ANALYSIS OF AN EMPIRICAL MODEL FOR SOIL HEAT FLUX UNDER A GROWING WHEAT CROP FOR ESTIMATING EVAPORATION BY AN INFRARED-TEMPERATURE BASED ENERGY BALANCE EQUATION

B.J. CHOUDHURY

Hydrological Sciences Branch, NASA/Goddard Space Flight Center, Greenbelt, MD 20771 (U.S.A.)

S.B. IDSO and R.J. REGINATO

U.S. Water Conservation Laboratory, Phoenix, AZ 85040 (U.S.A.)

(Received May 14, 1986; revision accepted September 12, 1986)

ABSTRACT

Choudhury, B.J., Idso, S.B. and Reginato, R.J., 1987. Analysis of an empirical model for soil heat flux under a growing wheat crop for estimating evaporation by an infrared-temperature based energy balance equation. Agric. For. Meteorol., 39: 283-297.

A net-radiation-based empirical model for soil heat flux (G) is analyzed for inclusion in a canopy-temperature-based energy balance equation to estimate evaporation from a growing wheat crop. The observed direct correlation between net radiation and soil heat flux for bare soils is extended to include the effect of growing vegetation by considering the canopy attenuation of net radiation. The parameters of the soil heat flux model are determined using observations of net radiation, evaporation and estimated sensible heat flux over Pavon wheat, while the empirical model is used to calculate and compare against the latent heat flux observations over Ciano wheat. Comparisons are done for 9 days of diurnal observations, which included clear and partially cloudy skies and the leaf area index varying from 0 (i.e., bare soil) to 4.7. The performance of the empirical model was not very satisfactory for 2 days of data over bare soil, primarily because of the phase difference in the diurnal variations of soil heat flux and net radiation. A linear regression analysis using the estimated and the observed latent heat fluxes for all 9 days gave a correlation coefficient of 0.97 and a standard error of 39 W m⁻². The results of a sensitivity analysis show that errors in estimating G translate directly into a bias in estimating the latent heat flux, and the magnitude of this bias decreases as the vegetation leaf area index increases.

INTRODUCTION

Application of infrared temperature in resistance-energy balance models to estimate evaporation has been discussed extensively (see recent reviews by Hatfield, 1983 and Jackson, 1985). A basic equation which has been applied is

$$\lambda E = R_{\rm n} - G - \varrho C_{\rm n} (T_{\rm s} - T_{\rm a}) / r_{\rm a} \tag{1}$$

where λE is the latent heat flux, R_n is the net radiation, G is the soil heat flux, ϱ is the density and C_p is the volumetric heat capacity of air, T_s and T_a are,

respectively, the surface and air temperatures and $r_{\rm a}$ is the aerodynamic resistance for heat transfer.

Infrared radiometer observations are assumed to provide $T_{\rm s}$. In most applications $R_{\rm n}$ has been a directly measured quantity, although Jackson (1984) and Jackson et al. (1985) have developed and tested a method for calculating $R_{\rm n}$ using multispectral observations, combined with ground station data for solar radiation, air temperature and humidity. The resistance, $r_{\rm a}$, has been calculated from $T_{\rm s}$, crop height and wind speed (Jackson, 1985; Choudhury et al., 1986), although in some studies $r_{\rm a}$ has been expressed in terms of an adjustable parameter for model calibration (Camillo et al., 1983). Since areally (field scale or somewhat larger) representative values could be obtained for $R_{\rm n}$, $T_{\rm s}$, $T_{\rm a}$ and $r_{\rm a}$, eq. 1 has a high appeal for providing estimates of λE (at least on a field scale) when G is known.

The soil heat flux (G) is generally measured by heat flux plates buried in the soil and then correcting the measured flux for heat storage between the flux plates and the soil surface. A remote sensing technique for estimating G is not currently available and Jackson (1985) points to the difficulty and high uncertainty of obtaining an areally representative value of G by flux plate observations.

A wide range of importance has been assigned to G in estimating λE . For example, Gurney and Hall (1983) used a detailed soil physics model for G in computing λE using satellite observed infrared temperature over a partially forested area, while Hatfield et al. (1984) assumed G=0 in estimating λE over several arable crops (alfalfa, cotton, soybean, sorghum and tomato) and found good agreement with the observed λE . The validity of the assumption G=0 in eq. 1 has not been tested over a growing wheat crop, i.e., as a bare soil progressively gets covered by vegetation. The actual importance of G in estimating λE over a growing crop is unclear.

