulse & Taylor 1975). It hereby passes crucial observational tests in the embedding of Newton's theory of gravitation by a mixed elliptic-hyperbolic system of equations (e.g. van Putten & Eardley (1996)), parameterized by Newton's constant G and the velocity of light c. The unit of luminosity $L_0 = c^5/G = 3.36\times10^{59}$ herein defines a characteristic scale for the final phase of black hole evaporation and, respectively, gravitational radiation from the coalescence of black hole binaries.

In modern cosmology, however, general relativity is faced with a mysterious cosmological constant Λ or dark energy and dark matter with no apparent microphysical origin (Riess et al. 1998; Perlmutter et al. 1999; Riess et al. 2004). In describing the large scale structure and evolution of the Universe described by the Friedmann-Robertson-Walker (FRW) equations of motion, they arise at weak gravity on the scale of the Hubble acceleration $a_H = cH_0$ defined by the cosmological horizon, where c denotes the velocity of light and H_0 denotes the Hubble constant.

Evidence for dark energy is found in the detection of an accelerated expansion of the Universe at high confidence levels in Type Ia supernova surveys (Riess et al. 1998; Perlmutter et al. 1999; Riess et al. 2004; John 2004). An accelerated Hubble flow points to weak but cosmologically significant forces of non-baryonic origin with a zero-crossing redshift $z_t \simeq 0.4$ for the deceleration parameter $q = q(z)$, defined by $q = -H^{-2}\ddot{a}/a$ in a FRW universe with scale factor $a = a(t)$ and dynamical Hubble constant $H = \dot{a}/a$. Recent supernova surveys with enlarged number of supernova detections extending to redshifts of order unity resolve the Taylor series expansion $q(z) = q_0 + (dq/dz)_0 z$ (Riess et al. 2004), that may serve as a constraint on theories of static or dynamic dark energy.

An excess of cold dark matter to baryonic matter appears by a factor of about six cosmologically in the standard model of Λ and cold dark matter (ΛCMD, Bahcall et al. (1999); Ade et al. (2013)), which appears pervasive across essentially all scales substantially greater than the solar system (e.g. Trippe (2014)). Notably, it appears in galaxy clusters (by a factor of about eight (Giodini et al. 2009)), in the Faber & Jackson (1976) and Tully & Fisher (1977) relations for stellar motion in galaxies, in globular clusters (Hernandez et al. 2013) and in ultra-wide stellar binaries (Hernandez et al. 2012).

There is a notorious tension between dark energy measurements in data of the cosmic microwave background (CMB) and Type Ia supernovae, covering high and, respectively, low redshift epochs of the Universe. Recently, Salvatelli et al. (2014) address the possibility of a dynamical dark energy. Based on CMB and supernova data, some tentative evidence is found favoring a possible interaction of dark energy with cold dark matter at relatively low redshift, while a constant dark energy in ΛCMD is ruled out at 99% confidence level.

In this Letter, we consider the remarkably universal appearance of dark energy as a consequence of the cosmological horizon as a Lorentz covariant boundary condition to general relativity. As a trapped surface, this apparent horizon defines the maximal size of holographic screens, whose surface area $A_H = 4\pi R_H^2$, where R_H denotes the Hubble radius. In setting the maximal radius of a holographic screen, it defines the number of degrees of freedom in the universe (Bekenstein 1981; 't Hooft 1993; Susskind 1995). The information represents the microphysical distribution of matter, which is presently nearly maximal within a factor of order unity (van Putten 2015a). This encoding is commonly envisioned in bits stored in discrete Planck sized surface elements of area l_p^2, $l_p = \sqrt{G\hbar/c^3}$, where $\hbar$ denotes the reduced Planck constant.

In a cosmological holographic interpretation ('t Hooft 1993; Susskind 1995; Easson et al. 2011), the aforementioned scale L_0 appears in a phantom pressure $p_0 = -L_0/cA_H$ in the universe enclosed by aforementioned cosmological holographic screen (van Putten 2015b). By Lorentz invariance of the cosmological horizon, it is accompanied by a dark energy $\rho_\Lambda = -p_0$ (e.g., Weinberg (1989)), manifest in an isotropically accelerated Hubble flow, whereby

$$\Omega_\Lambda = \frac{2}{3}, \tag{1}$$

where $\Omega_\Lambda = \rho_\Lambda/\rho_c$, $\rho_c = 3c^2H^2/8\pi G$. In what follows, we shall use geometrical units with $G = c = 1$ except when stated otherwise.

