CS388L: Introduction to Mathematical Logic
Quiz 5, Due April 1

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Prove by natural deduction:

$$((p \to q) \to p) \to p$$

Proof.

A1.	$(p \to q) \to p$.	
1 .	$A1 \Rightarrow (p \to q) \to p$	– axiom.
2 .	$\Rightarrow p \vee \neg p$	– axiom.
A2.	p.	
3 .	$A2 \Rightarrow p$	– axiom.
A3.	$\neg p$.	
4 .	$A3 \Rightarrow \neg p$	– axiom.
5 .	$A3, A2 \Rightarrow \bot$	– $(\neg E), 3, 4$.
6 .	$A3, A2 \Rightarrow q$	– $(C), 5$.
7 .	$A3 \Rightarrow p \to q$	– $(\to I), 6$.
8 .	$A1, A3 \Rightarrow p$	– $(\to E), 7, 1$.
9 .	$A1 \Rightarrow p$	– $(\vee E), 2, 3, 8$.
10 .	$\Rightarrow ((p \to q) \to p) \to p$	– $(\to I), 9$.

□