Feedback — Final Exam Part I

You submitted this quiz on **Sun 28 Feb 2016 11:19 AM PST**. You got a score of **9.30** out of **10.00**. You can attempt again, if you'd like.

```
To specify an array or sequence of values in an answer, separate the values in
the sequence by whitespace. For example, if the question asks for the first
ten powers of two (starting at 1), then the following answer is acceptable:

     1 2 4 8 16 32 64 128 256 512

If you wish to discuss a particular question and answer in the forums, please
post the entire question and answer, including the seed (which can be used by
the course staff to uniquely identify the question) and the explanation (which
contains the correct answer).
```

Question 1

```
(seed = 764897)
Suppose that you binary search for the key 13 in the following sorted array of size 15:

    14 25 26 36 38 47 53 55 59 67 78 84 89 90 97
```

Give the sequence of keys in the array that are compared with 13.
Your answer should be a sequence of integers, separated by whitespace.

You entered:

55 36 25 14

Your Answer		Score	Explanation
55 36 25 14	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

The correct answer is: 55 36 25 14

Here is the array to be searched after each compare:

```
     14 25 26 36 38 47 53 55 59 67 78 84 89 90 97
55:  14 25 26 36 38 47 53  -  -  -  -  -  -  -  -
36:  14 25 26  -  -  -  -  -  -  -  -  -  -  -  -
25:  14  -  -  -  -  -  -  -  -  -  -  -  -  -  -
14:   -  -  -  -  -  -  -  -  -  -  -  -  -  -  -
```

Question 2

(seed = 291530)
Suppose that you time a program as a function of N and produce the following table.

N	seconds
128	0.000
256	0.001
512	0.006
1024	0.033
2048	0.186
4096	0.975
8192	5.470
16384	29.828
32768	164.755
65536	914.359
131072	5031.788

Estimate the order of growth of the running time as a function of N. Assume that the running time obeys a power law T(N) ~ a N^b. For your answer, enter the constant b. Your answer will be marked as correct if it is within 1% of the target answer - we recommend using two digits after the decimal separator, e.g., 2.34.

You entered:

2.46

Your Answer		Score	Explanation
2.46	✔	1.00	
Total		1.00 / 1.00	

Question Explanation

The theoretical order-of-growth is N ^ (37/15) = 2.47

The empirical order-of-growth is N ^ (log_2 ratio)

N	seconds	ratio	log_2 ratio
128	0.000	-	-
256	0.001	-	-
512	0.006	6.00	2.58
1024	0.033	5.50	2.46
2048	0.186	5.64	2.49
4096	0.975	5.24	2.39
8192	5.470	5.61	2.49
16384	29.828	5.45	2.45
32768	164.755	5.52	2.47
65536	914.359	5.55	2.47
131072	5031.788	5.50	2.46

Question 3

(seed = 608675)
What is the order of growth of the worst case running time of the following code fragment as a function of N?

```
int sum = 0;
for (int i = 0; i*i*i < N; i++)
    for (int j = i+1; j*j*j < N; j++)
        for (int k = j+1; k*k*k < N; k++)
            sum++;
```

Your Answer		Score	Explanation
○ 1			
○ log N			
○ N^(1/2)			
◉ N	✔	1.00	
○ N log N			

- ○ `N^(3/2)`
- ○ `N^2`
- ○ `N^2 log N`
- ○ `N^(5/2)`
- ○ `N^3`
- ○ `N^4`
- ○ `N^5`
- ○ `N^6`
- ○ `N^7`

Total	1.00 / 1.00

Question Explanation

```
The answer is : N

The body of the inner loop is executed N^(1/3) choose 3 ~ N/6 times.
```

Question 4

```
(seed = 510921)
The column on the left contains the original input of 24 strings
to be sorted or shuffled; the column on the right contains the strings in sorted
order; the other 5 columns contain the contents at some intermediate
step during one of the 5 algorithms listed below (with different columns
corresponding to different algorithms).
```

