Castle Towers

Problem Code: CASTLE

Design Challenge

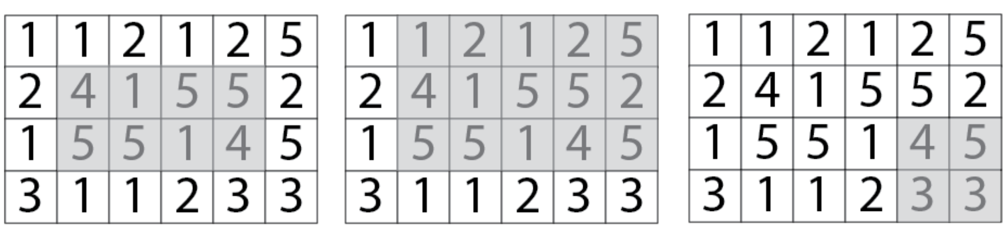
Task Description

The King has decided to build a castle on his fancy territory. There are hills across the land of the kindom so the heights of places vary. More specifically, the map of the kingdom can be regarded as a grid map of n rows and m columns. Each grid (i, j) (i-th row, j-th column) has a height value $h_{i,j}$. The castle has a rectangular shape. Its edges are parallel with the x- and y-axis.

The castle needs to be guarded, so the King would like to build four towers at the four corners of the rectangular caslte. To achieve the best viewpoint, it is desired that the towers are built on higher hills. To prevent dead corners of the watching guards, the King would like the lowest hill among the four hills selected for towers, to have the largest possible height.

The King does not care about the area of the castle. However, the castle shall be able to hold the four towers. In other words, the width and height of the castle must be no smaller than 2.

The kingdom map below consists of 4×6 grids. Each grid has its height written in it. The chosen castle on the left has its four towers built on hills of heights 4, 5, 5, 4. So the lowest hill of the towers is 4. This is best possible height for this map. The middle choice has hills of heights 1, 5, 5, 5 with the lowest being 1. The right choice has hills of heights 4, 5, 3, 3 with the lowest being 3. You cannot choose a single grid of height 5.



Constraints

$2 \leq m \leq n$.

Examples

Case 1: $\begin{bmatrix} 1 & 1 & 2 & 1 & 2 & 5 \\ 2 & 4 & 1 & 5 & 5 & 2 \\ 1 & 5 & 5 & 1 & 4 & 5 \\ 3 & 1 & 1 & 2 & 3 & 3 \end{bmatrix}$

Answer: 4

As explained above.

Case 2: $\begin{bmatrix} 1 & 3 & 2 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

Answer: 2

Case 3: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Answer: 1

Requirements

Time: $o(n^3)$ (little o) **Space:** $O(n^2)$