Observations of G for bare soils have shown strong linear correlations with $R_{\rm n}$; the slope of the regression has been variable. Idso et al. (1975) found the ratio G/R_n increased from 0.22 to 0.51 as a wet soil dried for about 2 weeks. Fuchs and Hadas (1972) and Novak and Black (1983), however, found the ratio to be moderately insensitive to the soil wetness, and the ratio was, respectively, 0.34 and 0.22. For mature arable crops G as a fraction of net radiation above the canopy was roughly 5% under sugarbeet (Brown, 1976), 10% under alfalfa (Clothier et al., 1986) and 7% each for wheat (Denmead, 1969), maize (Uchijima, 1976), sorghum (Szeicz et al., 1973) and soybeans (Baldocchi et al., 1985). For natural grasslands and an irrigated lawn, G was 10 to 20% of the net radiation (Nickerson and Smiley, 1975; Ripley and Redman, 1976; Oke, 1979; DeBruin and Holtslag, 1982). The above assessments of G, while not intended to be comprehensive, suggest that an assumption of G = 0 in eq. 1 might lead to an overestimate of λE , and the magnitude of the overestimate would be expected to vary with vegetation cover. For application of eq. 1 over a growing crop (i.e., as a bare soil progressively gets covered with vegetation) it is desirable to quantify G as a function of some plant parameter, such as canopy density or leaf area index.

In this paper we present an analysis of a net-radiation based empirical model for soil heat flux under a growing wheat crop. Observed direct correlation between net radiation and soil heat flux for bare soils is extended to include the effect of vegetation by considering the canopy attenuation of net radiation. The model parameters were determined from G values calculated as the residual component of eq. 1 using the observed λE , $R_{\rm n}$, infrared and air temperatures and estimated $r_{\rm a}$ from crop and meteorological data. The empirical model is then used in eq. 1 to estimate λE with an independent data set of diurnal observations for 9 days from planting to maturity of wheat and these estimates are compared against the lysimeter data.

Detailed soil physics models exist (e.g., Lin, 1980; Camillo et al., 1983) for accurate calculation of G for bare soils. In developing the empirical equation for G, our objective was to retain the simplicity of eq. 1 in providing λE over a growing crop.

METHODS

Field observations

The present analysis is based upon two varieties of spring wheat (*Triticum aestivum* L. cvs. Pavon and Ciano) grown in adjacent lysimeters at the U.S. Water Conservation Laboratory in Phoenix, AZ. The lysimeters were seeded during mid-December 1982, emergence was on Day 13, 1983, and at full emergence the stand count was 250 plants/m². Rows were oriented in a north–south direction and spaced at 0.18 m. The lysimeters were of one square meter area, and the soil was Avondale loam (Anthropic Torrifluvent, fine loamy, mixed, calcareous, hyperthermic). There were six irrigations, in addition to the irrigation just after planting. As a result of these irrigations, it is unlikely that the crop was under water-stress.

Air temperature at 1.5 m was measured with shielded thermocouples, vapor pressure by ceramic wick psychrometers, wind speed by Young* cup anemometers and net radiation by Fritschen-type radiometers. These measurements were logged automatically as half-hourly averages throughout a day, approximately 5 m south of the lysimeters.

Surface temperatures in the lysimeters were measured by nadir-viewing infrared radiometers (Everest* Interscience model 110) equipped with $10.5-12.5\,\mu\mathrm{m}$ filters. These observations were also recorded automatically as half-hourly averages throughout a day. The surface emissivity was assumed to be 0.98.

The empirical relationship between G and R_n (to be discussed in the next section) was calibrated using observations over Pavon wheat, while the testing of this relationship is done using the data over Ciano wheat. Additionally, in

^{*}Trade name and company names are included only for the benefit of the reader and imply no endorsement or preferential treatment by either NASA or USDA.

testing the empirical equation, we have excluded those days for which the data were used in calibrating the empirical equation.