moth	kiwi	bass	gnat	bass	bass	bass
bass	gnat	bull	bass	bull	bull	bull
dove	hake	calf	dove	calf	crab	calf
bull	frog	clam	bull	crab	dove	clam
sole	dove	crab	lynx	dove	goat	crab
hake	goat	crow	hake	goat	hake	crow
mule	crow	dove	crow	hake	loon	dove
loon	bull	frog	loon	loon	mink	frog
crab	calf	gnat	crab	mink	mole	gnat
mole	bass	goat	mole	mole	moth	goat
mink	clam	hake	mink	moth	mule	hake
goat	crab	kiwi	goat	mule	sole	kiwi
calf	loon	mule	calf	slug	calf	loon
toad	lynx	toad	kiwi	sole	clam	lynx
slug	mink	slug	clam	toad	crow	mink
worm	mole	worm	frog	worm	frog	mole
gnat	moth	sole	moth	gnat	gnat	moth
frog	mule	loon	worm	frog	kiwi	mule
clam	puma	moth	slug	clam	lynx	puma
kiwi	slug	mole	toad	kiwi	puma	slug
crow	sole	mink	mule	crow	slug	sole
puma	toad	puma	puma	puma	toad	toad
lynx	worm	lynx	sole	lynx	worm	worm
wren	wren	wren	wren	wren	wren	wren
----	----	----	----	----	----	----
0	?	?	?	?	?	6

Match up each column with the corresponding algorithm from the given list:

```
     0. Original input
     1. Selection sort
     2. Insertion sort
     3. Mergesort (top-down)
     4. Quicksort (standard, no shuffle)
     5. Heapsort
     6. Sorted

Use each number exactly once. That is, your answer should be a permutation of
the 7 integers between 0 and 6 (starting with 0 and ending with 6),
separated by whitespace.

Hint: think about algorithm invariants. Do not trace code.
```

You entered:

```
0 5 1 4 2 3 6
```

Your Answer		Score	Explanation
0	✔	0.14	
5	✔	0.14	
1	✔	0.14	
4	✔	0.14	
2	✔	0.14	
3	✔	0.14	
6	✔	0.14	
Total		1.00 / 1.00	

Question Explanation

```
The correct answer is: 0 5 1 4 2 3 6

 0: Original input
 5: Heapsort after heap construction phase and putting 12 keys into place
 1: Selection sort after 12 iterations
 4: Quicksort (standard, no shuffle) after first partitioning step
 2: Insertion sort after 16 iterations
 3: Mergesort (top-down) just before the last call to merge()
 6: Sorted
```

Question 5

(seed = 147178)
Match up each of the following 5 sorting algorithms

___ randomized quicksort (standard)

___ heapsort

___ mergesort (top-down)

___ mergesort (bottom-up)

___ insertion sort

with the order of growth of its expected running time to sort an input of size N whose keys are all equal
by choosing from the following 6 options:

A. 1
B. log N
C. N
D. N log N
E. N^4/3
F. N^2

Your answer should be a sequence of 5 letters (each between A and F), separated by whitespace. You may use each letter once, more than once, or not at all.

Assume that the sorting algorithms are the pure, unoptimized versions.

You entered:

D C D D C

Your Answer		Score	Explanation
D	✔	0.20	
C	✔	0.20	
D	✔	0.20	
D	✔	0.20	
C	✔	0.20	
Total		1.00 / 1.00	

Question Explanation

The correct answer is: D C D D C

Question 6

(seed = 191202)
Suppose that you have a data type for a sequence of N items and that it is represented internally using a singly-linked list (maintaing a pointer to the first and last nodes and where the ith item in the sequence is stored the ith node of the list). Assume that the data type is implemented in an efficient and natural manner given the specified representation.

Match up each of the following 6 operations

___ remove and return the ith item in the sequence

___ remove and return the last item in the sequence

___ remove and return the first item in the sequence

___ return the index of the first occurrence of a specified item

___ replace the ith item in the sequence with a specified item

___ return the last item in the sequence

with their worst-case running time, by choosing from the following 5 options:

```
A.  1
B.  log N
C.  N
D.  N log N
E.  N^2


Your answer should be a sequence of 6 letters (each between A and E),
separated by whitespace. You may use each letter once, more than once,
or not at all.
```

You entered:

C C A C A A

Your Answer		Score	Explanation
C	✔	0.17	
C	✔	0.17	
A	✔	0.17	
C	✔	0.17	
A	✖	0.00	
A	✔	0.17	
Total		0.83 / 1.00	

Question Explanation

The correct answer is: C C A C C A

Question 7

(seed = 385625)
Suppose that you have a priority queue containing N comparable keys that is represented internally using an array in descending order (with the ith largest key at a[i]). Assume that the data type is implemented in an efficient and natural manner given the specified representation.

