1. Alg for palindrome:

```
isPalindrome(S):
    ptrA = 0
    ptrB = len(S)-1
    while (ptrA < ptrB):
        if (S[ptrA] == S[ptrB]):
            ptrA++;
            ptrB--;
        else
            return false
    return true
```

3. Change for 87 cents using 25, 10, 5, 1.

	Output:
87 - 25	25
62 - 25	25
37 - 25	25
12 - 10	10
2 - 1	1
1 - 1	1
0	

4. $f(x) = O(g(x))$?

Means: $\exists c$ st. $f(x) \le c\,g(x)$

$f(x) = x^3$

- $g(x) = x^2$

  $x^3 \le \square\, x^2$

  $\boxed{\text{NO.}}$

- $g(x) = x^3$

  $x^3 \le \square\, x^3$

  $\boxed{\text{YES}}$

  $(c = 5)$

- $g(x) = x^2 + x^3$

  $x^3 \le \square\,(x^2 + x^3)$

  $\boxed{\text{YES}}$ $c = 1$

- $g(x) = x^3/2$

  $x^3 \le \square\,(x^3/2)$

  $\boxed{\text{YES}}$

  $c = 3$

⑤ Same order means

$$f(x) = \Theta(g(x))$$

$$\Rightarrow c_1 |g(x)| \le |f(x)| \le c_2 |g(x)|$$

a $f(x) = 3x + 7$
$g(x) = x$

$$c_1 x \le 3x + 7 \le c_2 x$$

$c_1 = 1 \quad c_2 = 4$

★ If $f(x) = \Theta(g(x))$,
$g(x) = \Theta(f(x))$.

b $f(x) = \lfloor x + 1/2 \rfloor$

$g(x) = x$

$$c_1 x \le \left\lfloor \frac{x+1}{2} \right\rfloor \le c_2 x$$

$c_1 = .25 \quad c_2 = 2$ ✓

c $f(x) = 2x^2 + x - 7$, $g(x) = x^2$

$$c_1 x^2 \le 2x^2 + x - 7 \le c_2 x^2$$

$c_1 = 1 \quad c_2 = 3$ ✓

⑥ Give Big O estimate:

a $t = 0$
for $i = 1$ to 3:
  for $j = 1$ to 4:
    $t = t + ij$

How many times is this instruction executed?

$$\sum_{i=1}^{3} \sum_{j=1}^{4} 1 \Rightarrow \sum_{i=1}^{3} 4 \Rightarrow 12$$

Runtime $= 12. \Rightarrow O(12) = \boxed{O(1)}$

★ $O(100) = O(5)$.
$O(n) = O(3n)$

b $i = 1$
$t = 0$
while $i \le n$:
  $t = t + i$
  $i = 2i$

Tricky!
i grows exponentially, so loop is run logarithmic times.

Runtime : $\boxed{O(\log_2 n)}$

7. Prove $1\cdot 1! + 2\cdot 2! + 3\cdot 3! + \dots n\cdot n!$
$= (n+1)! - 1$
(when n is positive).

Base Case:

$n=1: \; 1\cdot 1! = (1+1)! - 1$

$1 = (2-1)$ ✓

Assume for n:

$$\sum_{i=1}^{n} i\cdot i! = (n+1)! - 1$$

Show it holds for $n+1$:

Replace all ns w/ $(n+1)$

$$\sum_{i=1}^{n+1} i\cdot i! = ((n+1)+1)! - 1$$

looking for where I can substitute this

$$(n+1)(n+1)! + \sum_{i=1}^{n} i\cdot i! = (n+2)! - 1$$

$$(n+1)(n+1)! + (n+1)! - \cancel{1} = (n+2)! - \cancel{1}$$

$$(n+1)!((n+1)+1) = (n+2)!$$

$$(n+2)(n+1)! = (n+2)! \quad ✓$$

8. $$\sum_{i=1}^{n} 2i = ?$$

2, $+4$, 6, $+6$, 12, $+8$, 20, $+10$, 30, $+12$, 42, $\vdots$

$$\Rightarrow 2\cdot\sum_{i=1}^{n} i = 2\cdot\frac{n(n+1)}{2}$$

$$= n(n+1)$$

Candidate formula.

