CAB203
Tutorial 1

Foundations

1. Some simple algebra shows $(x+1)(x+2) = x^2 + 3x + 2$. Apply the definition of divides $(a|b)$ to obtain two new statements.
2. A *square number* is a natural number n such that $n = x^2$ for some integer x. Apply this definition to show that 25 and 121 are square numbers.
3. Using only the partial list of axioms on slide 25 of Lecture 1, show that $(0+a)+0 = a$.

Modular arithmetic

Calculate:

1. $17 \mod 5$
2. $36 \mod 7$
3. $(5+7) \mod 3$
4. $(5 \cdot 4) \mod 3$
5. Write a short Python function that takes two numbers, a and b, and returns True when $a|b$.
6. Write a short Python function that takes three numbers, a, b, c and returns True when $a \equiv b \pmod{c}$.
7. *Stretch question* Show that if r is the remainder when a is divided by b (i.e. $a = bq + r$) then $a \equiv r \pmod{b}$.
8. *Stretch question* Apply the definition of modular equivalence to show that if $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$ then $a \equiv c \pmod{n}$.

Exponents

Use the laws of exponents and logarithms to calculate the following (don't use a calculator):

1. $2^{15} \cdot 2^3$ bits expressed in kilobits
2. Express 8^5 with base 2
3. $\log_2 \frac{256}{16}$
4. $\log_2 8^3$

5. Write a short Python program that takes a number of addresses n and returns the minimum number of bits required to express that many unique addresses. Note that n might not be a power of 2. You may wish to us `math.ceil()`.

6. *Stretch question* Use the fact that $n^x \cdot n^y = n^{x+y}$ to show that $\log_n ab = \log_n a + \log_n b$ using only the basic properties of logarithms (i.e. $\log_n n^x = x$ and $x = n^{\log_n x}$).