Soil heat flux

For calibration of the empirical equation, the soil heat flux was calulated as the residual component of the surface energy balance

$$G = R_{\rm n} - \lambda E - \varrho C_{\rm p} (T_{\rm s} - T_{\rm a}) / r_{\rm a}$$
 (2)

The aerodynamic resistance was calculated by stability and bluff-body corrected equations given by Choudhury et al. (1986). The aerodynamic surface roughness of bare soil is assumed to be 10 mm, while for the growing crop the roughness heights for momentum and zero-plane displacement were calculated following Shaw and Pereira (1982). These roughness heights and zero-plane displacements are consistent with others (Monteith, 1973; van Bavel and Hillel, 1976; Legg and Long, 1975). The roughness height for heat is related to that for momentum according to Garratt (1978).

EMPIRICAL RELATIONSHIP

We attempted to establish the functional relationship between G and R_n on somewhat of a rational basis. The observed high correlations for bare soils between G and R_n at varied stages of drying suggest that for vegetated soils also G might be expressed in terms of R_n at the soil surface. Thus, if we express the relationship between soil heat flux (G_0) and net radiation for a bare soil as

$$G_0 = \alpha_{\rm g} R_{\rm n} \tag{3a}$$

Then, for a vegetated soil, one can express G as

$$G = \alpha_{\alpha} \tau R_{n} \tag{3b}$$

where α_g is a proportionality factor and τ is the fraction of net radiation exchanged at the soil surface, which might be described by an exponential relation as

$$\tau = \exp(-\beta L) \tag{3c}$$

where L is the leaf area index and β is an empirical constant. Thus, the above approach to relating G and R_n requires determination of two empirical constants, namely α_g and β . An equation of the form (3b) has been postulated by Shuttleworth and Wallace (1985), although we considered this equation independently.

For 9 days of observations over Pavon wheat (Day of the year 9, 31, 40, 45, 48, 52, 57, 58 and 59) the calculated ratio of G and R_n (averaged over the mid-day hours) is shown in Fig. 1 as a function of L. By a linear regression analysis between $\ln(G/R_n)$ and L, we find the describing equation (r = 0.93)

$$G = 0.4 \exp\left(-0.5L\right) R_{\rm n} \tag{4}$$

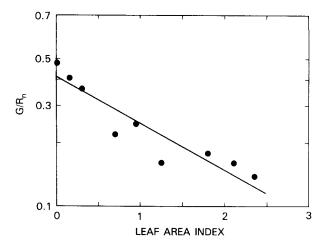


Fig. 1. Calculated ratio G/R_n for Pavon wheat as a function of leaf area index. The fitted line is also shown.

The computed α_g value of 0.4 is substantially in agreement with the observations of Fuchs and Hadas (1972) ($\alpha_g = 0.34$) and Idso et al. (1975) ($\alpha_g = 0.22$ to 0.51), and the computed extinction coefficient $\beta = 0.5$ is within the range of observations, 0.45 to 0.65 for various crops (Monteith, 1973). Thus, eq. 3b provides a realistic description of average soil heat flux under wheat.

We restricted the number of days of data used in assessing the parameters in eq. 4 because we wanted to exclude these days in testing the applicability of this equation more objectively. It is pertinent to note that spectral reflectance in the visible and near-infrared might provide an estimate of the leaf area index (see, for example, Asrar et al., 1984). For an attempt along this direction see Clothier et al. (1986).

ESTIMATION OF LATENT HEAT FLUX

The empirical equation for G (eq. 4) was used in eq. 1, in addition to infrared temperature and meteorologic observations, to calculate λE for Ciano wheat. The computed λE , together with the observed λE , infrared temperature and meteorologic data are shown in Fig. 2 (A to I). Four pairs of consecutive days are shown (Days 6, 7, 42, 43, 49, 50, 70, 71), varying in surface cover from bare soil to an L value of 4.7. Day 56 remains unpaired, because Day 57 was used in developing the empirical equation and the observations during the daytime were incomplete for Day 55. The results of linear correlation between the computed and the observed λE during the daytime hours (when λE is greater than zero) are given in Table I. This table also contains a comparison of daytime total evaporations.

