

$$\begin{aligned}
 R_1 = & - \left[ 2\sigma_1^2 M_{11} \frac{\partial^2 X_1}{\partial S_1^2} + 2\sigma_2^2 M_{11} \frac{\partial^2 X_1}{\partial S_2^2} \right] \\
 & - \frac{\partial M_{11}}{\partial X_1} \left[ \sigma_1^2 \frac{\partial X_1}{\partial S_1} \frac{\partial X_1}{\partial S_1} + \sigma_2^2 \frac{\partial X_1}{\partial S_2} \frac{\partial X_1}{\partial S_2} \right] \\
 & - 2 \frac{\partial M_{11}}{\partial X_2} \left[ \sigma_1^2 \frac{\partial X_2}{\partial S_1} \frac{\partial X_1}{\partial S_1} + \sigma_2^2 \frac{\partial X_2}{\partial S_2} \frac{\partial X_1}{\partial S_2} \right] \\
 & + \frac{\partial M_{22}}{\partial X_1} \left[ \sigma_1^2 \frac{\partial X_2}{\partial S_1} \frac{\partial X_2}{\partial S_1} + \sigma_2^2 \frac{\partial X_2}{\partial S_2} \frac{\partial X_2}{\partial S_2} \right] = 0
 \end{aligned}$$

$$D_2(X_1) = - \left[ 2\sigma_1^2 M_{11} \frac{\partial^2 X_1}{\partial S_1^2} + 2\sigma_2^2 M_{11} \frac{\partial^2 X_1}{\partial S_2^2} \right]$$

$$D_2(\Delta X_1) = - \left[ 2\sigma_1^2 M_{11} \frac{\partial^2 \Delta X_1}{\partial S_1^2} + 2\sigma_2^2 M_{11} \frac{\partial^2 \Delta X_1}{\partial S_2^2} \right]$$

$$\begin{aligned}
 D_1 = & - \frac{\partial M_{11}}{\partial X_1} \left[ \sigma_1^2 \left( \frac{\partial X_1}{\partial S_1} \right)^2 + 2 \frac{\partial X_1}{\partial S_1} \frac{\partial \Delta X_1}{\partial S_1} \right) + \sigma_2^2 \left( \frac{\partial X_1}{\partial S_2} \right)^2 + 2 \frac{\partial X_1}{\partial S_2} \frac{\partial \Delta X_1}{\partial S_2} \right] \\
 & - 2 \frac{\partial M_{11}}{\partial X_2} \left[ \sigma_1^2 \left( \frac{\partial X_2}{\partial S_1} \frac{\partial X_1}{\partial S_1} + \frac{\partial X_2}{\partial S_1} \frac{\partial \Delta X_1}{\partial S_1} + \frac{\partial X_1}{\partial S_1} \frac{\partial \Delta X_2}{\partial S_1} \right) \right. \\
 & \quad \left. + \sigma_2^2 \left( \frac{\partial X_2}{\partial S_2} \frac{\partial X_1}{\partial S_2} + \frac{\partial X_2}{\partial S_2} \frac{\partial \Delta X_1}{\partial S_2} + \frac{\partial X_1}{\partial S_2} \frac{\partial \Delta X_2}{\partial S_2} \right) \right] \\
 & + \frac{\partial M_{22}}{\partial X_1} \left[ \sigma_1^2 \left( \frac{\partial X_2}{\partial S_1} \right)^2 + 2 \frac{\partial X_2}{\partial S_1} \frac{\partial \Delta X_2}{\partial S_1} \right) + \sigma_2^2 \left( \frac{\partial X_2}{\partial S_2} \right)^2 + 2 \frac{\partial X_2}{\partial S_2} \frac{\partial \Delta X_2}{\partial S_2} \right]
 \end{aligned}$$

