

# Model Evidence

2017-02-18

## Preparation

We will use the data in the *mnSchools.csv* file. These data include institutional-level attributes for several Minnesota colleges and universities. The source of these data is: <http://www.collegeresults.org>. The attributes include:

- **id**: Institution ID number
- **name**: Institution name
- **gradRate**: Six-year graduation rate. This measure represents the proportion of first-time, full-time, bachelor's or equivalent degree-seeking students who started in Fall 2005 and graduated within 6 years.
- **public**: Dummy variable indicating educational sector (0 = private institution; 1 = public institution)
- **sat**: Estimated median SAT score for incoming freshmen at the institution
- **tuition**: Cost of attendance for full-time, first-time degree/certificate-seeking in-state undergraduate students living on campus for academic year 2013-14.

```
mn = read.csv(file = "~/Google Drive/Documents/github/EPsy-8252/data/mnSchools.csv")
head(mn)
```

id	name	gradRate	public	sat	tuition
1	Augsburg College	65.2	0	1030	39294
3	Bethany Lutheran College	52.6	0	1065	30480
4	Bethel University, Saint Paul, MN	73.3	0	1145	39400
5	Carleton College	92.6	0	1400	54265
6	College of Saint Benedict	81.1	0	1185	43198
7	Concordia College at Moorhead	69.4	0	1145	36590

## Fit Candidate Models

```
lm.a = lm(gradRate ~ 1 + public, data = mn)
lm.b = lm(gradRate ~ 1 + sat, data = mn)
lm.c = lm(gradRate ~ 1 + tuition, data = mn)
lm.d = lm(gradRate ~ 1 + public + sat, data = mn)
lm.e = lm(gradRate ~ 1 + tuition + sat, data = mn)
lm.f = lm(gradRate ~ 1 + public + tuition, data = mn)
lm.g = lm(gradRate ~ 1 + public + sat + tuition, data = mn)
```

## Model Evidence: AICc

In the last set of notes, we computed the AICc for the six models. Based on these values, we adopted Model E; it was the most likely model given the data and the six candidate models. It can be quite useful to put these values in a table to help organize the information. Here are the six models, and their AICc values.

Model	k	AICc
Model E	4	223
Model G	5	226

Model	k	AICc
Model D	4	227
Model B	3	234
Model F	4	253
Model C	3	258
Model A	3	280

## $\Delta\text{AICc}$

It can be useful to compute the difference between each model's AICc value and the model with the minimum AICc (Model E). This gives us an indication of how much empirical support there is for each of the models. This is typically referred to as  $\Delta\text{AICc}$ . For example to compute the  $\Delta\text{AICc}$  value for Model G,

$$226 - 223 = 3$$

Model	k	AICc	$\Delta\text{AICc}$
Model E	4	223	0.00
Model G	5	226	2.77
Model D	4	227	4.02
Model B	3	234	10.92
Model F	4	253	29.90
Model C	3	258	34.63
Model A	3	280	56.77

Using the criteria in Figure 2 on p. 25 of Burnham, Anderson, & Huyvaert (2011):

- Model E has a lot of empirical support.
- Models G and D have less empirical support than Model A, but, nonetheless still some empirical support.
- Model B has considerably less empirical support than Model A.
- Models F, C, and A have essentially no empirical support.

## Relative Likelihood

The relative likelihood provides the likelihood of each of the candidate models, given the set of candidate models. We compute this as

$$e^{-\frac{1}{2}(\Delta\text{AICc})}$$

For example, to compute the relative likelihood of Model G,

$$e^{-\frac{1}{2}(3)} = 0.223$$

Using R,

```
exp(-0.5 * 3)
```

```
## [1] 0.223
```

Computing these for all six models we find

Model	k	AICc	$\Delta\text{AICc}$	Rel. Lik.
Model E	4	223	0.00	1.000
Model G	5	226	2.77	0.250
Model D	4	227	4.02	0.134
Model B	3	234	10.92	0.004
Model F	4	253	29.90	0.000
Model C	3	258	34.63	0.000
Model A	3	280	56.77	0.000

We can use these values to make relative statements among the models. For example, relative to the other five candidate models, Model E has the highest relative likelihood, making it the most likely model. Model G is relatively more likely than Model D. Models B, F, C, and A are all unlikely relative to the other models in the set.

