1. **[Writing] Having filled in the spreadsheet, answer the following question about your results:**
   (a) **Which one tends to be more accurate: in-sample or out-sample pricing? Is there a bias one way or the other between the two? Give an intuitive explanation for the bias**

   In-sample pricing typically yields more precise results. Although all estimates tend to undervalue the true figure, in-sample forward pricing usually offers a closer approximation.

   This can be intuitively understood as a potential overfitting issue within the model, causing the in-sample pricing to appear more accurate.

   (b) **Do out-sample results have a bias one way or the other (compared to the true value)? Explain where this bias comes from and why it is less noticeable in-sample.**

   Yes. Out-sample pricing results bias smaller than the true value. It is less noticeable in-sample because it is trained from in-sample pricing.

   (c) **Which of Tsitsiklis–Van Roy and Longstaff–Schwartz produce more accurate results in-sample? Is there a bias one way or the other between the two? Find an intuitive explanation for the bias**

   Tsitsiklis-Van Roy is more accurate in-sample. Longstaff-Schwartz tend to be smaller.

   (d) **Does a polynomial regression with the monomial basis benefit from preconditioning? Explain your answer by looking at the difference between backward and forward prices in-sample.**

   No, preconditioning gives the same result

   (e) **Does a polynomial regression with the Hermite basis benefit from preconditioning? Does it benefit from standardization? Or both?**

   It benefits from standardization.

   (f) **Explain why the pricers typically produce higher values as you increase the degree of the polynomial.**

   Because the fitted Ct tend to be higher if I increase the degree.

   (g) **Which configuration would you use (basis, degree, preconditioning, TVR/LS, forward/backward, in-sample, out-of-sample) in production and why?**

   I will definitely not use Monte-Carlo methods for American options because it is not accurate. If I have to choose, I will use forward in-sample method.

2. **[Writing] Read the Section 1 and the beginning of Section 2 of Longstaff and Schwartz's original paper [LS01]. They use a non-polynomial basis for their regressions. Is this basis orthogonal under any probability distribution? If so, name**

the distribution. Does their simulation data come from such a distribution? Would their choice of basis would be appropriate for a pricing call option

This basis is orthogonal under the given exponential distribution.

The simulation data doesn't come from such a distribution

The choice of a regression basis must not only be based on the distribution of data but also on its ability to capture significant features of the data. In the context of option pricing, this includes capturing the continuation value of the current stock price effectively.
Given that Longstaff and Schwartz's chosen basis is adept at capturing the continuation value of the current stock price, it becomes suitable for pricing both call and put options, irrespective of the original distribution of the basis functions.

3. **[Writing] Let's assume you've generated paths and priced an American option. How would you calculate a confidence interval that contains the option's true value with 95% probability. (Without repeatedly running the pricer) How would you calculate the option's delta?**

Price the Option, We should first compute the mean $\overline{X}$ and standard deviation SE = $\frac{S}{\sqrt{N}}$. Then we calculate the confidence interval of 95% probability by:

$$[\overline{X} - 1.96 * \mathrm{SE}\,, \overline{X} + 1.96 * \mathrm{SE}]$$

Calculate Delta:
Finite Difference Method:
One common approach, as described by Shreve and others, is to use a finite difference method. You'll perturb the initial asset price slightly upward (S+ΔS) and downward (S−ΔS), and then reprice the option for each perturbed price. The delta (Δ) is then approximated as:

$$\Delta = \frac{\text{OptionPrice }(S + \Delta S) - \text{ OptionPrice }(S - \Delta S)}{2 \times \Delta S}$$

Data Interpolation:
To get a more accurate delta, especially when the option pricing function is not smooth or when the underlying asset's price changes are large, data interpolation techniques can be used. By fitting a curve (e.g., polynomial or spline) to the option prices for a range of underlying asset prices, you can derive a smoother estimate of the option's delta.