Now prove it:

Base case:

$n = 1.$ $\quad \sum_{i=1}^{1} 2i = 2 = 1(1+1)$ ✓

Assume for n.

Show for $n+1$:

$$\sum_{i=1}^{n+1} 2i = (n+1)(n+2)$$

$$2(n+1) + \sum_{i=1}^{n} 2i = (n+1)(n+2)$$

$$2(n+1) + n(n+1) = (n+1)(n+2)$$

$$(n+1)(2+n) = (n+1)(n+2) \quad ✓$$

(9) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\frac{1}{2^n}=\sum_{i=1}^{n}\frac{1}{2^i}$

Testing some numbers to find a pattern...

$n=1: \frac{2-1}{2}=\frac{1}{2}$

$n=2: \frac{2}{4}+\frac{1}{4}=\frac{3}{4}$

$n=3: \frac{4}{8}+\frac{2}{8}+\frac{1}{8}=\frac{7}{8}$

$n=4: \frac{8}{16}+\frac{4}{16}+\frac{2}{16}+\frac{1}{16}=\frac{15}{16}$

$\vdots$

Try: $\sum_{i=1}^{n}\frac{1}{2^i}=\frac{2^n-1}{2^n}$

Base Case: $n=1.\ \frac{2^1-1}{2^1}=\frac{1}{2}$ ✓

Assume for n: $\sum_{i=1}^{n}\frac{1}{2^i}=\frac{2^n-1}{2^n}$

Show for $n+1$:

$$\sum_{i=1}^{n+1}\frac{1}{2^i}=\frac{2^{(n+1)}-1}{2^{(n+1)}}$$

$$\frac{1}{2^{n+1}}+\sum_{i=1}^{n}\frac{1}{2^i}=\frac{2^{n+1}-1}{2^{n+1}}$$

$$\frac{1}{2^{n+1}}+\frac{2^n-1}{2^n}=\frac{2^{n+1}-1}{2^{n+1}}$$ Substituting our assumption.

$$\frac{1}{2\cdot 2^n}+\frac{2^n-1}{2^n}=\frac{2^{n+1}-1}{2\cdot 2^n}$$ Rewriting

$$\frac{2^n-1}{2^n}=\frac{2^{n+1}-1}{2\cdot 2^n}-\frac{1}{2\cdot 2^n}$$ Move term from LHS → RHS

$$=\frac{\cancel{2}\cdot 2^{n}-\cancel{2}}{\cancel{2}\cdot 2^n}$$ Cancel 2s.

$$\frac{2^n-1}{2^n}=\frac{2^n-1}{2^n}$$ ✓

(12) Recursive alg for finding min of array:

Observe: min of array is the smallest of either the first elem or the smallest of all the rest.

```
MinArray(A):
    if len(A) = 1:
        return A[0]
    return min(A[0], MinArray(A[1:n]))
```

(14)

```
FindMode(A):
    If (len(A) = 1):
        return A[0]
    m1 = FindMode(Left(A))
    m2 = FindMode(Right(A))
    if (count(A, m1) > count(A, m2))
        return m1
    else return m2;
```

* doesn't deal w/ no modes/ all elems are unique, or multiple modes.

(15)

Prove:

```
Mult(x, y):
    if (y=0) return 0;
    if (y=1) return 1;
    if (y is even):
        return (2 · Mult(x, y/2))
    else
        return (2 · Mult(x, y/2) + x)
```

$x=3,\ y=8$:

Mult(3, 8)

$2 \cdot$ Mult(3, 4)

$2 \cdot 2 \cdot$ Mult(3, 2)

$2 \cdot 2 \cdot 2 \cdot$ Mult(3, 1)

$2 \cdot 2 \cdot 2 \cdot 3$ ✓

Mult(3, 7):

$2 \cdot$ Mult(3, 3) + 3

$2 \cdot 2 \cdot$ Mult(3, 1) + 3 + 3

$(2 \cdot 2 \cdot 3) + 3 + 3 + 3$ ✓

(17) Prove correct:

$y = 1$
$z = x + y$

$x = 0$
$z = 0 + 1 = 1.$ ✓

(18) Use a loop invariant to prove:

```
power = 1
i = 1
while i ≤ n:
    power = power * x
    i = i + 1
```

Candidate invariant:

$$\text{power} = x^{i-1}$$