Approximation Algorithms Part II: week 1 assignment

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Primal-dual algorithm for Set Cover

Q1.

$$\begin{array}{lrl} \text{maximize} & \sum_{i=1}^{n} y_i & \\ \text{s.t.} & \sum_{i:e_i \in S_j} \leq w_j & \forall j = 1, \ldots, m \\ & y_i \geq 0 & \forall j = 1, \ldots, m \end{array}$$

Q2. At most n iterations.

Q3. Each iteration covers at least one different elements, and there are n elements to cover. Therefore, the algorithm terminates with all elements covered.

Q4. $\text{val}(y^*) \leq \text{OPT}$.

Q5. Due to step 3 in the algorithm, the constraints in dual problem is always satisfied, thus solution y is feasible for the dual.

Q6. $\text{val}(y) \leq \text{val}(y^*) \leq \text{OPT}$.

Q7. $\sum_{i:e_i \in S_j} y_i = w_j$ for all $j \in I$.

Q8. $\sum_{j \in I} \sum_{i:e_i \in S_j} y_i = \sum_{j \in I} w_j$.

Q9.

$$
\begin{aligned}
\sum_{j \in I} w_j = \sum_{j \in I} w_j x_j &= \sum_{j \in I} \sum_{i : e_i \in S_j} x_j y_i \\
&= \sum_{j} \sum_{i : e_i \in S_j} x_j y_i \\
&= \sum_{i} \left(\sum_{j : e_i \in S_j} x_j \right) y_i \\
&= f \sum_{i} y_i \\
&= f \mathrm{val}(y)
\end{aligned}
$$

Q10. Therefore,

$$
\sum_{j \in I} w_j \leq f \cdot \mathrm{val}(y) \leq f \cdot \mathrm{OPT}
$$

So, this algorithm gives f-approximation.