The computed λE for bare soils tends to be higher (lower) than the lysimeter values during the morning (afternoon) periods (Figs. 2A and 2B). A proportion-

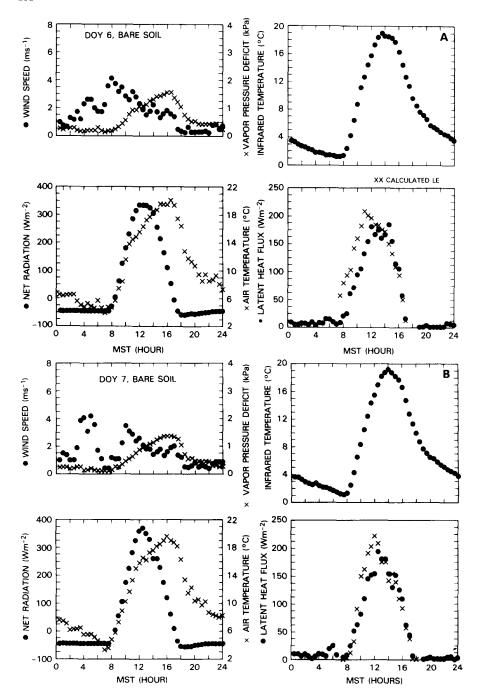


Fig. 2. Diurnal meteorologic data, infrared temperature, and computed ($\times \times$) and observed (\bullet) latent heat flux for nine days of the year (DOY).

