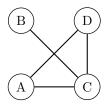


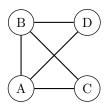
## 1 Graphs

1. For each of the following graphs, determine the neighbourhoods and degrees of each vertex. Verify that the handshaking lemma holds for each graph.

(a)

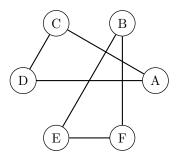


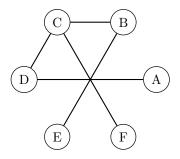
(b)



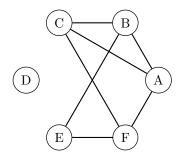
- 2. Draw the graph on the vertices  $\{0,1,2,3,4,5\}$  where u is adjacent to v if  $u \equiv v+1 \pmod 6$  or  $u \equiv v+2 \pmod 6$ .
- 3. For each of the following graphs and answer these questions:
  - i. Is A, B, C a path?
  - ii. Is A, B, C, D, A a cycle?
  - iii. What is the length of the longest path?
  - iv. What are the connected components?

(a)



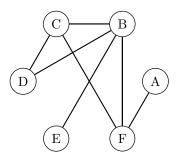


(c)

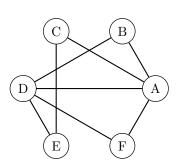


4. Let D(u,v) be the distance between vertices u and v. For the following graphs, form find the distance classes from vertex A,  $D_j = \{v \in V : D(A,v) = j\}$ .

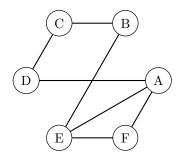
(a)



(b)



(c)



- 5. For each problem, describe a graph and a graph theoretic problem that describes the original problem. Describe an approach to solving the graph theoretic problem.
  - (a) A travel company wants to develop a website that allows users to select an airport and lists the minimum number of stops-overs required to fly to all other airports.
  - (b) A robot is being programmed to solve a maze. The maze consists of several intersections with corridors between them, plus an entrance and an exit. The robot is able to follow corridors and remember intersections and corridors in the maze that it has visited. The goal of the robot is to travel from the entrance to the exit.
  - (c) A student is studying Rubik's cubes. She wants to write a program that, given a starting configuration of the cube, determines the minimum number of moves required to solve the cube, or determines that it is impossible (perhaps someone has moved the stickers!)
- 6. Modify the distanceClasses function from the lecture to return the distance between two points rather than returning the distance classes from a particular vertex.