To study the observational consequences of (1), we consider its implications for general relativity in an isotropic and homogeneous universe described by the FRW line-element. The specific back reaction to general relativity will be expressed in terms of modified FRW equations. In obtaining a second order equation, it represents a singular perturbation of the original Hamiltonian energy constraint in general relativity. It describes an evolution equation for the Hubble radius in response to the presence of the dimensionless matter density Ω_m.

We here study the implications for an evolving dark energy in terms of the associated accelerated expansion and confronted with the transition redshift z_t, $q(z_t) = 0$, and the observed confidence region of $(q_0, (dq/dz)_0)$ obtained from Type Ia supernova surveys. In the present approach, the dynamical dynamical dark energy is co-evolving with Ω_m with no direct interaction, different from the proposed Ansatz in Salvatelli et al. (2014). We consider only the cosmology evolution after the radiation dominated epoch, since baryon nucleosynthesis is not affected in the present approach.

2 HOLOGRAPHIC PHANTOM ENERGY

In the approximation of a homogeneous and isotropic universe, we consider the FRW the line-element 3+1 $ds^2 = -dt^2 + a^2(t)h_{ij}dx^idx^j$ in spherical coordinates

$$ds^2 = -dt^2 + a(t)^2\left(\frac{dr}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta\right). \tag{2}$$

Here, r denotes the comoving radial coordinate scaled by $a(t)$ with dynamical Hubble parameter $H = \dot{a}/a$. Surfaces of constant world-time t have intrinsic Ricci scalar curvature ${}^{(3)}R = 6k/a^2$, $k = \{-1, 0, 1\}$, and extrinsic curvature $K_{ij} = -a\dot{a}h_{ij}$. At a radius R_H, a congruence of outgoing null-geodesics has vanishing expansion, where the unit space-like normal s^i satisfies $D_is^i - K + s^is^jK_{ij} = 0$ (York 1979; Baumgarte et al. 2003), i.e. $R_H = c/\sqrt{H^2 + k/a^2}$ (e.g. Easson et al. (2011)). We shall consider a three-flat universe ($k = 0$), as expected from primordial inflation (Liddle & Lyth 2000). The cosmological horizon assumes the radius $R_H = c/H$. We here consider a generalization of the de Sitter temperature (Gibbons & Hawking 1977) given by the Unruh (1976) temperature of its surface gravity (adapted from Cai & Kim (2005))

$$k_BT_H = \frac{H\hbar}{2\pi}\left(\frac{1-q}{2}\right), \tag{3}$$

where $q = -a\ddot{a}/\dot{a}^2$ is the deceleration parameter and k_B denotes the Boltzmann constant. As a null-surface, the cosmological horizon has a Bekenstein-Hawking entropy $S/k_B = (1/4)A_H/l_p^2$, here identified with the number of degrees of freedom in the phase space in the visible Universe.

In a holographic interpretation the de Sitter temperature (3) of the cosmological horizon introduces, by Lorentz invariance, a dark energy density from a phantom pressure (Easson et al. 2011) from virtual displacements in accord with Gibbs' principle (e.g. Verlinde (2011); van Putten (2012)),

$$-p = A_H^{-1}T_H\frac{dS}{dR} = \frac{k_BT_H}{2R_Hl_p^2}. \tag{4}$$

With (3), (4) reduces to the equivalent local expression $p = -\ddot{A}/(2A)$ in terms of accelerated growth of the number of degrees of freedom in the comoving volume within a comoving surface area $A(t,r) = 4\pi a^2(t)r^2$ of constant r. With $q_0 = -1$ of de Sitter space, (4) recovers (1). As a null-surface, the cosmological horizon is Lorentz invariant, whereby (4) introduces a dynamical dark energy $\rho_\Lambda = -p$, satisfying

$$\Omega_\Lambda = \frac{2}{3}\left(\frac{1-q}{2}\right) \tag{5}$$

as a generalization of (1). For instance, $\Omega_\Lambda \simeq \{0, \frac{1}{2}, \frac{2}{3}, 1\}$ in, respectively, a radiation dominated, matter dominated, present day and Λ dominated epoch.