## Evidence Ratio

By themselves, the relative likelihoods values do not give us really any more information than we already had by examining the  $\Delta\text{AICc}$  values. One way to make use of these values is to compute *evidence ratios*. An evidence ratio is simply the ratio of the relative likelihoods between any two models. For example, to compute the evidence ratio between Model G and Model D,

$$\text{Evidence Ratio} = \frac{0.250}{0.134} = 1.87$$

We can interpret this as, given the data and the set of candidate models, the empirical support for Model G is 1.87 times that for Model D.

Typically the evidence ratios between the best candidate model and every other candidate model is computed and reported.

Model	k	AICc	$\Delta\text{AICc}$	Rel. Lik.	Evidence Ratio
Model E	4	223	0.00	1.000	1.00
Model G	5	226	2.77	0.250	4.00
Model D	4	227	4.02	0.134	7.47
Model B	3	234	10.92	0.004	235.28
Model F	4	253	29.90	0.000	3103711.04
Model C	3	258	34.63	0.000	33103302.71
Model A	3	280	56.77	0.000	2124896922761.96

- Given the data, and the set of candidate models, the empirical support for Model E is 4 times that for Model G.
- Given the data, and the set of candidate models, the empirical support for Model E is 7.5 times that for Model D.
- Given the data, and the set of candidate models, the empirical support for Model E is 235 times that for Model B.

## Model Probabilities (Akaike Weights)

Model probabilities, also referred to as Akaike Weights, provide a numerical measure of the probability of each model given the data and the candidate set of models. We can use them to actually make statements such as, *the probability of Model X is 0.1 given the data and candidate set of models*. We compute model probabilities using the relative likelihood values. In general the model probability for a particular model is

$$\text{Model Probability} = \frac{\text{Relative Likelihood for the model}}{\text{Sum of all Relative Likelihoods}}$$

For example, to compute the model probability of Model G,

$$\text{Model Probability} = \frac{0.25}{1 + 0.25 + 0.134 + 0.004 + 0 + 0 + 0} = 0.18$$

We could interpret this as, given the data and the set of candidate models, the probability of Model G is 0.18. Below we add the model probabilities to our table. Here we refer to the model probabilities as AICc weights, which is common in the literature.

Model	k	AICc	$\Delta\text{AICc}$	Rel. Lik.	Evidence Ratio	Akaike Wt.
Model E	4	223	0.00	1.000	1.00	1.000
Model G	5	226	2.77	0.250	4.00	0.250
Model D	4	227	4.02	0.134	7.47	0.134
Model B	3	234	10.92	0.004	235.28	0.004
Model F	4	253	29.90	0.000	3103711.04	0.000
Model C	3	258	34.63	0.000	33103302.71	0.000
Model A	3	280	56.77	0.000	2124896922761.96	0.000

## Using Built-In Functions in R to Compute Evidence Values

We will use the `aictab()` function from the `AICcmodavg` package to compute these values directly from the `lm()` fitted models. This function takes a list of models in the candidate set (it actually has to be an R list). The optional argument `modnames=` is a vector of model names associated with the models in the candidate set.

```
library(AICcmodavg)

# AICc Table for Model Selection
myAIC = aictab(
  cand.set = list(lm.a, lm.b, lm.c, lm.d, lm.e, lm.f, lm.g),
  modnames = c("Model A", "Model B", "Model C", "Model D", "Model E", "Model F", "Model G")
)

# View table
myAIC
```

	Modnames	K	AICc	Delta_AICc	ModelLik	AICcWt	LL	Cum.Wt
5	Model E	4	223	0.00	1.000	0.720	-107	0.720
7	Model G	5	226	2.77	0.250	0.180	-107	0.900
4	Model D	4	227	4.02	0.134	0.096	-109	0.997
2	Model B	3	234	10.92	0.004	0.003	-114	1.000
6	Model F	4	253	29.90	0.000	0.000	-122	1.000
3	Model C	3	258	34.63	0.000	0.000	-125	1.000
1	Model A	3	280	56.77	0.000	0.000	-136	1.000