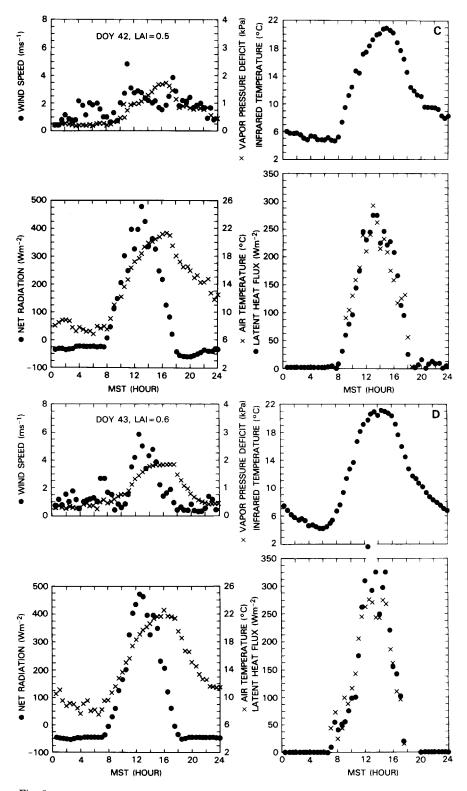


Fig. 2

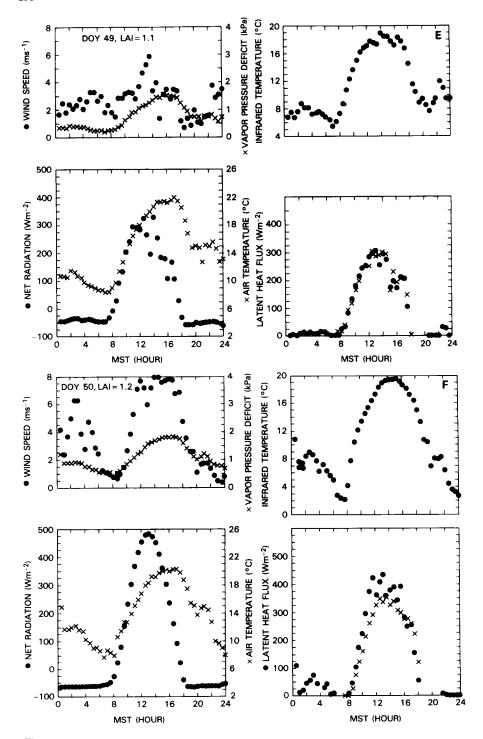


Fig. 2

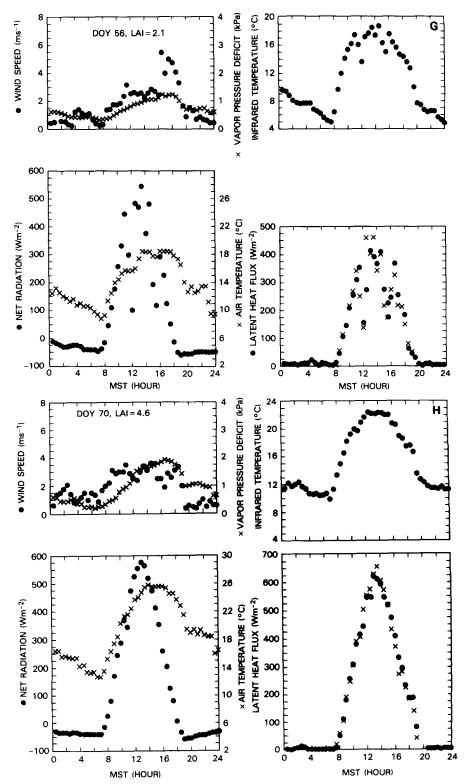


Fig. 2

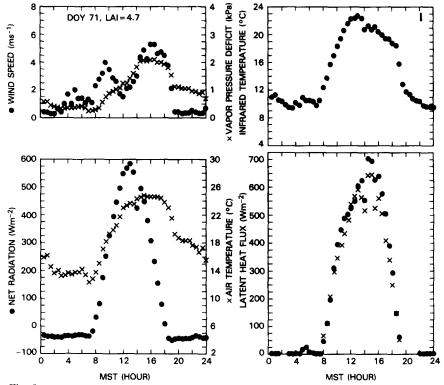


Fig. 2

al relationship between G and R_n (as is the case with eq. 4) cannot account for the diurnal phase differences between these two variables. The hysteresis in the G vs. R_n for a bare soil is such that during the morning, G values are higher compared to those in the afternoon at identical R_n (van Bavel and Hillel, 1976; Fuchs and Hadas, 1972). Thus, eq. (4) would overestimate G values during the afternoon, and underestimate G during the morning. Considering that the observed λE has a standard error of about 30 W m⁻², we estimate the error in estimating G due to hysteresis is also about 30 W m⁻² (Fuchs and Hadas, 1972). During 0730 to 1130 hours on Day 6 the computed λE values are consistently higher than the observations by 50 to 60 W m⁻², while the computed λE values are consistently lower during 1400 to 1630 hours by 10 to 30 W m⁻². Similar hysteresis type error is also seen for Day 7. An improvement over eq. 4 would be to consider α_g in eq. 3a varying diurnally to offset the phase difference.

The diurnal trends of computed and observed λE are in fairly good agreement for the other 7 days, for which the soil is not bare (Figs., 2C to 2I). The magnitude of G decreases progressively as L increases, and therefore any incurred error in G due to hysteresis would also be modulated as L increases. These 7 days were partially cloudy except for Day 50 and 70. One can discern a correlation between the fluctuations in R_n and λE . For the present case of irrigated wheat, it is reasonable to expect that evaporation would depend fairly

TABLE I

Results of linear regression between the computed (Y) and lysimeter observed (X) LE: Y = a + bX. Other symbols are: N (number of observations), σ (standard deviation) and r (correlation coefficient) of regression. Also given are the daytime total evaporations

Day of the year	N	<i>a</i> (W m ⁻²)	b	σ (W m ⁻²)	r	Daytime evaporation (mm)		
						Observed	Computed	Percent error
6	20	12	1.06	29	0.93	1.6	2.0	25
7	20	-1	1.11	24	0.95	1.6	1.7	6
42	22	-8	0.99	29	0.96	2.5	2.5	0
43	22	-4	0.92	37	0.94	2.7	2.5	- 7
49	21	-3	1.05	22	0.98	2.9	2.9	0
50	21	-5	0.85	30	0.98	4.2	3.6	-14
56	23	-13	1.03	68	0.91	3.8	3.8	0
70	24	-25	0.98	83	0.93	6.0	6.0	0
71	24	-28	0.88	101	0.91	7.3	6.7	-8

strongly on the net radiation. However, it is pertinent to note that R_n alone does not determine λE , since during the midday hours the ratio of λE and R_n is about 0.7 for Day 42 and 43, while the ratio is about 1.1 for Day 70 and 71. Indeed, Hatfield et al. (1984) have shown that an energy balance model which is driven primarily by the net radiation tends to make substantial errors in estimating λE , but the errors are generally acceptable for models, such as eq. 1, which depend fairly strongly upon the surface temperature and the net radiation. Blad and Rosenberg (1976) tested eq. 1 over alfalfa for 6 days, showing good agreement for daily evaporation on both clear and cloudy days. However, the agreement was less satisfactory for 15-min average fluxes. From

