Hw4

1. Find MLE : proportion, 70 purchases, 58 (women), 12 (men). Find M.L.E of $\hat{p}$.

$$\begin{cases} x \in \{0, 1\}, \ f(x) = p^x (1-p)^{1-x}, \\ \ell(P, x_1, \dots, x_{70}) = \sum x_i \log(P) + (70 - \sum x_i) \log(1-P). \end{cases}$$

$$\frac{\Delta \ell}{\Delta P} = \frac{\sum x_i}{P} - \frac{70 - \sum x_i}{1-P} = 0 \Rightarrow \hat{p} = \frac{\sum x_i}{n} = \boxed{\frac{6}{35}}$$

2. $X_1, \dots, X_n$, Bernoulli with parameter θ, except $0 < \theta < 1$. Show MLE of θ DNE.

$$\begin{cases} x \in \{0, 1\} \ f(x) = \theta^x (1-\theta)^{1-x} \\ \ell(\theta; x_1, \dots, x_n) = \sum x_i \log(\theta) + (n - \sum x_i) \log(1-\theta) \end{cases}$$

$\frac{\Delta \ell}{\Delta \theta} = 0 \Rightarrow \hat{\theta} = \frac{\sum x_i}{n}$. if $X_1 = X_2 = \dots = X_n = 0 \Rightarrow \hat{\theta} = 0$

if $X_1 = X_2 = \dots = X_n = 1$, $\hat{\theta} = 1$.

~~However~~, Note, $0 < \theta < 1$, $\therefore$ $\boxed{\text{MLE of } \theta \text{ DNE}}$

$\boxed{\text{if observation} = 0 \text{ or } 1.}$

3. $X_i \sim P(\lambda)$ $f(x) = \frac{e^{-x} \lambda^x}{x!}$

(a) $\ell(\lambda, X_1, \dots X_n) = -\lambda n + \sum x_i \log(\lambda) - \log[\prod_{i=1}^{n} (x_i!)]$

$\therefore \frac{\Delta \ell}{\Delta \lambda} = \frac{\sum x_i}{\lambda} - n = 0 \Rightarrow \hat{\lambda} = \frac{\sum x_i}{n}$

(b) If $X_1 = X_2 = \dots = X_n = 0$, then $\hat{\lambda} = 0$ when $\lambda > 0$.

$\therefore$ $\boxed{\text{MLE of } \lambda \text{ DNE when every value is } 0}$

4. $N(\mu, \sigma^2)$ $f(x) = e^{-\frac{1}{2\sigma^2}(x_i-\mu)^2} \frac{1}{\sqrt{2\pi}}$

$L(\mu, \sigma, x_1, \dots x_n) = \prod_{i=1}^{n} \left[\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x_i-\mu)^2} \right]$

$l = n \log\left(\frac{1}{\sqrt{2\pi}}\right) - \frac{n}{2}\log(\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$

$\therefore \frac{\Delta l}{\Delta \sigma} = 0 = -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{i=1}^{n}(x_i-\mu)^2$

$\Rightarrow \hat{\sigma}^2 = \sum_{i=1}^{n}(x_i-\mu)^2/n$.