

Set theory

1. Determine whether each element is in the set:

- (a) Is $5 \in \{2x : x \in \mathbb{Z}\}$?
- (b) Is $10 \in \{x \in \mathbb{Z} : x \text{ divides } 60\}$?

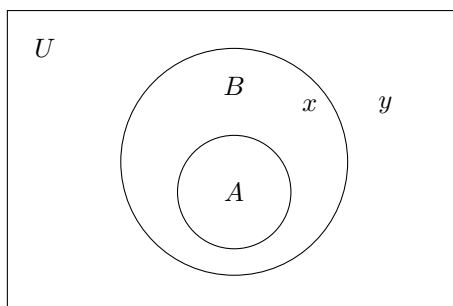
2. Are these subsets?

- (a) Is $\{2, 5\} \subseteq \mathbb{Z}$?
- (b) Is $\{2, 3, 6\} \subseteq \{2x : x \in \mathbb{Z}\}$?
- (c) Is $\{4x : x \in \mathbb{Z}\} \subseteq \{2x : x \in \mathbb{Z}\}$?

3. What are these sets?

- (a) $\{1, 3, 5\} \cup \{2, 4, 6\}$
- (b) $\{4, 7, 9\} \cap \{9, 7, 3, 6\}$
- (c) $\{1, 2, 3\} \setminus \{1, 2, 3, 4, 5\}$
- (d) $\mathbb{Z}_{\geq 0} \setminus \{2x : x \in \mathbb{Z}\}$
- (e) $\overline{\{x : x = 2y\}}$ with universe $U = \mathbb{Z}_{\geq 0}$

4. Look at the Venn diagram and answer the questions.



- (a) Is $B \subseteq A$?
- (b) Is $x \in A$?
- (c) Is $x \in B \setminus A$?
- (d) Is $y \in \overline{B}$?
- (e) Is $x \in U$?
- (f) Is $x \in \overline{A \cap B}$?

5. (Stretch question) For some universe U with $A, B \subseteq U$, show:

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

6. Write Python programs for the following

- (a) Define a function that takes a number x and returns a set containing all non-negative integers less than 100 that are divisible by x
- (b) Define a function that implements complements by taking a set S and a universe U and returning the complement of S in U

Syllogisms

1. Write the syllogism type for the following syllogism. Is it a valid type?:

$$\begin{array}{l} \text{All trees are plants} \\ \text{Pines are trees} \\ \hline \text{Pines are plants} \end{array}$$

2. Is this syllogism type valid? If yes, draw a Venn diagram illustrating the sets and elements. If not give a counter-example:

$$\begin{array}{l} x \in A \\ B \subseteq A \\ \hline x \in B \end{array}$$

3. Is this syllogism type valid? If yes, draw a Venn diagram illustrating the sets and elements. If not give a counter-example:

$$\begin{array}{l} x \in B \\ B \subseteq A \\ \hline x \in A \end{array}$$

4. The following syllogism has a false conclusion. Explain two problems with the syllogism:

$$\begin{array}{l} \text{All mothers are human} \\ \text{Socrates is human} \\ \hline \text{Socrates is a mother} \end{array}$$