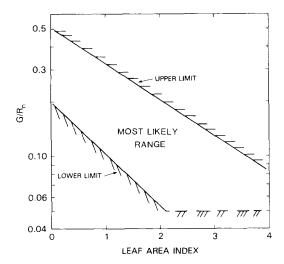


Fig. 3. The likely range for the ratio G/R_n as a function of leaf area index.

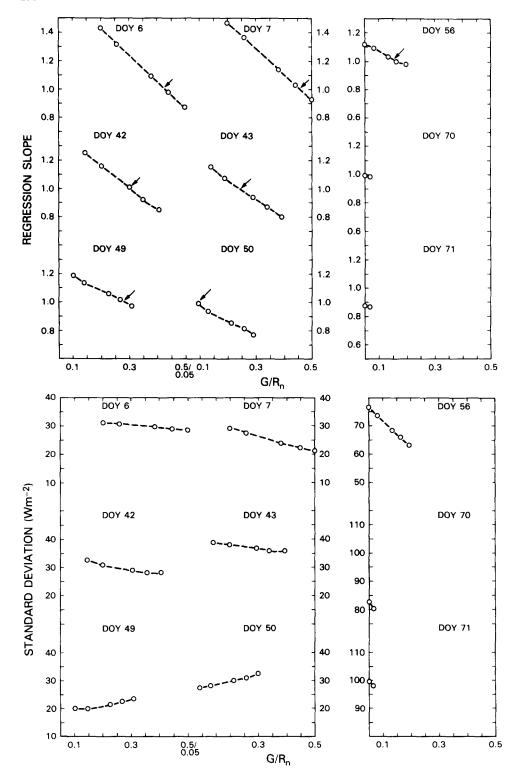


Table I it can be seen that the daily evaporation totals generally do not differ by more than 10%, while the standard deviation for the fluxes range from about $25~\rm W~m^{-2}$ to about $100~\rm W~m^{-2}$. Thus, the calculated 30-min average fluxes are less satisfactory than the daily evaporations. Considering that the range of λE values involved in the regression analysis (from 0 to about $700~\rm W~m^{-2}$), the overall performance of the model is good, since the intercept for any day is close to zero, and the slope of the regression is generally within 15% of the 1:1 line. For 9 days combined, the regression analysis gave a correlation coefficient of 0.97, the slope being 0.90, intercept being $+17~\rm W~m^{-2}$ and the standard error being $39~\rm W~m^{-2}$.

SENSITIVITY ANALYSIS

Sensitivity analysis of eq. 1 has been discussed by several workers (see, for example, Reginato et al., 1985). The present sensitivity analysis would address the effect of errors in G (specifically the parameters $\alpha_{\rm g}$ and β in eqs. 3b and 3c) in estimating λE . The point is that the observations of Fuchs and Hadas (1972), Idso et al. (1975) and Novak and Black (1983) give $\alpha_{\rm g}$ values ranging from about 0.2 to 0.5, and it has been suggested that $\alpha_{\rm g}$ may vary with latitude, meteorological conditions and the soil type. Specifically, we address the question: How would the slope and the standard deviation of the estimated λE change under regression with the observed λE when G is uncertain?

The observed range of α_g is 0.2 to 0.5, while the range for β is 0.45 to 0.65 for arable crops. Taking the lower limit of α_g and the upper limit of β , and then the upper limit of α_g and the lower limit of β we find that the likely range of G for a growing crop would be (Fig. 3)

$$0.2 \exp(-0.65L)R_{\rm n} \leq G \leq 0.5 \exp(-0.45L)R_{\rm n}$$
 (5)

However, because the observed minimum G/R_n under a crop is 0.05, we will also restrict the present sensitivity analysis at this minimum value.