3 SIXFOLD ENHANCED COUPLING TO Ω_M

The Einstein equations for a universe with cosmological constant and stress-energy tensor T_{ab} of matter are

$$G_{ab} + \Lambda g_{ab} = 8\pi T_{ab}, \tag{6}$$

where G_{ab} denotes the Einstein tensor. Ignoring the holographic back reaction from the cosmological event horizon, we recall for a three-flat space the pair of FRW equations

$$\frac{\ddot{a}}{a} = \frac{1}{3}\Lambda - \frac{1}{2}H^2\Omega_m \tag{7}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho_m + 3p) + \frac{8\pi}{3}\rho_\Lambda. \tag{8}$$

Here $\Omega_m = \rho_m/\rho_c = \omega_m(a_0/a)^3$ is cold ($p = 0$) dark matter of $\rho_m = \rho_0(a_0/a)^3$ and $\omega_m = \rho_0/\rho_c^0$ at present-day closure density $\rho_c^0 = 3H_0^2/8\pi$ defined by the Hubble constant H_0 at $z = 0$; and $\Lambda = 8\pi\rho_\Lambda$. Matched to CMB data, it gives a ΛCDM concordance with $\omega_m \simeq 0.31$ (Ade et al. 2013). For $T_{ab} = \rho_mu_au_b + p(g_{ab} + u_au_b)$ describing matter with a total mass-energy density ρ_m and unit velocity four-vector u^b in the presence of a pressure p, (6) implies more generally a Hamiltonian energy constraint ${}^{(3)}R - K : K + K^2 = 16\pi\rho_m + 16\pi\rho_\Lambda$ in a 3+1 line element with three-curvature ${}^{(3)}R$ and extrinsic curvature K_{ij}. For (2) with $k = 0$ and $K_{ij} = -a\dot{a}\delta_{ij}$, it implies

$$\Omega_m + \Omega_\Lambda = 1, \tag{9}$$

where $\Omega_m = \rho_m/\rho_c$.

Our dynamical dark energy $\Lambda = 8\pi\rho_\Lambda = H^2(1-q)$ in (6), i.e.,

$$G_{ab} = 8\pi T_{ab} - H^2(1-q)g_{ab}, \tag{10}$$

splits up into two parts: a source term H^2 and $-qH^2 = \ddot{a}/a$. Second order in time, the latter modifies the principle part of the Einstein tensor. Moving second-order terms on the left hand-side and leaving first-order terms on the right hand side, we have, for the FRW line-element under consideration, $\tilde{G}_{ab} = 8\pi T_{ab} - H^2 g_{ab}$, where $\tilde{G}_{ab} = G_{ab} + (\ddot{a}/a)g_{ab}$. Specifically, (5) implies that the constraint (9) is now second-order in time, $\ddot{a}/a = 2H^2 - 8\pi\rho_m$, i.e.,

$$\frac{\ddot{a}}{a} = 2H^2 - 3H^2\Omega_m. \tag{11}$$

The general equation (11) is succinctly expressed in terms of an evolution equation for the Hubble radius [1]

$$\dot{R}_H = 1 + q = -1 + 3\Omega_m, \tag{12}$$

A radiation dominated epoch has $q = 1$ with Ω_m replaced by $\Omega_r = 1$, whereas a cold matter dominated epoch has $q = 1/2$ with $\Omega_m = 5/6$ accompanied by dynamical dark energy fraction $\Omega_\Lambda = 1/6$ with late time runaway solution $a = a_0/(t_* - t)$ with t^* on the order of a Hubble time. We have $q = -2$ and $\dot{R}_H = -1$ upon approaching $R_H(t_*) = 0$.

Remarkably, coupling to matter in (11) is *six times stronger* than in the original FRW equation (7). For purposes of numerical integration, we normalize it as

$$\frac{a''}{a} = 2h^2 - 3\omega_m\left(\frac{a_0}{a}\right)^3, \tag{13}$$

where $a = a(\tau)$ as a function of dimensionless time $\tau = tH_0$ and $h = H/H_0$.

