

# Artificial Intelligence – Spring 2017

## Homework 5

Issued: May 16<sup>th</sup>, 2017

Due: May 30<sup>th</sup>, 2017

**Problem 1:** Exercises 14.14 a, b, c in AI textbook

**Problem 2:** Exercises 14.15

**Problem 3:** Exercises 15.13

**Problem 4:**

Consider the training data below.  $X_1$  and  $X_2$  are binary-valued features and  $Y$  is the label you'd like to classify.

$Y$	$X_1$	$X_2$
+1	0	0
-1	1	0
+1	1	1
+1	0	0
-1	0	1
-1	1	0

(a) Assuming a Naive Bayes model, fill in the quantities learned from the training data in the tables below (no smoothing).

$Y$	$P(Y)$	$X_1$	$P(X_1 Y = -1)$	$P(X_1 Y = +1)$	$X_2$	$P(X_2 Y = -1)$	$P(X_2 Y = +1)$
-1		0			0		
+1		1			1		

(b) Fill in the learned quantities below as in (a), but with add- $k$  (Laplace) smoothing, with  $k = 1$ .

$Y$	$P(Y)$	$X_1$	$P(X_1 Y = -1)$	$P(X_1 Y = +1)$	$X_2$	$P(X_2 Y = -1)$	$P(X_2 Y = +1)$
-1		0			0		
+1		1			1		

(c) Use your model in (b) to calculate  $P(Y|X_1 = 0, X_2 = 0)$ .

(d) What does  $P(Y|X_1 = 0, X_2 = 0)$  approach as  $k \rightarrow \infty$ ?

(e) Circle the feature sets that would enable a linear binary classifier to classify the training data perfectly.  $I$  indicates an indicator function; i.e. equal to 1 if and only if the given conditions holds.

- (i)  $\{X_1\}$
- (ii)  $\{X_2\}$
- (iii)  $\{X_1, X_2\}$
- (iv)  $\{1, X_1, X_2\}$
- (v)  $\{1, \text{abs}(X_1 - X_2)\}$
- (vi)  $\{1, X_1, X_2, X_1 + X_2\}$
- (vii)  $\{1, X_1, X_2, \max(X_1, X_2)\}$
- (viii)  $\{X_1, X_2, I(X_1 = X_2)\}$
- (ix)  $\{1, X_1, (X_1 X_2)\}$

**Problem 5:**

Construct a support vector machine that computes the XOR function. Use values of +1 and -1 (instead of 1 and 0) for both inputs and outputs, so that an example looks like  $([-1, 1], 1)$  or  $([-1, -1], -1)$ . Map the input  $[x_1, x_2]$  into a space consisting of  $x_1$  and  $x_1 x_2$ . Draw the four input points in this space, and the maximal margin separator. What is the margin? Now draw the separating line back in the original Euclidean input space.