MTH 9821 Numerical Methods for Finance

Fall 2023

Midterm Exam
October 2

This midterm is to be done in teams of two (the teams will be assigned). All coding is to be done in Python, exclusively using ChatGPT to start your coding. You can add specific lines of code or files as needed, but the vast majority of the coding should be ChatGPT–generated.

Your need to submit both the blueprint solution file AND all the code in a way that our teaching assistants can run it and verify your output.

Note that, from now on, you can use ChatGPT freely in your homeworks; in fact, you are encouraged to do so.

Monte Carlo Pricing and Greeks Estimations for Plain Vanilla European Options

Consider seven months European call and put options with strike \$52 on a lognormally distributed underlying asset with spot price \$49 and volatility 27%, paying 2% dividends continuously. Assume that the risk-free rate is constant at 4.50%.

Use Monte Carlo simulations to calculate the values of the options, as well as the values of the Delta, Gamma, vega, Theta, and rho of the options. All of these are to be computed with four decimal digits accuracy.

Monte Carlo Implied Volatility Estimation for Plain Vanilla European Options

A ten months European put with strike \$66 on an asset with spot price \$62 paying dividends continuously at rate 1.50% is worth \$6.36. The risk free interest rates are constant at 4.50%.

Use Monte Carlo simulations to compute the implied Volatility of this option with two decimal digits accuracy, e.g., 26.73%.

Trinomial Trees Pricing and Greeks Estimations
for
American Options

Use the following parameterizations for trinomial tree methods:

$$u = e^{\sigma\sqrt{3\delta t}}; \qquad d = e^{-\sigma\sqrt{3\delta t}}; \tag{1}$$

$$p_u = \frac{1}{6} + (r - q - \frac{\sigma^2}{2}) \sqrt{\frac{\delta t}{12\sigma^2}}; \tag{2}$$

$$p_m = \frac{2}{3}; \tag{3}$$

$$p_d = \frac{1}{6} - (r - q - \frac{\sigma^2}{2}) \sqrt{\frac{\delta t}{12\sigma^2}}. \tag{4}$$

Consider ten months American call and put options with strike \$32 on a lognormally distributed underlying asset with spot price \$33 and volatility 24%, paying 2% dividends continuously. Assume that the risk-free rate is constant at 4.50%.

Use trinomial trees with $N \in \{10, 20, 40, \ldots, 1280\}$ time steps to calculate the values of the options, as well as the values of the Delta, Gamma, vega, Theta, and rho of the options. All of these are to be computed with four decimal digits accuracy.

Implied Volatility Computation with Trinomial Trees

An eight months American put with strike \$60 on an asset with spot price \$56 paying dividends continuously at rate 1.50% is worth \$6.95. The risk free interest rates are constant at 4.50%.

Use trinomial trees to compute the implied Volatility of this option with two decimal digits accuracy, e.g., 26.73%.

(i) Identify the number of time steps N_{fixed} in the trinomial tree pricer which gives the value of a four months put option within 10^{-6} tolerance. Start with 10 time steps for the given option and double the time steps until two consecutive approximations are within tolerance of each other. The finest tree considered corresponds to N_{fixed} time steps.

(ii) Use the secant method with initial volatility approximations being equal to the Stefanica–Radoicic approximation minus two percent and the Stefanica–Radoicic approximation plus two percent. Use 10^{-4} tolerance. Report all the implied volatility approximations from the secant method recursion as well as the corresponding trinomial tree approximate values of the option.