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Solvability conditions and general solution for mixed Sylvester equations☆

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ABSTRACT

In this paper, we give some necessary and sufficient solvability conditions for the mixed Sylvester matrix equations, and parameterize general solution when it is solvable. Moreover, we investigate the maximal and minimal ranks of the general solution, and maximal and minimal ranks and inertias of Hermitian part of solution, respectively.

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1. Introduction

The Sylvester matrix equation has found huge applications in H_2-optimal control, H_∞-optimal control (Saberi, Stoorvogel, & Sannuti, 2003), robust control (Cavinlii & Bhattacharyya, 1983; Varga, 2000), singular system control (Shahzad, Jones, Kerrigan, & Constantinides, 2011), control theory (Castelan & Gomes da Silva, 2005; Ding & Chen, 2006; Duan & Zhou, 2006; Tsui, 1987; Wimmer, 1994), and neural network (Zhang, Jiang, & Wang, 2002).

Here we consider the mixed Sylvester matrix equations

$$A_1X - YB_1 = C_1, \qquad A_2Z - YB_2 = C_2. \tag{1}$$

Recently, Lee and Vu (2012) proved that the mixed Sylvester matrix Eq. (1) is consistent if and only if there exist invertible matrices R_1, R_2 and S such that

$$\begin{aligned} \begin{bmatrix} A_1 & C_1 \\ 0 & B_1 \end{bmatrix} R_1 &= S \begin{bmatrix} A_1 & 0 \\ 0 & B_1 \end{bmatrix}, \\ \begin{bmatrix} A_2 & C_2 \\ 0 & B_2 \end{bmatrix} R_2 &= S \begin{bmatrix} A_2 & 0 \\ 0 & B_2 \end{bmatrix}. \end{aligned} \tag{2}$$

Liu (2006) derived a solvability condition of (1). However, the expression of the general solution to (1) was not available then. On the other hand, it is difficult to determine whether the solvability condition (2) holds. Naturally, it is interesting to consider new solvability conditions and an expression of the general solution to the mixed Sylvester matrix equations (1).

Our first aim in this paper is to give new necessary and sufficient conditions for the existence of a solution to (1). Then, we present an expression of the general solution to (1) when it is solvable. Finally, we consider some rank and inertia characterizations of the general solution of (1).

The remainder of this paper is organized as follows. In Section 2, we review some known results and lemmas. In Section 3, we consider the solvability conditions and the expression of the general solution to the mixed Sylvester matrix equations (1). In Section 4, we derive the maximal and minimal ranks of the general solution to the mixed Sylvester matrix equations (1). In Section 5, we give the maximal and minimal ranks and inertias of the Hermitian part of the solution of (1).

Throughout this paper, we denote the complex number field by $\mathbb{C}$. The notations $\mathbb{C}^{m\times n}$ and $\mathbb{C}_h^{m\times m}$ stand for the sets of all $m \times n$ complex matrices and all $m \times m$ complex Hermitian matrices, respectively. The identity matrix with an appropriate size is denoted by I. For a complex matrix A, the symbols A^* and $r(A)$ stand for the conjugate transpose and the rank of A, respectively. The Moore–Penrose inverse of $A \in \mathbb{C}^{m\times n}$, denoted by $A^\dagger$, is defined to be the unique solution X to the following four matrix equations

$$AXA = A, \qquad XAX = X,$$
$$(AX)^* = AX, \qquad (XA)^* = XA.$$

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