CAB203
Mathematical foundations quiz

For multiple choice questions select the *best* answer indicate your selection in the Blackboard Quiz. For short answer questions, enter your final answer only into the corresponding answer field in the Blackboard Quiz. Each question is worth 1 point. Answer all 25 questions.

Please note that there is an appendix at the end of this paper that includes a table for hexadecimal values and an ASCII character chart. You are allowed to consult all unit material and material on the internet. However you are not allowed to communicate with others (students or otherwise) during the quiz.

1. Consider the following problem: Given two baskets of apples, we combine them into a single basket of apples. How can we know how many apples we have in the final basket? The traditional solution is to add the number of apples in each of the original baskets.

   What role should *counting* apples play in *this* solution?

   (a) Counting is the abstraction of the quantity of apples
   (b) Counting is a model of what it means to combine two baskets of apples.
   (c) Counting establishes the relationship between the baskets of apples and natural (i.e. positive integer) numbers.
   (d) We can count the apple in the final basket to get the answer.

2. According to the laws of logarithms, which of the following is equal to:

$$\log_8 \frac{(2^a)^b}{2^c}$$

   (a) 8^{ab-c}
   (b) $\frac{1}{3}(2^{ab-c})$
   (c) $\frac{1}{3}(ab-c)$
   (d) $3(a+b-c)$

3. Numbers have many special properties that are not necessarily shared by other mathematical objects. Put differently, each mathematical theory has its own axioms which are often quite different from the axioms of the natural numbers. So it is important not to take the properties of numbers for granted.

   Matrices (which we will study in more depth later) are mathematical objects that can be added, subtracted, and multiplied much like numbers. They have the following properties for all matrices x, y, z (assuming square matrices of the same dimension):

   i. $x + y = y + x$
   ii. $x + (y + z) = (x + y) + z$
   iii. $x(y + z) = xy + xz$

   Let a, b, c, d be matrices. Given the above properties *only*, $a(b+c)+d$ is always equal to three of the following. Which is the one that cannot be shown to be always equal to $a(b + c) + d$?

   Hint: Each of the options (other than the one which is not equal) can be obtained from $a(b + c) + d$ using at most two of the listed properties.

   (a) $ab + (ac + d)$
   (b) $d + (ab + ca)$
   (c) $(ac + ab) + d$
   (d) $d + a(c + b)$

4. Which of the following is *not* equivalent to the others?

   (a) $x|y$
   (b) x is divisible by y
   (c) There exists an integer c such that $cx = y$
   (d) x divides y

5. Applying mathematical definitions from the unit, which of the following is equivalent to

$$s \equiv t \pmod r$$

   (a) t is the smallest integer equivalent to s modulo r
   (b) $s - t$ divides r
   (c) There is some integer z such that $s - t = rz$
   (d) $(s - t)|r$

6. What is the value stored in x after the following operations? Give your answer in hexadecimal, using upper case letters if required, and omit any leading 0's. (i.e. your answer could look something like 3F.)

```
x = 0x35
x = x << 4
x = x & 0xFF
mask = 1 << 3
x = x | mask
```

7. The *reverse* of a bit string is that same bit string, but written down backwards. That is to say, the reverse of $\overline{x} = x_{n-1} \ldots x_1 x_0$ is $x_0 x_1 \ldots x_{n-1}$. For example, the reverse of `0011` is `1100`. We define a bitstring to be *skew-symmetric* if it is equal to the complement of its reverse. That is to say, if you reverse the string and then apply NOT to every bit you will obtain the original string. For example `1100` is skew-symmetric.

   Which of the following hexadecimal — when written down as a 16-bit binary string (with leading 0's if necessary) — is skew symmetric?

   (a) F170
   (b) EE95
   (c) 0048
   (d) EC50

8. Which line of Python code can be used to set bit 4 of `x`?

   (a) `x << 4`
   (b) `x = x | (1 << 4)`
   (c) `x = 1 << 3`
   (d) `x = x & ~(1 << 4)`

9. *Binary coded decimal* (BCD) is a scheme for encoding integers into bits which was previously popular with 8-bit computers, some of which even had dedicated instructions for using it. A number is encoded in BCD using the following steps:

   1. Write down the number in base-10
   2. Convert each numeral in the base-10 representation into a 4-bit binary string. Eg. 0 becomes 0000, 7 becomes 0111 etc.
   3. Concatenate all the 4-bit strings to obtain the final string.

   Which of the following is true about BCD?