Note the output includes the number of parameters ( $k$ ) and AICc value for each candidate model, and prints them in order from most likely to least likely based on the AICc. Note that we have to compute the evidence ratios separately.

```
# Evidence Ratios
evidence(myAIC, model.high = "Model E", model.low = "Model G")
```

```
##
## Evidence ratio between models 'Model E' and 'Model G':
## 4
```

```
evidence(myAIC, model.high = "Model E", model.low = "Model D")
```

```
##
## Evidence ratio between models 'Model E' and 'Model D':
## 7.47
```

```
evidence(myAIC, model.high = "Model E", model.low = "Model B")
```

```
##
## Evidence ratio between models 'Model E' and 'Model B':
## 235
```

```
evidence(myAIC, model.high = "Model E", model.low = "Model F")
```

```
##
## Evidence ratio between models 'Model E' and 'Model F':
## 3103711
```

```
evidence(myAIC, model.high = "Model E", model.low = "Model C")
```

```
##
## Evidence ratio between models 'Model E' and 'Model C':
## 33103303
```

```
evidence(myAIC, model.high = "Model E", model.low = "Model A")
```

```
##
## Evidence ratio between models 'Model E' and 'Model A':
## 2124896922762
```

```
# Examine object
str(myAIC)
```

```
## Classes 'aictab' and 'data.frame': 7 obs. of 8 variables:
## $ Modnames : Factor w/ 7 levels "Model A","Model B",...: 5 7 4 2 6 3 1
## $ K : num 4 5 4 3 4 3 3
## $ AICc : num 223 226 227 234 253 ...
## $ Delta_AICc: num 0 2.77 4.02 10.92 29.9 ...
## $ ModelLik : num 1 0.25004277 0.13392295 0.004250287 0.000000322 ...
## $ AICcWt : num 0.720348808 0.180118012 0.096471237 0.003061689 0.000000232 ...
## $ LL : num -107 -107 -109 -114 -122 ...
## $ Cum.Wt : num 0.72 0.9 0.997 1 1 ...
```

Since `myAIC` is a data frame, we can add columns, use indexing, submit results to `ggplot()`, etc. Here we compute the evidence ratios using information from the `myAIC` object, and add them as an additional column.

```
# Create new column
myAIC$ER = max(myAIC$AICcWt) / myAIC$AICcWt
myAIC
```

	Modnames	K	AICc	Delta_AICc	ModelLik	AICcWt	LL	Cum.Wt	ER
5	Model E	4	223	0.00	1.000	0.720	-107	0.720	1.00
7	Model G	5	226	2.77	0.250	0.180	-107	0.900	4.00
4	Model D	4	227	4.02	0.134	0.096	-109	0.997	7.47
2	Model B	3	234	10.92	0.004	0.003	-114	1.000	235.28
6	Model F	4	253	29.90	0.000	0.000	-122	1.000	3103711.04
3	Model C	3	258	34.63	0.000	0.000	-125	1.000	33103302.71
1	Model A	3	280	56.77	0.000	0.000	-136	1.000	2124896922761.96

We can also customize the output to only include information we are interested in. For example, we could remove columns, re-order columns, or change column names.

```
# Get existing column names
names(myAIC)
```

```
## [1] "Modnames" "K" "AICc" "Delta_AICc" "ModelLik"
## [6] "AICcWt" "LL" "Cum.Wt" "ER"
```

```
# Select and order columns for better printing
myAIC = myAIC[, c("Modnames", "LL", "K", "AICc", "Delta_AICc", "ER", "AICcWt")]
```

```
# Rename columns for better printing
names(myAIC) = c("Model", "Log Lik.", "k", "AICc", "$\\Delta$AICc", "Evidence Ratio", "Akaike Wt.")
```

Table 1

*Model Evidence for Six Candidate Models Predicting Graduation Rates for 32 Colleges and Universities in Minnesota. Evidence Ratios are Reported as a Comparison with the Model having the Highest Relative Likelihood (Model E).*

Model	Log Lik.	k	AICc	$\Delta$ AICc	Evidence Ratio	Akaike Wt.
Model E	-107	4	223	0.00	1.00	0.720
Model G	-107	5	226	2.77	4.00	0.180
Model D	-109	4	227	4.02	7.47	0.096
Model B	-114	3	234	10.92	235.28	0.003
Model F	-122	4	253	29.90	3103711.04	0.000
Model C	-125	3	258	34.63	33103302.71	0.000
Model A	-136	3	280	56.77	2124896922761.96	0.000

All of this evidence points to Model E as the best candidate model given the data. Models G and D have some tenability, but are not nearly as empirically supported as Model E. Finally, Models B, F, C, and A seem completely unsupported.

## References

Burnham, K. P., Anderson, D. R., & Huyvaert, K. P. (2011). AIC model selection and multimodel inference in behavioral ecology: Some background, observations, and comparisons. *Behavioral Ecology and Sociobiology*, 65(1), 23–35.