On the right hand side of the second FRW equation (8), we have $H^2\left[-\frac{1}{2}\Omega_m - \frac{3}{2}\Omega_p + \Omega_\Lambda\right] = H^2\left[-\frac{1}{2} - \frac{3}{2}\Omega_p + \frac{3}{2}\Omega_\Lambda\right]$, where $\Omega_p = p/\rho_c$. By (5), it reduces to $8\pi p = -\ddot{a}/a$. Here, the appearance of a logarithmic acceleration $\ddot{a}/a$ appears natural in the face of the high symmetry of a FRW cosmology. In Gowdy T^3 cosmologies, for instance, polarized waves are described by a linear wave equation for the logarithm a diagonal metric (Berger & Moncrief 1993; van Putten 1997). The result is a linear relation

$$q = 3\Omega_p, \tag{14}$$

between deceleration and pressure. Since q drops below zero at present, p is currently negative. In our interpretation, p has a corresponding positive two-dimensional pressure in cosmological holographic screens, whereby (14) defines a positive correlation between acceleration and pressure. It suggests that (14) is an inertial equation in cosmological evolution. While p and dark matter are both evolving in time, (14) has no direct coupling between them. It appears that the total phantom pressure in the universe consists of a thermal component, due to (1) and its extension (5), and the inertial component (14) with, at present, no small parameters.

[1] Allowing for curvature, $\Omega_k = -k/aH^2$, the general expression is $\dot{R}_H = 2\beta - 3 + 3\beta^2\Omega_m$, where $\beta = HR_H/c = \sqrt{1 - k(R_H/a)}$ ($k = \{\pm 1, 0\}$).

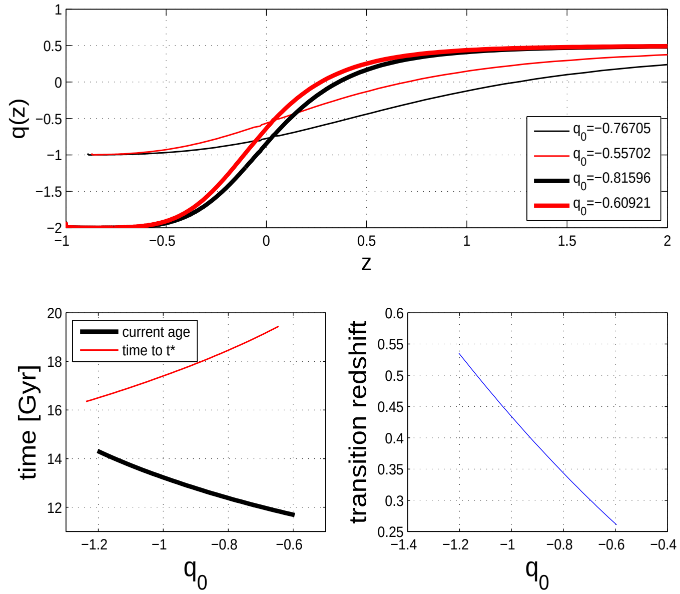

Figure 1. Evolution of the deceleration parameter $q(z)$ of a three-flat cosmology as a function of redshift z (top) according to general relativity with (thick lines) and without (thin lines) back reaction from the cosmological horizon. As function of $q_0 = q(0)$, the age of the universe, the time remaining to t^* (left bottom) and the transition value z_t at crossing $q(z_t; q_0) = 0$ (right bottom).

4 ACCELERATED EXPANSION

The modified FRW equation (12) has observational consequences for weak gravitational interactions on the scale a_H, that are in dramatic contrast to what is naively expected based on (7). Numerical integration of (13) gives a graph $q_0(z)$, independent of $H_0^2\omega_m$, since different choices for $H_0^2\omega_m$ in (12) can be absorbed in a rescaling of time. Fig. 1 shows a transition redshift $q(z_t; q_0) = 0$ as a function of the deceleration parameter at the present epoch $z = 0$, satisfying

$$z_t(q_0) = 0.43 - 0.24(1 + q_0). \tag{15}$$

This is a model independent result consistent with the observed values $z_t = 0.46 \pm 0.13$ and $q_0(0) \simeq -0.8$ inferred from the gold and silver sample in the supernova survey of Riess et al. (2004).

To facilitate our confrontation of with data in the $(q, (dq/dz)_0)$ plane, we consider $(q, (D_h^+ q)_{z_0})$ using the approximation one-sided finite difference

$$D_h^+ q(z_0) = \frac{q(z_0 + h) - q(z_0)}{\Delta z} = (dq/dz)_{z_0} + O\left(h\right) \tag{16}$$

for z_0 close to zero and moderate $h = \Delta z$, permitted by the redshift range of the data. Even though measurement of $q(z)$ is quite challenging, taking $0 < \Delta z < 1$ is expected to allow for a reasonable estimate with moderate dependence on choice of Δz for both model alternatives. Here, (16) is calculated using a linear fit by the method of least square error. Results for different values of h serve to indicate the window of uncertainty in the extraction of dq/dz from the data.

