ODE solver by neural network

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1 Goal

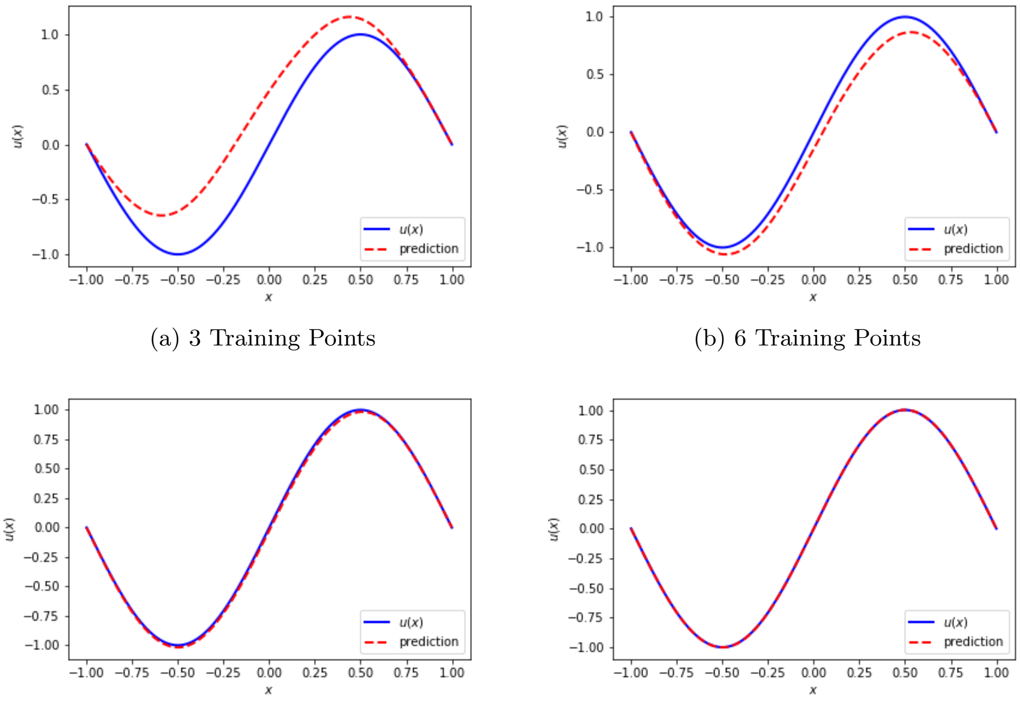
Train a deep neural network by TensorFlow with two hidden layers, 50 neurons per layer, and a hyperbolic tangent activation function to solve ODE

$$\frac{\partial^2 u}{\partial x^2} - u = -(\pi^2 + 1)\sin(\pi x)$$

with boundary conditions $u(-1) = u(1) = 0$. Check convergence of number of training points. Plot the approximation error in the L2 norm as the number of training points is increased.

2 Results

The following figures are directly generated by NN_ODE.py, which is the main script. It imports NeuralNetwork from PDEsolver_tf.py. At the beginning of NN_ODE.py, number of training data can be defined. 1000 test data is used. 40000 iteration is used to train the model. Batch size is full. Fig.1 shows the error of prediction with respect to different numbers of training points. This plot shows convergence.