$$\begin{aligned}
D_1^{X_1} &= -\frac{\partial M_{11}}{\partial X_1} \left[ \sigma_1^2 \left( \frac{\partial X_1}{\partial S_1} \right)^2 + \sigma_2^2 \left( \frac{\partial X_1}{\partial S_2} \right)^2 \right] \\
&\quad - 2 \frac{\partial M_{11}}{\partial X_2} \left[ \sigma_1^2 \left( \frac{\partial X_2}{\partial S_1} \frac{\partial X_1}{\partial S_1} \right) + \sigma_2^2 \left( \frac{\partial X_2}{\partial S_2} \frac{\partial X_1}{\partial S_2} \right) \right] \\
&\quad + \frac{\partial M_{22}}{\partial X_1} \left[ \sigma_1^2 \left( \frac{\partial X_2}{\partial S_1} \right)^2 + \sigma_2^2 \left( \frac{\partial X_2}{\partial S_2} \right)^2 \right] \\
D_1^{\Delta X_1} &= -\frac{\partial M_{11}}{\partial X_1} \left[ \sigma_1^2 2 \frac{\partial X_1}{\partial S_1} \frac{\partial \Delta X_1}{\partial S_1} + \sigma_2^2 2 \frac{\partial X_1}{\partial S_2} \frac{\partial \Delta X_1}{\partial S_2} \right] \\
&\quad - 2 \frac{\partial M_{11}}{\partial X_2} \left[ \sigma_1^2 \left( \frac{\partial X_2}{\partial S_1} \frac{\partial \Delta X_1}{\partial S_1} + \frac{\partial X_1}{\partial S_1} \frac{\partial \Delta X_2}{\partial S_1} \right) \right. \\
&\quad \left. + \sigma_2^2 \left( \frac{\partial X_2}{\partial S_2} \frac{\partial \Delta X_1}{\partial S_2} + \frac{\partial X_1}{\partial S_2} \frac{\partial \Delta X_2}{\partial S_2} \right) \right] \\
&\quad + \frac{\partial M_{22}}{\partial X_1} \left[ \sigma_1^2 2 \frac{\partial X_2}{\partial S_1} \frac{\partial \Delta X_2}{\partial S_1} + \sigma_2^2 2 \frac{\partial X_2}{\partial S_2} \frac{\partial \Delta X_2}{\partial S_2} \right]
\end{aligned}$$

$$\begin{aligned}
D_1^{X_2} &= -\frac{\partial M_{22}}{\partial X_2} \left[ \sigma_1^2 \left( \frac{\partial X_2}{\partial S_1} \right)^2 + \sigma_2^2 \left( \frac{\partial X_2}{\partial S_2} \right)^2 \right] \\
&\quad - 2 \frac{\partial M_{12}}{\partial X_1} \left[ \sigma_1^2 \left( \frac{\partial X_1}{\partial S_1} \frac{\partial X_2}{\partial S_1} \right) + \sigma_2^2 \left( \frac{\partial X_1}{\partial S_2} \frac{\partial X_2}{\partial S_2} \right) \right] \\
&\quad + \frac{\partial M_{11}}{\partial X_2} \left[ \sigma_1^2 \left( \frac{\partial X_1}{\partial S_1} \right)^2 + \sigma_2^2 \left( \frac{\partial X_1}{\partial S_2} \right)^2 \right]
\end{aligned}$$

$$\begin{aligned}
D_1^{\Delta X_2} &= -\frac{\partial M_{22}}{\partial X_2} \left[ \sigma_1^2 2 \frac{\partial X_2}{\partial S_1} \frac{\partial \Delta X_2}{\partial S_1} + \sigma_2^2 2 \frac{\partial X_2}{\partial S_2} \frac{\partial \Delta X_2}{\partial S_2} \right] \\
&\quad - 2 \frac{\partial M_{22}}{\partial X_1} \left[ \sigma_1^2 \left( \frac{\partial X_1}{\partial S_1} \frac{\partial \Delta X_2}{\partial S_1} + \frac{\partial X_2}{\partial S_1} \frac{\partial \Delta X_1}{\partial S_1} \right) \right. \\
&\quad \left. + \sigma_2^2 \left( \frac{\partial X_1}{\partial S_2} \frac{\partial \Delta X_2}{\partial S_2} + \frac{\partial X_2}{\partial S_2} \frac{\partial \Delta X_1}{\partial S_2} \right) \right] \\
&\quad + \frac{\partial M_{11}}{\partial X_2} \left[ \sigma_1^2 2 \frac{\partial X_1}{\partial S_1} \frac{\partial \Delta X_1}{\partial S_1} + \sigma_2^2 2 \frac{\partial X_1}{\partial S_2} \frac{\partial \Delta X_1}{\partial S_2} \right]
\end{aligned}$$

Old :  $x^{n+1} \cdot n = x^n \cdot n$

New :  $\Delta x \cdot n = 0$

Old :  $\frac{\partial}{\partial s_i} (x^{n+1} \cdot t) = 0$

New :  $\frac{\partial}{\partial s_i} (\Delta x \cdot t) = -\frac{\partial}{\partial s_i} (x^n \cdot t)$