The results of sensitivity analysis are shown in Figs. 4A and 4B. Errors in (G/R_n) translate linearly into the corresponding change in the slope of the linear regression between the estimated and observed λE , where the latter is the independent variable. This linear relationship between the errors is perhaps understandable because λE is a linear function of G (Verma and Rosenberg, 1975; Reginato et al., 1985). For the pair of Days 6 and 7, and 42 and 43, the slope is equal to one at roughly identical ratios of G and R_n ($G/R_n \simeq 0.45$ for the pair of Days 6 and 7, and $G/R_n \simeq 0.27$ for Days 42 and 43). However, for the pair of Days 49 and 50 the slope is equal to one at substantially different values of the ratio G/R_n . For Day 70, the slope is very close to one, and could be made closer to one by assuming G=0. However, for Day 71, the slope

Fig. 4. The slope (top) and the standard deviation (bottom) of the linear regression between the computed and the observed latent heat fluxes (the latter being the independent variable) for a range of values for the ratio G/R_n .

remained close to 0.9 when the calculations were done assuming G=0. This discrepancy of the slopes is not presently understood.

The standard deviation of the linear regression appears to be relatively insensitive to the errors in G/R_n (Fig. 4B). The standard deviation generally increases as the ratio G/R_n decreases. These sensitivity results suggest that an accurate estimation of G is more important for removing the bias in the calculation of λE (i.e., keeping the slope of the regression close to one) than for reducing the standard error in λE .

SUMMARY

The soil heat flux (G) under a growing wheat crop is expressed in terms of net radiation above the canopy (R_n) and an exponentially decreasing function of leaf area index (L). The diurnal trends of latent heat flux (λE) computed using the equation for G agreed well with the lysimeter observations except for bare soils, which showed hysteresis-type errors. A more soil-physics-based model for G is needed for accurately computing bare soil λE . The results of sensitivity analysis showed that errors in estimating G/R_n ratio translates linearly into a bias in estimating λE . Thus, a constant G/R_n ratio throughout the growing season of a crop may not be an acceptable assumption for estimating λE . The calculated λE is likely to be an overestimate when G is assumed to be zero, unless eq. 1 is calibrated under this assumption by adjusting the aerodynamic resistance. For remote sensing applications one must judge the acceptable errors in computing G against the errors in computing $R_{\rm n}$ and $r_{\rm a}$, and recognize that infrared temperature will not, in general, be the aerodynamic temperature. Nevertheless, we have demonstrated the feasibility of retaining the simplicity of eq. 1 by means of an equally simple equation for G.

REFERENCES

- Asrar, G., Fuchs, M., Kanemasu, E.T. and Hatfield, J.L., 1984. Estimating absorbed photosynthetic radiation and leaf area index from spectral reflectance of wheat. Agron. J., 76: 300–306.
- Baldocchi, D.C., Verma, S.B. and Rosenberg, N.,J., 1985. Water use efficiency in a soybean field: influence of plant water stress. Agr. For. Meteorol., 34: 53-65.
- Blad, B.L. and Roseberg, N.J., 1976. Evaluation of resistance and mass transport evapotranspiration models requiring canopy temperature data. Agron. J., 68: 764-769.
- Brown, K.W., 1976. Sugar Beet and Potatoes. In: J.L. Monteith (Editor), Vegetation and the Atmosphere, Vol. 2, Academic Press, New York, pp. 65-86.
- Camillo, P.J., Gurney, R.J. and Schmugge, T.J., 1983. A soil and atmosphere boundary layer model for evapotranspiration and soil moisture studies. Water Resour. Res., 19: 371–380.
- Choudhury, B.J., Reginato, R.J. and Idso, S.B., 1986. An analysis of infrared temperature observations over wheat and calculation of latent heat flux. Agric. For. Meteorol., 37: 75-88.
- Clothier, B.E., Clawson, K.L., Pinter, P.J., Jr., Moran, M.S., Reginato, R.J. and Jackson, R.D., 1986.
 Estimation of soil heat flux from net radiation during the growth of alfalfa. Agric. For. Meteorol., 37: 319-329.
- De Bruin, H.A.R. and Holtslag, A.A.M., 1982. A simple parameterization of the surface fluxes of sensible and latent heat during daytime compared with the Penman-Monteith concept. J Appl. Meteorol., 21: 1610-1621.

