# HW2

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1. $Y=1,2,3,4,6,7,8$

$$P_{Y(K)}\begin{cases}\frac{1}{42} & K=1\\ \frac{2}{42} & K=2\\ \frac{6}{42} & K=3\\ \frac{4}{42} & K=4\\ \frac{6}{42} & K=6\\ \frac{7}{42} & K=7\\ \frac{16}{42} & K=8\end{cases}$$

$$\therefore E(Y)=\sum_{K=1,2,3,4,6,8} P_{Y(K)}\cdot K=\frac{1+2+6+4+6+7+16}{42}=\frac{252}{42}=6$$

2. $$E(X,Y)=\int_0^1\int_0^1 f(x,y)\,xy\,dy\,dx=\int_0^1\int_0^1 12xy^3\,dx\,dy$$

$$=\int_0^1 (6y^3x^2)\Big|_0^1\,dy=\int_0^1 6y^3\,dy=\frac{3}{2}y^4\Big|_0^1=\frac{3}{2}$$

3. $$E[(X_1-X_2+X_3)^2]=E(X_1^2+4X_2^2+X_3^2-4X_1X_2-4X_2X_3+2X_1X_3)$$

$$=6E(X_1^2)-6E(X_1X_2)=6E(X_1^2)-6(EX_1)^2=6\,Var\,X_1=\frac{1}{2}$$

4. $$E(e^{\frac{3}{4}X})=\int_0^\infty e^{\frac{3}{4}x}\cdot e^{-x}\,dx=\int_0^\infty e^{-\frac{1}{4}x}\,dx$$

$$=-4e^{-\frac{1}{4}x}\Big|_0^\infty=4$$

5.

| X | y | f(x) |
|---|---|---|
| 1 | 3 | $\frac{1}{6}$ |
| 2 | 9 | $\frac{1}{6}$ |
| 3 | 19 | $\frac{1}{6}$ |
| 4 | 33 | $\frac{1}{6}$ |
| 5 | 51 | $\frac{1}{6}$ |
| 6 | 73 | $\frac{1}{6}$ |

$E(Y) = \sum (2x^2+1)\, dF(x)$ ~~$= E(2X^2+1)$~~

~~$X=1,2,3,4,5,6 \quad = 2E[X^2]+1$~~

~~$\sum (2x^2+1) P(X=x)$~~

~~$X=1,2,3,4,5,6$~~

$= \frac{1}{6}(3+9+19+\cdots+73) = \frac{94}{3}$

6. $Y = 2X+1 \Rightarrow Y^2 = 4X^2+4X+1.$

$E[Y^2] = E[4X^2+4X+1] = \int_0^1 (2-2x)(4x^2+4x+1)\,dy$

$= \int_0^1 (-8x^3+6x+2)\,dx = -2x^4+3x^2+2x\Big|_0^1 = 3.$

7. $E[(aX+b)^n] = (aE[x]+b)^n$

$= \sum_{i=0}^{n} \binom{n}{i} (aE(x))^{x-i} \cdot b^i = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i E(x^{n-i})$

8. $E(X-Y) = E[X-(n-X)] = E[2X-n] = 2E(X)-n$

$\because \lambda = np$

$F(X=k) = \frac{(np)^k}{k!} e^{-np} \xrightarrow{E(X)=\lambda=np} E(X-Y) = 2np-n = n(2p-1)$

When $n=20$, $p=5\%$, $E(X-Y) = 20(2\times 5\%-1)$

$= 20(0.1-1) = 20\cdot(-0.9) = -18.$

Answers, If we select 20 parts from shipment rando
and the propotion of detective parts is 5%,
We could expect good parts are 18 more than
defectives parts on averdge.