ENM 540: Data-driven modeling and probabilistic scientific computing

Lecture #3: Linear regression

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$$f : \mathcal{X} \to \mathcal{Y}$$

Supervised learning

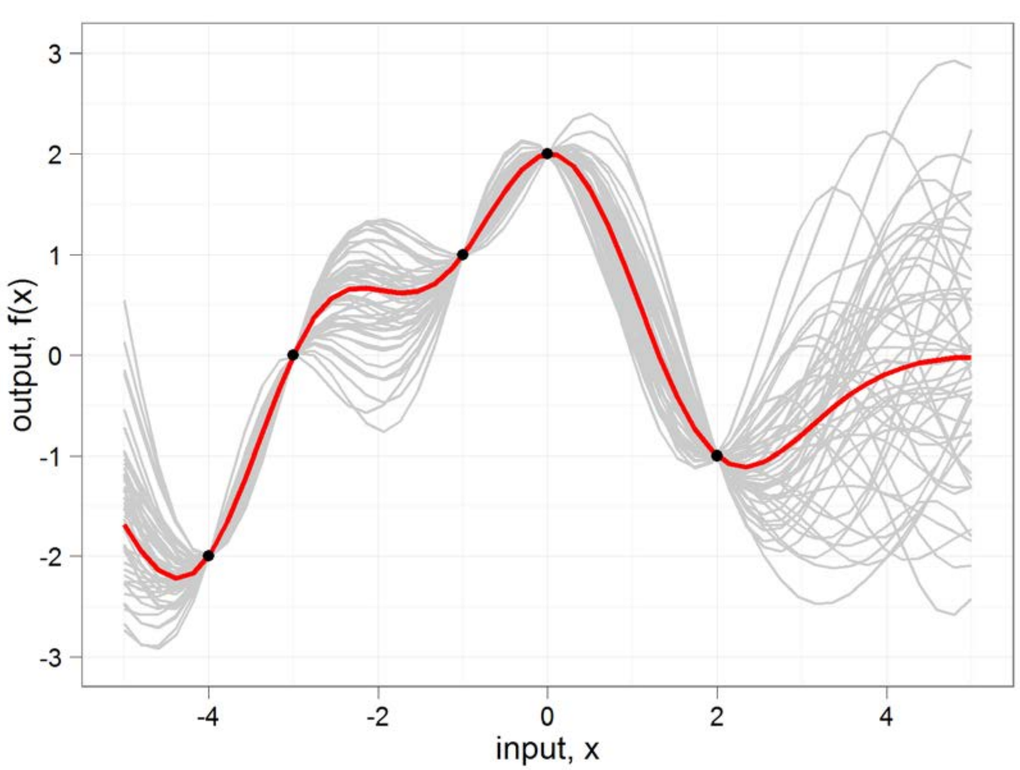
$$f : \mathcal{X} \rightarrow \mathcal{Y}$$

$$\mathcal{D} = \{\boldsymbol{x}, \boldsymbol{y}\},\ \boldsymbol{x} \in \mathcal{X},\ \boldsymbol{y} \in \mathcal{Y}$$