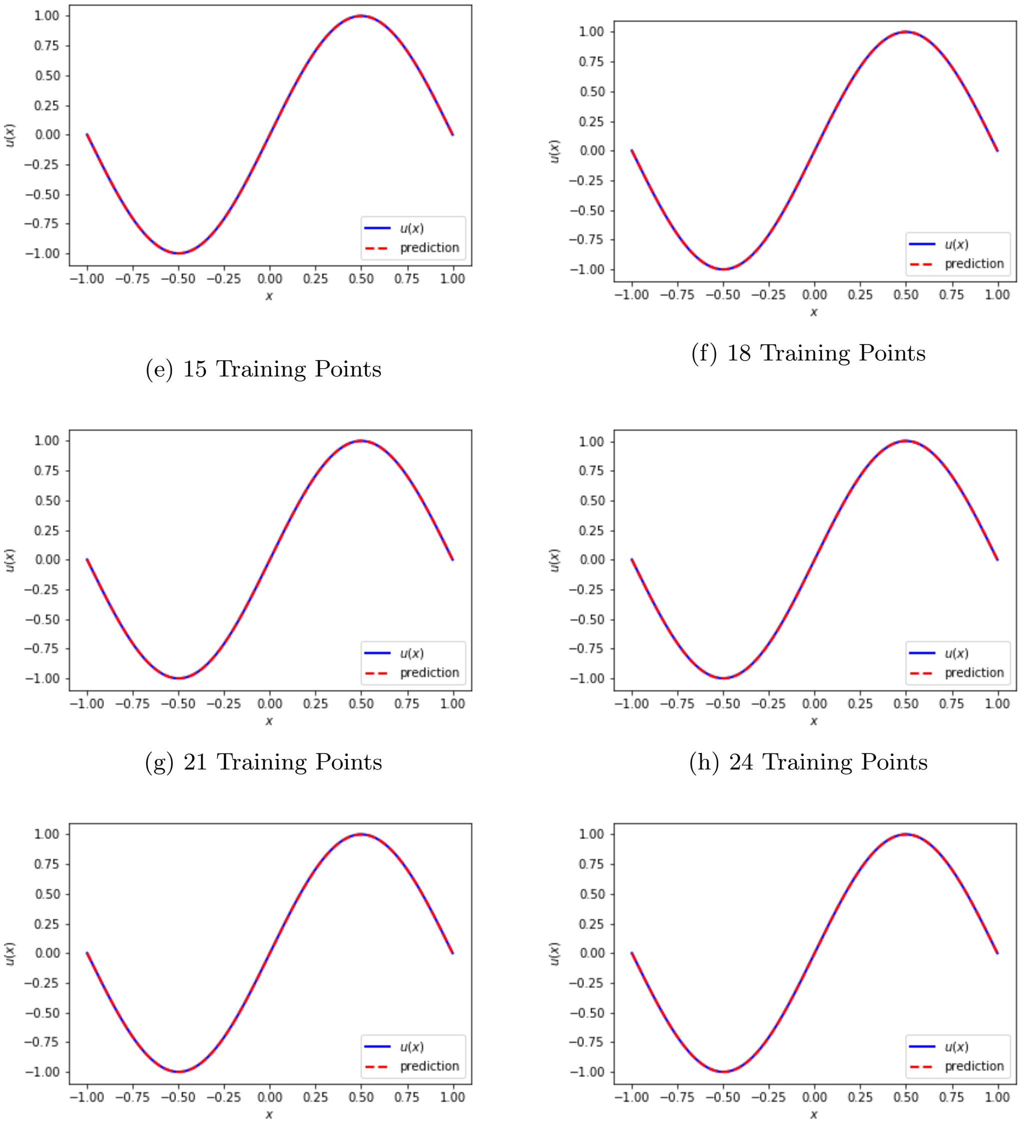


(a) 3 Training Points

(b) 6 Training Points

(c) 9 Training Points

(d) 12 Training Points



(e) 15 Training Points

(f) 18 Training Points

(g) 21 Training Points

(h) 24 Training Points

(i) 27 Training Points

(j) 30 Training Points

Figure 1: Prediction results with different numbers of training data

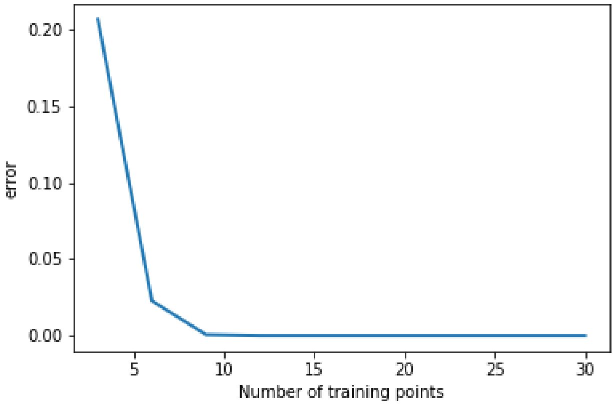


Figure 2: Prediction error decreases with increasing number of training data