CAB203
Tutorial 6

1 Relations

1. Write out the following sets in full:
   (a) $\{0, 1\} \times \{0, 1\}$
   (b) $\{a, b\} \times \{0, 1\}$
   (c) $\{\alpha, \beta, \gamma\} \times \{a, b\}$

2. For each of the following relations determine if it is symmetric, reflexive, transitive, anti-symmetric or irreflexive, and whether it is a equivalence relation, partial ordering or total ordering. If it is an equivalence relation, describe the equivalence classes.
   (a) $R = \{(0, 0), (0, 1), (1, 1), (1, 2), (0, 2), (2, 2)\}$ on the set $\{0, 1, 2\}$
   (b) $R = \{(0, 0), (2, 2), (1, 1), (3, 3), (0, 1), (1, 0), (2, 3), (3, 2)\}$ on the set $\{0, 1, 2, 3\}$
   (c) $R = \{(0, 0), (1, 0), (2, 0), (1, 1), (2, 2)\}$ on the set $\{0, 1, 2\}$
   (d) $R = \{(a, b) \in \mathbb{N}^2 : \exists c \in \mathbb{N}\ ac = b\}$ on the set $\mathbb{N}$
   (e) R defined on the set of all cars where aRb if a and b have the same size of tyres
   (f) R defined on the set of all people, where aRb if b has lived in all the cities that a has lived in.
   (g) R defined on the set of cities, where aRb if there is a regularly scheduled direct flight from a to b or from b to a
   (h) *(Stretch question)* Define S by the set of formulas of the form $f(x) = ax + b$ where $a, b \in \mathbb{R}$. Then define the relation R on S by fRg if $f = g$ or if there does not exist $x \in \mathbb{R}$ such that $f(x) = g(x)$.
   (i) *(Stretch question)* Define S by the set of formulas $f(x) = ax^2 + bx + c$, and the relation R on S by fRg if $\forall x \in \mathbb{R}\ f(x) \leq g(x)$.
   (j) *(Stretch question)* Let S be a non-empty set and let $f : S \to \mathbb{Z}$ where for each $x \in \mathbb{Z}$ there is at most one $s \in S$ such that $f(s) = x$. To put it differently, if $f(s) = f(t)$ then $s = t$. Define a relation R on S by sRt when $f(s) \leq f(t)$.

3. *(Stretch question)* There is at least one relation on any set A that is symmetric, antisymmetric, transitive, and irreflexive. What is it? Is it the only one? Is there any relation that is both reflexive and irreflexive?

4. In Python, we might encode a relation over a set A as a Python set containing Python tuples, each of which is length 2 where the both elements are in A. For example, a relation might look like:

   ```
   R = { (1, 3), (2, 1), (3, 4) }
   ```

   Write a short Python function that takes a relation as above and returns the set A that R is over. Note that relations do not have to refer to every element of the set that they are over, (i.e. they do not need to have some (a, b) or (b, a) for every $a \in A$) so the set you return will just be the set of elements that R refers to.

5. Write a Python function which, given a relation in the format described in the previous question, and returns a pair (a, b) where a is True if the relation is symmetric and False otherwise, and b is True if the relation is anti-symmetric and False otherwise.

6. Write a Python function which, given a relation in the format used in the previous questions, returns True if the relation is transitive, otherwise False.

2 Functions

1. For each of the following, determine whether the given relation is a function on the given set. If it is a function, determine whether it has an inverse, and the range.
   (a) $\{(0,1),(1,2),(2,0)\}$ on the set $\{0,1,2\}$
   (b) $\{(1,0),(2,1),(1,2),(0,1)\}$ on $\{0,1,2\}$
   (c) The set $R \subseteq V \times C$, where V is the set of voters in some election, and C is the set of candidates, so that (v,c) is in R if v voted for c. (In this election, voters can vote for at most one candidate. To be considered a voter, a person must vote for some candidate.) Assume $|V| > |C|$.
   (d) With the above definition, does anything change if we allow a voter to vote for more than one candidate? How about if we include people who don't vote for anyone?
   (e) The set $\{(x,y) \in \mathbb{R}^2 : 2x + y = 3\}$ on the set $\mathbb{R}$
   (f) The set $\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 4\}$ on the set $\mathbb{R}$
2. Write a Python function which, given a relation in the format used in the previous section, a set A, a set B returns (a,b) where a is True if the relation is a function from A to B, otherwise False, and b is True if the relation is a function from A to B and has an inverse, otherwise False.

3 Sequences

1. For each of the following, which is the better choice, a list or a tuple?
   (a) x,y,z-coordinates for vertices of a 3D model
   (b) Bus numbers for busses arriving at a bus stop.
   (c) First name, Last name, student number
2. Write a collection of functions that implement a mini student database. The functions are:
   (a) `createD()` Return an empty database
   (b) `addStudent(D, student)`: Add a student to the database
   (c) `studentName(D, number)`: retrieve the students first and last name from the database by student number

   `D` is some data structure (you get to decide what it should be) representing the database. `student` is a data structure representing an individual student, and should include student number, first name and last name.

   To keep things simple, your functions are allowed to have undefined behaviour when given inappropriate inputs. You can assume, for example, that a student number querried will always be in the database, student numbers are unique, etc..