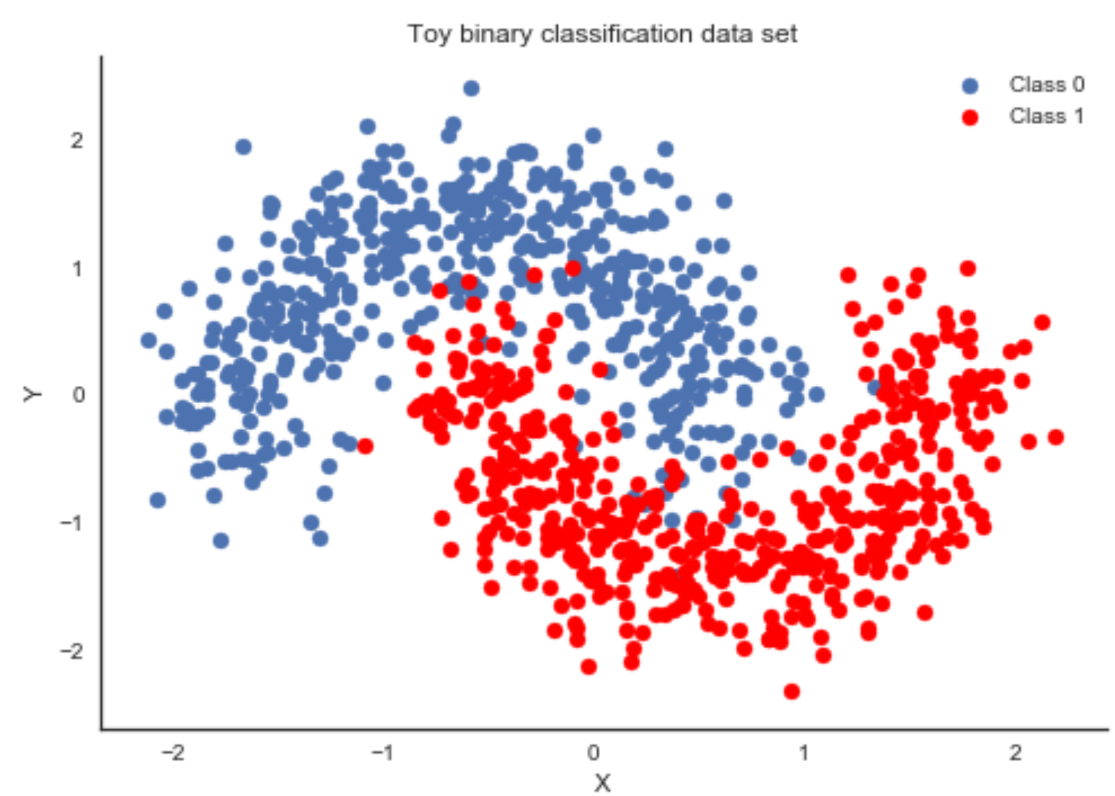
$$\boldsymbol{y} = f(\boldsymbol{x}) + \epsilon$$

