CS388L Quiz 4

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Let Γ be a positive program containing the rule $p \leftarrow q$. Show that if p doesn't occur in the heads of the other rules of Γ then every minimal model of Γ satisfies the formula $p \leftrightarrow q$.

Proof. For any minimal model M of Γ,

- If $q \in M$, since the rule $p \leftarrow q$ must be satisfied by M, then $p \in M$ as well. Clearly, M satisfies $p \leftrightarrow q$.
- If $q \notin M$, the rule $p \leftarrow q$ is satisfied. Suppose $p \in M$, define $M' = M \setminus \{p\}$. Clearly, the rule $p \leftarrow q$ is satisfied by M' as well. For all other rules $H \leftarrow B$ in Γ:
  - If M satisfies H, since p doesn't occur in H, M' satisfies H as well. Then M' satisfies this rule.
  - If M doesn't satisfy H, since M satisfies $H \leftarrow B$, M must not satisfy B. By problem 24, M', a subset of M, doesn't satisfy B as well. Thus, M' also satisfies this rule.

  This is to say, M' satisfies Γ, which contradicts that M is a minimal model of Γ. Thus, our assumption that $p \in M$ must be false. This is to say, if $q \notin M$, $p \notin M$ as well. Clearly, M satisfies $p \leftrightarrow q$.

To sum up, if p doesn't occur in the heads of the other rules of Γ then every minimal model of Γ satisfies the formula $p \leftrightarrow q$.

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