Match up each of the following 6 operations

___ delete and return a maximum key

___ iterate over the keys in ascending order

___ insert a key

___ does the priority queue contain a specified key?

___ return a minimum key

___ delete and return a minimum key

with their amortized running time, by choosing from the following 5 options:

A. 1
B. log N
C. N
D. N log N
E. N^2

Your answer should be a sequence of 6 letters (each between A and E), separated by whitespace. You may use each letter once, more than once, or not at all.

You entered:

C C C B A A

Your Answer		Score	Explanation
C	✔	0.17	
C	✔	0.17	
C	✔	0.17	
B	✔	0.17	
A	✔	0.17	
A	✔	0.17	

Total	1.00 / 1.00

Question Explanation

The correct answer is: C C C B A A

Question 8

(seed = 468797)
Suppose that you have a data type that represents a set of N distinct items and that the set of items is re
presented internally using an ordered array (with the the ith smallest key in a[i]). Assume that the data t
ype is implemented in an efficient and natural manner given the specified representation.

Match up each of the following 6 operations

___ return the maximum key

___ remove a specified key from the set

___ insert a specified key into the set

___ return the largest key <= a specified key

___ return the number of keys <= a specified key

```
  ___  return the number of keys


with their amortized running times, by choosing from the following 5 options:

   A.  1
   B.  log N
   C.  N
   D.  N log N
   E.  N^2


Your answer should be a sequence of 6 letters (each between A and E),
separated by whitespace. You may use each letter once, more than once,
or not at all.
```

You entered:

A C C B B A

Your Answer		Score	Explanation
A	✔	0.17	
C	✔	0.17	
C	✔	0.17	
B	✔	0.17	

B	✔	0.17
A	✔	0.17
Total		1.00 / 1.00

Question Explanation

The correct answer is: A C C B B A

Question 9

(seed = 945331)
Match each of the following 6 quantities

___ Min height of a weighted quick-union forest with N sites

___ Min height of a (perfectly balanced) 2-3-4 tree (where each node has 2-4 children) with N keys

___ Max height of an (unweighted) quick-union forest with N sites

___ Min height of a binary heap with N keys

___ Max function-call stack size when (top-down) mergesorting an array of N keys

___ Max height of a (perfectly balanced) 4-way heap (each node has 4 children) with N keys

by choosing from the following 10 options:

A. 0 or constant
B. ~ lg* N
C. ~ 1/2 lg N
D. ~ log_3 N
E. ~ ln N
F. ~ lg N
G. ~ 2 lg N
H. ~ 2 ln N
I. ~ 4.311 ln N
J. ~ N

Your answer should be a sequence of 6 letters (each between A and J),
separated by whitespace. You may use each letter once, more than once, or not at all.

Recall that lg N denotes the binary logarithm (log_2 N); ln N denotes the
natural logarithm (log_e N); and lg* N denotes the binary iterated logarithm.

You entered:

A C J F J G

Your Answer		Score	Explanation
A	✔	0.17	
C	✔	0.17	
J	✔	0.17	
F	✔	0.17	
J	✖	0.00	
G	✖	0.00	
Total		0.67 / 1.00	

Question Explanation

The correct answer is: A C J F F C

Question 10

(seed = 603736)
You are applying for a job at a new software technology company. Your interviewer asks you to identify which of the following graph-processing tasks are possible using techniques discussed in Algorithms, Part I. Check all that apply.

Your Answer		Score	Explanation
☑ Given a set of N points in the plane, determine whether there are 5 collinear points among them in N^(1.5) time.	✖	0.00	It is conjectured that the best possible algorithm is quadratic in the worst case.
☑ Determine how many keys in a left-leaning red-black BST are less than a given key in logarithmic time.	✔	0.20	The rank algorithm discussed in the BST lecture takes time proportional to the height of the BST.
☑ Output the keys in a left-leaning red-black BST in sorted order in linear time.	✔	0.20	An inorder traversal of a (red-black) BST examines the keys in ascending order.
☑ Find the kth smallest in an array of N comparable keys faster than by using a compare-based sorting algorithm to sort the array.	✔	0.20	The expected running time of randomized quick-select is linear; any compare-based sorting algorithm must make a linearithmic number of compares in the worst case.
☑ Given N keys, construct a (perfectly balanced) 2-3 tree containing those keys that has height less than or equal to lg N.	✔	0.20	The height of any 2-3 tree is at most lg N.
Total		0.80 / 1.00	