$$p(f(\boldsymbol{x}^*)|\boldsymbol{x}^*, \mathcal{D})$$

Regression



Classification



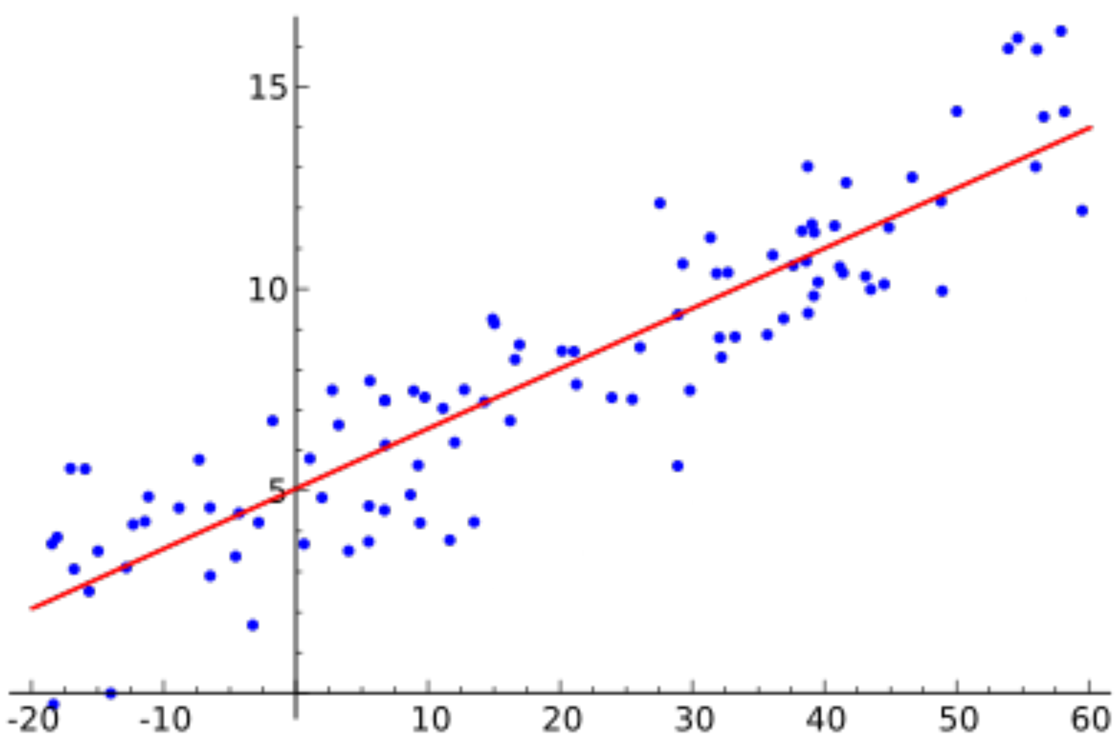
Linear regression

$$f : \mathcal{X} \rightarrow \mathcal{Y}$$

$$\mathcal{D} = \{\boldsymbol{x}, \boldsymbol{y}\}, \; \boldsymbol{x} \in \mathcal{X}, \; \boldsymbol{y} \in \mathcal{Y}$$

$$\boldsymbol{y} = f(\boldsymbol{x}) + \epsilon$$

$$f(\boldsymbol{x}) = w^T \boldsymbol{x}$$



"It's <u>not</u> just about lines and planes!"

Linear regression

Nonlinear functions can be approximating using basis functions (or features)