   (a) In hexadecimal, a number encoded in BCD is the same as the number itself written in base-10. Eg. 2022 when encoded in BCD becomes `0x2022` in hexadecimal.
   (b) BCD is used because the operations for addition etc. are simpler as they are basically just regular base-10 arithmetic, although BCD requires more bits than 2's complement.
   (c) BCD is used because it is more compact than 2's complement, although operations are more complex in hardware.
   (d) (a) and (b) but not (c)

10. Suppose that we have a 4-bit CPU and we are using 2's complement encoding. What is the result of adding bit strings 1110 and 0101? Assume that the carry flag is ignored. Give your answer in base-10.

11. Let A be a Boolean formula. Three of the following statements are equivalent to each other, meaning that they are all true for A or all false. Which one is *not* equivalent to the other three?

    (a) A is a tautology
    (b) $A \leftrightarrow T$ is satisfiable
    (c) $A \equiv T$
    (d) In a truth table for A, the A column has all T entries.

12. For which of the following values of A do we have $A \vDash \neg p \vee q$?

    (a) $A = (p \wedge q) \vee (\neg q \wedge \neg p)$
    (b) $A = q \rightarrow p$
    (c) $A = \neg\neg p$
    (d) $A = r \rightarrow (\neg p \vee q)$

13. Which of the following is *not* a well-formed Boolean formula?

    (a) $((x \vee z) \wedge (z \vee \neg x))$
    (b) $\neg\neg a$
    (c) $\neg(a \vee b)$
    (d) $(a \vee b)(c \wedge d)$

14. According to the propositional logic meaning of if ... then..., which of the following is *false*?

    (a) If cats are reptiles then pigs can fly.
    (b) If cats are mammals then Brisbane is on Mars.
    (c) If Brisbane is a city then cats are mammals.
    (d) If cats are reptiles then cats are mammals.

15. Concerning logical equivalence and logical implication, which of the following is true?

    (a) If $A \vDash B$ then $A \equiv B$.
    (b) If $A \equiv B$ then $A \vDash B$.
    (c) If $A \equiv B$ and $B \vDash C$ then $C \vDash A$.
    (d) If $A \equiv B$ then we have no idea whether $B \vDash A$ or not.

16. Which of the following sets is equal to:
$$\{3x : x \in \{3, 6, 9\}\}$$

    (a) $\{x \in \{1, 2, 3\} : 3x \in \mathbb{Z}\}$
    (b) $\{x \in \mathbb{Z} : 3|x\}$
    (c) $\{1, 2, 3\}$
    (d) $\{9x : x \in \{1, 2, 3\}\}$

17. Which of the following is the set of all non-negative integers that divide 12?

    (a) $\{x \in \mathbb{Z}_{\geq 0} : x|12\}$
    (b) $\{x \in \mathbb{Z}_{\geq 0} : 12|x\}$
    (c) $\{1, 2, 3, 4, 6\}$
    (d) $(\{1, 2, 3, 4, 5, 6, \} \cup \{1, 12, \}) \setminus \{3, 5\}$

18. Which of the following is *not* a subset of $\{1, 2, 3, 4, 5, 6\}$?

    (a) $\{1, 3, 7\} \setminus \{7, 8, 9\}$
    (b) $\mathbb{R} \cap \{1, 2, 3, 4, 5, 6\}$
    (c) $\{x \in \mathbb{Z} : x|6\}$
    (d) $\{x \in \mathbb{Z} : x|1 \wedge x|0\}$

19. Which of the following explains the relationship between set theory and the Zermelo-Fraenkel axioms?

    (a) The Zermelo-Fraenkel axioms describe what sets are.
    (b) The Zermelo-Fraenkel axioms are independent of set theory.
    (c) The Zermelo-Fraenkel axioms describe the relationships that exist between the objects in set theory.
    (d) The Zermelo-Fraenkel axioms are insufficient to guarantee the existance of unions of sets.

20. Consider the following syllogism:

$$\begin{array}{l} \text{All mothers are female.} \\ \text{Some humans are mothers.} \\ \hline \text{Some humans are female.} \end{array}$$

    Which of the following is the *type* of this syllogism?

    (a)
$$\begin{array}{l} M \subseteq F \\ H \cap M \neq \emptyset \\ \hline H \cap F \neq \emptyset \end{array}$$

    (b)
$$\begin{array}{l} M \subseteq F \\ H \cup M \neq \emptyset \\ \hline H \cup F \neq \emptyset \end{array}$$

    (c)
$$\begin{array}{l} M \subseteq F \\ H \subseteq M \\ \hline H \subseteq F \end{array}$$

    (d)
$$\begin{array}{l} M \subseteq F \\ H \cap M = \emptyset \\ \hline H \cap F = \emptyset \end{array}$$

21. The predicate $x \in \mathbb{Z} \to x \in \mathbb{R}$:

    (a) is true.
    (b) is fully quantified.
    (c) is not well formed.
    (d) has one free parameter.