- Denmead, O.T., 1969. Comparative micrometeorology of a wheat field and a forest of *Pinus Radiata*. Agric. Meteorol., 6: 357–371.
- Fuchs, M. and Hadas, A., 1972. The heat flux density in a non-homogeneous bare loessial soil. Boundary Layer Meteorol., 3: 191-200.
- Garratt, J.R., 1978. Transfer characteristics for a heterogeneous surface of large aerodynamic roughness. Q. J. R. Meteorol. Soc., 104: 491-502.
- Gurney, R.J. and Hall, D.K., 1983. Satellite-derived surface energy balance estimates in the Alaskan Sub-Arctic. J. Climate Appl. Meteorol., 22: 115-125.
- Hatfield, J.L., 1983. Evapotranspiration obtained from remote sensing methods. In: D. Hillel (Editor), Advances in Irrigation, Vol. 2, Academic Press, New York, pp. 395-416.
- Hatfield, J.L., Reginato, R.J. and Idso, S.B., 1984. Evaluation of canopy temperature evapotranspiration models over various crops. Agric. For. Meteorol., 32: 41-53.
- Idso, S.B., Aase, J.K. and Jackson, R.D., 1975. Net radiation-soil heat flux relations as influenced by soil water content variations. Boundary Layer Meteorol., 9: 113-122.
- Jackson, R.D., 1984. Total reflected solar radiation calculated from multi-band sensor data. Agric. For. Meteorol., 33: 163-165.
- Jackson, R.D., 1985. Evaluating evapotranspiration at local and regional scales. Proc. IEEE, 73: 1086–1096.
- Jackson, R.D. Pinter, P.J., Jr. and Reginato, R.J., 1985. Net radiation calculated from remote multispectral and ground station meteorological data. Agric. For. Meteorol., 35: 153-164.
- Legg. B.J. and Long, I.F., 1975. Turbulence diffusion within wheat canopies. II. Results and interpretation. Q. J. R. Meteorol. Soc., 101: 611-628.
- Lin, J.D., 1980. On the force-restore method for prediction of ground temperature. J. Geophys. Res., 83: 3251–3254.
- Monteith, J.L., 1973. Principles of Environmental Physics. Edward Arnold, London.
- Nickerson, E.C. and Smiley, V.E., 1975. Surface layer and energy budget parameterizations for mesoscale models. J. Appl. Meteorol., 14: 297–300.
- Novak, M.D. and Black, T.A., 1983. The surface heat flux density of a bare soil. Atmosphere-Ocean., 21: 431–443.
- Oke, T.R., 1979. Advectively-assisted evapotranspiration from irrigated urban vegetation. Boundary Layer Meteorol., 17: 167-173.
- Reginato, R.J., Jackson, R.D. and Pinter, P.J., Jr., 1985. Evapotranspiration calculated from remote multispectral and ground station meteorological data. Remote Sens. Environ., 18: 75–89.
- Ripley, E.A. and Redmann, R.E., 1976. Grassland. In: J.L. Monteith (Editor), Vegetation and the Atmosphere. Academic Press, New York, pp. 349-398.
- Shaw, R.H. and Pereira, A.R., 1982. Aerodynamic roughness of a plant canopy: a numerical experiment. Agric. Meteorol., 26: 51–65.
- Shuttleworth, J.W. and Wallace, J.S., 1985. Evaporation from sparse crops an energy combination theory. Q. J. R. Meteorol. Soc., 111: 839–855.
- Szeicz, G., van Bavel, C.H.M. and Takami, S., 1973. Stomatal factor in the water use and dry matter production by sorghum. Agric. Meteorol., 12: 361–389.
- Van Bavel, C.H.M. and Hillel, D.I., 1976. Calculating potential and actual evaporation from a bare soil surface by simulation of concurrent flow of water and heat. Agric. Meteorol., 17: 453–476.
- Verma, S.B. and Rosenberg, N.J., 1975. Accuracy of lysimetric, energy balance, and stability-corrected aerodynamic methods of estimating above-canopy flux of CO₂. Agron. J., 67: 699-704.
- Uchijima, Z., 1976. Maize and Rice. In: J.L. Monteith (Editor), Vegetation and the Atmosphere, Academic Press, New York, pp. 33-64.