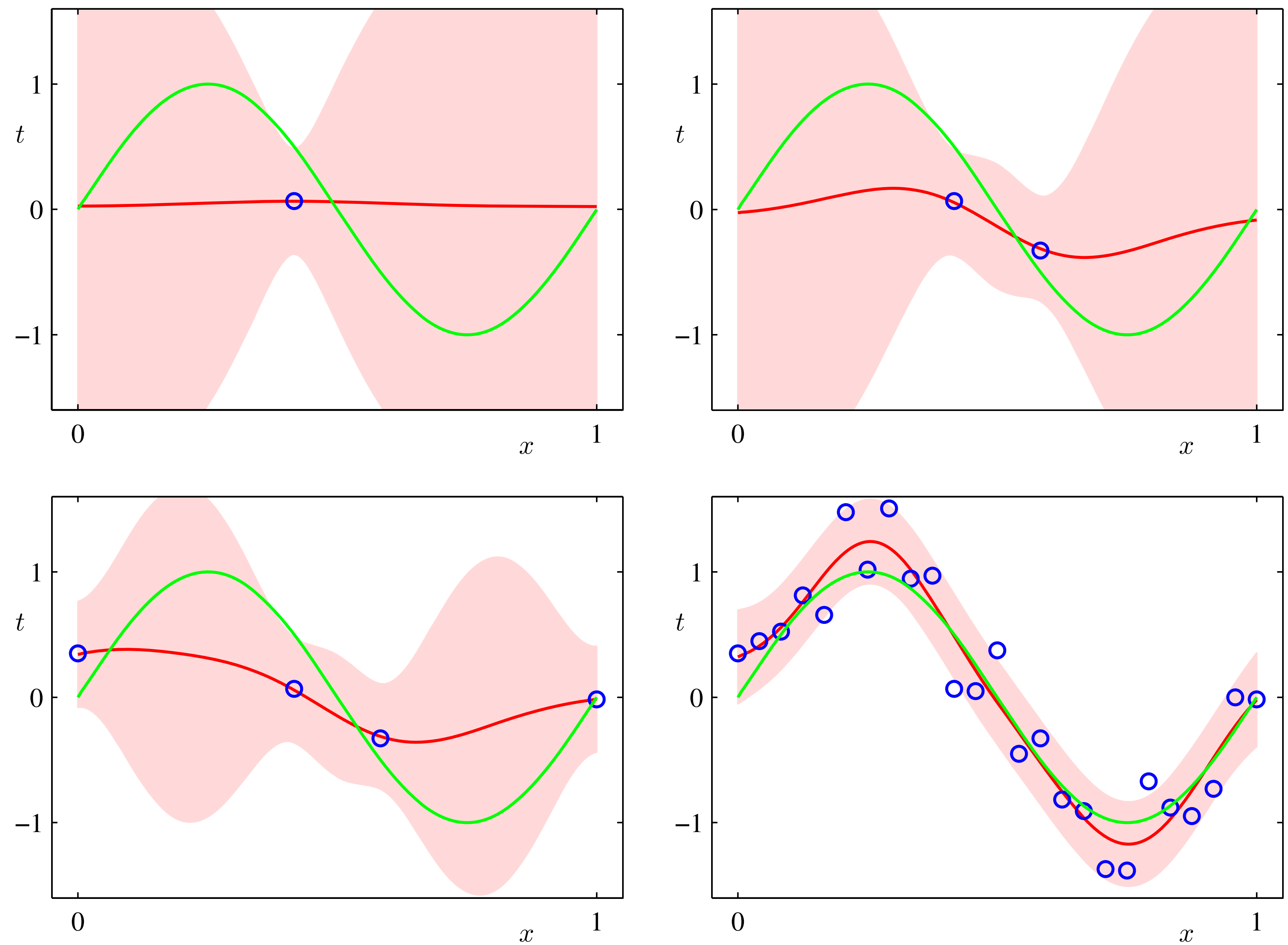


Figure 3.8 Examples of the predictive distribution (3.58) for a model consisting of 9 Gaussian basis functions of the form (3.4) using the synthetic sinusoidal data set of Section 1.1. See the text for a detailed discussion.

$$\boldsymbol{y} = w^T \phi(\boldsymbol{x}) + \epsilon$$

Linear regression

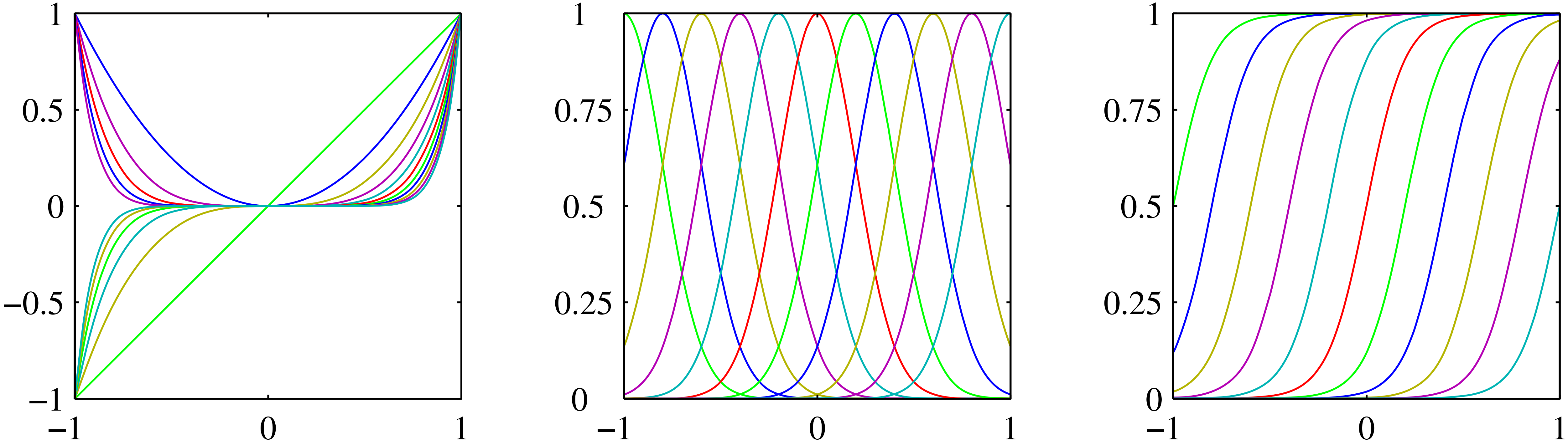


Figure 3.1 Examples of basis functions, showing polynomials on the left, Gaussians of the form (3.4) in the centre, and sigmoidal of the form (3.5) on the right.

Linear regression

Figure 3.2 Geometrical interpretation of the least-squares solution, in an N-dimensional space whose axes are the values of $t_1, \ldots, t_N$. The least-squares regression function is obtained by finding the orthogonal projection of the data vector $\mathbf{t}$ onto the subspace spanned by the basis functions $\phi_j(\mathbf{x})$ in which each basis function is viewed as a vector $\boldsymbol{\varphi}_j$ of length N with elements $\phi_j(\mathbf{x}_n)$.