Fig. 2 shows our observational test of (11) and (7) with evolving, respectively, static dark energy in ΛCMD for a

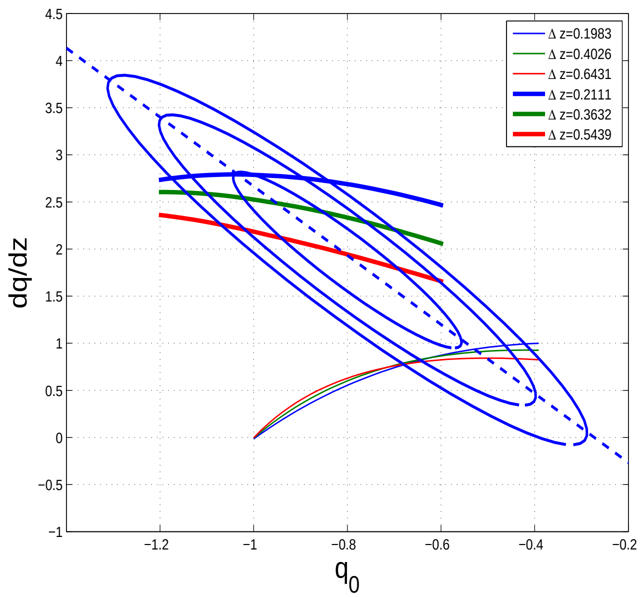

Figure 2. Model graphs $(q_0, D_h^+ q(z_0))$ ($z_0 = 0.008$) are compared with confidence regions (1, 2 and 3σ ellipses) of the Type Ia supernovae survey (gold and silver sample) of Riess et al. (2004) for a three-flat cosmological evolution according to general relativity with (thick lines) and without (thin lines: ΛCDM) back reaction from the cosmological event horizon. Shown are the one-sided finite-difference estimates $D_h^+ q(z_0)$ for dq/dz for various choices of $h = \Delta z$, in simulated extraction from data from nearby events. The curves shown for different choices of h point to a window of about $-1 < q_0 < -0.7$.

three-flat cosmology (Ade et al. 2013), overlaid with the observed confidence region in the $(q_0, (dq/dz)_0)$ plane obtained from the supernova survey of Riess et al. (2004). The latter points to a deceleration parameter value about $q_0 \simeq -0.8$ in the range $-1 < q_0 < -0.7$ of John (2004) (see also Trimble et al. (2006)). In this particular range of q_0, Fig. 2 shows a satisfactory agreement between data and (11), while ruling out (7) at 3 σ.

5 CONCLUSIONS

A holographic back reaction of the cosmological horizon is shown to give a singular perturbation of the FRW equations, described by a dynamical dark energy and a sixfold enhancement in coupling to matter. The observational consequences present a striking departure to what is expected based on general relativity alone. Without fine-tuning, these consequences are in agreement with observations, on the lifetime of the Universe and the transition redshift at zero-crossing of the deceleration parameter (Fig. 1) as well as confidence intervals on the latter and its first derivative (Fig. 2).

While Fig. 2 shows a tension with ΛCDM in the full sample of gold and silver, this appears less so in gold alone. It seems prudent to improve our understanding of these samples in a future analysis.

We emphasize that in a preceding radiation dominated era, $q = 1$, $\Omega_\Lambda = 0$ in (5) and there is no radiative correction to the Einstein tensor. As nucleosynthesis takes place in a radiation dominated era, its results are left unchanged in the present theory.

As holographic quantities, our dark energy and dark matter appear uniformly on a cosmological scale in (12), driving the overall evolution of the universe in co-evolution with but without direct interaction with baryonic matter. Of holographic origin, Λ in (10) represents a Lorentz invariant contribution from the dynamical evolution of the number of degrees of freedom in comoving phase space. Based on these theoretical and observational consequences, holographic cosmology gives some novel justification for a holographic origin of the observed three-dimensional phase space (Bekenstein 1981; t Hooft 1993; Susskind 1995).

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