22. Recall that $\mathbb{Z}$ is the set of integers, i.e. $\{\cdots - 2, -1, 0, 1, 2, \ldots\}$. Which of the following is true?

    (a) $\exists x \in \mathbb{Z}\, \forall y \in \mathbb{Z}\; x + y = 0$
    (b) $\forall x \in \mathbb{Z}\, \exists y \in \mathbb{Z}\; x + y = 0$
    (c) $\forall x \in \mathbb{Z}\, \forall y \in \mathbb{Z}\; x + y = 0$
    (d) $\exists y \in \mathbb{Z}\, \forall x \in \mathbb{Z}\; x + y = 0$

23. Consider the following Python program:

```
def smoosh(S, T, q):
 r = { q(x,y) for x in S for y in T }
 return False in r
```

where `S` and `T` are Python sets and `q` is a Python function that takes two parameters and returns `True` or `False`.

Which of the following best describes what the function `smoosh` returns?

(a) $\neg\forall x \in S\ \forall y \in T\ q(x, y)$

(b) $\exists x \in S\ \exists y \in T\ q(x, y)$

(c) $\forall x \in S\ \forall y \in T\ \neg q(x, y)$

(d) $\neg\forall x \in S\ \forall y \in T\ \neg q(x, y)$

24. Which of the following is logically equivalent to $\forall x \in \mathbb{Z}\ \exists y \in \mathbb{Z}\ \neg p(x, z)$?

(a) $\neg(\exists y \in \mathbb{Z}\ \forall x \in \mathbb{Z}\ \neg p(x, z))$

(b) $\neg(\exists x \in \mathbb{Z}\ \forall y \in \mathbb{Z}\ \neg p(x, z))$

(c) $\neg(\exists x \in \mathbb{Z}\ \neg(\exists y \in \mathbb{Z}\ \neg p(x, z)))$

(d) $\neg(\exists x \in \mathbb{Z}\ \neg(\forall y \in \mathbb{Z}\ p(x, z))$

25. Consider the following syllogism:

All trees have leaves.
The oak in my yard is a tree.

The oak in my yard has leaves.

Which of the following logical implications is *most* helpful in proving that the type for this syllogism is valid?

(a) $s \in S \wedge p(s) \vDash \exists x \in S\ p(x)$

(b) $(\forall x \in S\ p(x)) \wedge (s \in S) \vDash p(s)$

(c) $\forall x \in S\ (p(x) \wedge q(x)) \vDash (\forall x \in S\ p(x)) \vee (\forall x \in S\ q(x))$

(d) $(\forall x \in S\ p(x)) \wedge (S \neq \emptyset) \vDash \exists x \in S\ p(x)$

Due date: Friday, 22 April 2021, 11:59:00pm

A Tables

Base-10	Hexadecimal	4-bit binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

USASCII code chart

Bits b_7 b_6 b_5					0 0 0	0 0 1	0 1 0	0 1 1	1 0 0	1 0 1	1 1 0	1 1 1
b_4	b_3	b_2	b_1	Column → / Row ↓	0	1	2	3	4	5	6	7
0	0	0	0	0	NUL	DLE	SP	0	@	P	`	p
0	0	0	1	1	SOH	DC1	!	1	A	Q	a	q
0	0	1	0	2	STX	DC2	"	2	B	R	b	r
0	0	1	1	3	ETX	DC3	#	3	C	S	c	s
0	1	0	0	4	EOT	DC4	$	4	D	T	d	t
0	1	0	1	5	ENQ	NAK	%	5	E	U	e	u
0	1	1	0	6	ACK	SYN	&	6	F	V	f	v
0	1	1	1	7	BEL	ETB	'	7	G	W	g	w
1	0	0	0	8	BS	CAN	(	8	H	X	h	x
1	0	0	1	9	HT	EM	)	9	I	Y	i	y
1	0	1	0	10	LF	SUB	*	:	J	Z	j	z
1	0	1	1	11	VT	ESC	+	;	K	[	k	{
1	1	0	0	12	FF	FS	,	<	L	\	l	|
1	1	0	1	13	CR	GS	-	=	M	]	m	}
1	1	1	0	14	SO	RS	.	>	N	^	n	~
1	1	1	1	15	SI	US	/	?	O	_	o	DEL