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The η-Anti-Hermitian Solution to a System of Constrained Matrix Equations over the Generalized Segre Quaternion Algebra

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Abstract: In this paper, we propose three real representations of a generalized Segre quaternion matrix. We establish necessary and sufficient conditions for the existence of the η-anti-Hermitian solution to a system of constrained matrix equations over the generalized Segre quaternion algebra. We also obtain the expression of the general η-anti-Hermitian solution to the system when it is solvable. Finally, we provide a numerical example to verify the main results of this paper.

Keywords: generalized Segre quaternion algebra; η-anti-Hermitian solution; real representation

1. Introduction

In 1843, Hamilton [1] discovered the real quaternions

$$\mathbb{H} = \{q = q_0 + q_1 i + q_2 j + q_3 k : \ i^2 = j^2 = k^2 = -1, \ ijk = -1, \ q_0, q_1, q_2, q_3 \in \mathbb{R}\}$$

which is a four-dimensional non-commutative division algebra over the real number field $\mathbb{R}$.

The real quaternions have played an important role in many fields such as quantum physics, computer graphics and signal processing [2–4]. In these areas, the real quaternion algebra is more useful than the usual algebra. For example, Ling et al. [5] presented a new algorithm for solving the linear least squares problem over the quaternions. By means of direct quaternion arithmetics, the algorithm does not make the scale of the problem dilate exponentially, compared to the conventional real or complex representation methods. However, the multiplication of real quaternions is non-commutative; therefore, to avoid non-commutativity, the commutative quaternion algebra was introduced.

In 1892, Segre [6] defined the commutative quaternions

$$\mathbb{S} = \{a = a_0 + a_1 i + a_2 j + a_3 k : \ i^2 = -1, \ j^2 = 1, ij = ji = k, \ a_0, a_1, a_2, qa_3 \in \mathbb{R}\}$$

which is a four-dimensional commutative algebra that is not divisible over $\mathbb{R}$.

The commutative quaternions have been widely applied in various fields. For color image processing, Pei et al. [7] defined a simplified commutative quaternion polar form to represent color images, which is useful in the brightness–hue–saturation color space. After this, Pei et al. [8] developed the algorithms for calculating the eigenvalues, the eigenvectors and the singular value decompositions of commutative quaternion matrices. They employed the singular value decompositions of commutative quaternion matrices to implement a color image which reduces the computational complexity to one-forth of the conventional. Guo et al. [9] defined the reduced canonical transform of commutative quaternions which is the generalization of reduced Fourier transform of commutative quaternions. Lin et al. [10] established a commutative quaternion valued neural network (CQVNN) and studied the asymptotic stability of CQVNN.

The commutative quaternion matrix equations have been studied extensively. In [11], Kosal et al. studied some algebraic properties of commutative quaternion matrices

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