# Confusion, Phillips Curves and De-anchored Inflation*

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November 12, 2024

*Preliminary*

**Abstract**

We investigate inflation dynamics when firms are uncertain about the causes of aggregate fluctuations and use prices and output as learning tools. During periods of low inflation, firms observing increased output attribute this change partly to positive demand and partly to positive supply shocks, resulting in a dampened pricing response. Consequently, demand shocks are near non-inflationary while supply shocks are strongly inflation. As inflation escalates, firms observing increased inflation attribute this change partly to positive demand and partly to negative supply shocks. However, no matter which case they believe, they raise prices for sure. This triggers even higher inflation and, therefore, a self-fulfilling cycle of de-anchored inflation. Supported by survey evidence, our endogenous information New Keynesian model (with or without explicit nominal rigidity) can generate realistic monetary non-neutrality and explains occasional inflation de-anchoring. This model explains the business cycle puzzle of inflation disconnect and flattened Phillips curve (Angeletos et al. (2020)), and also offers new insights into inflation dynamics and monetary policy implications.

*We are grateful for comments and discussions with George-Marios Angeletos, Lawrence Christiano, Martin Eichenbaum, Alessandro Pavan, Matthew Rognlie, Diego Kanzig and Huo Zhen.

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# 1 Introduction

Understanding inflation dynamics and the effectiveness of monetary policy has been a cornerstone of business cycle research. Historically, theories from Hume (1752) to Lucas (1972) have highlighted the significant role of information frictions in shaping effectiveness of monetary policy. Despite advancements, the precise nature of the Phillips curve—representing the relationship between inflation and unemployment—remains contentious, especially given its varying slope over different periods. In recent times, the problem seems to get worse. Angeletos et al. (2020) and Del Negro et al. (2020), among other research, have documented the fact that inflation seems to be disconnected from most of business cycle's variation in turns of output, consumption, investment and other real variables. More explicitly, shocks that drives real variables does not cause much inflation, while shocks that drives inflation does not cause much movement in real variables.

It has been challenging for models with explicit nominal rigidities[1], such as Calvo pricing, to explain the inflation disconnect, flattened/steepened Phillips curve and micro evidence on pricing. First, the estimated slope of the Phillips curve is one to two orders of magnitude smaller than implied from the frequency of price changes using micro data (Bils and Klenow (2004)). Second, the Phillips curve was relatively steep before 1990 but has significantly flattened before the COVID-19 pandemic (Del Negro et al. (2020), Hazell et al. (2022)). However, during periods of economic turmoil such as the 1970s and the COVID-19 pandemic, the Phillips curve generally started to steepened again. This suggests that traditional models may be missing key elements to jointly explain the inflation disconnect, and the flattened Phillips curve.

We propose that the puzzles surrounding the slope of the Phillips curve and the periodic de-anchoring of inflation can be better explained through a framework incorporates endogenous learning, and uncertainty about the origins of economic fluctuations. Inspired on the insights of Lucas (1972), we develop a new type of New Keynesian model that features endogenously learning with multiple origins of shocks. Our model posits that the Phillips curve is not a fixed relationship but is driven by agents' expectations, which evolve based on their observations and interpretations of economic signals. This also allows demand and supply shocks to have different

[1]We refer explicit nominal rigidity to models that are monetary non-neutral under full information rational expectation.

inflationary responses: demand shocks can be near non-inflationary while supply shocks can be strongly inflationary. Our result holds with or without explicit nominal rigidity [2].

**Mechanism of Flattened and Shock Dependent Phillips Curve** In reality, households and firms do not have complete knowledge about the aggregate state of the economy. They form their beliefs about economic fundamentals from observable such as supermarket and gas prices, demand variations at their firms, wages, news, and other endogenous and exogenous objects. Rational households and firms would learn more from output than inflation when inflation is low and uninformative. During such times, agents learn from information primarily related to aggregate output, such as unemployment rates, product demand, and stock prices. From classic supply and demand relationships, an increase in output can result from either a positive demand shock or a positive supply shock. Optimally, firms shall increase prices in response to a positive demand shock and decrease prices in response to a positive supply shock. Firms, observing increased demand, may interpret it as an idiosyncratic demand shock, a positive aggregate demand shock, or a positive supply shock. Compared to the standard New Keynesian model, this will leads firms to adjust their prices less than they should when demand shocks happen. However, supply shocks will give rise to higher inflation since supply shocks directly reduce firms' marginal costs, while expectations about aggregate inflation and output dampen the desire to increase prices from the wage channel.

**De-anchored Inflation** During salient events such as supply chain disruptions or wars, the Phillips curve's slope could increases significantly and potentially lead to de-anchored inflation. The underlying mechanism is similar: supply shocks cause higher inflation, prompting households and firms to pay more attention to inflation. Classic supply and demand curves indicate that an increase in prices can stem from either a negative supply shock or a positive demand shock. But now, firms shall raise their prices regardless of these two shock's nature. The observation of higher prices encourages further price increases, creating a self-fulfilling inflationary spiral.

This observation dates back to econometric identification challenges: it is difficult for econometricians to disentangle supply and demand curves from equilibrium outcomes. Economists often rely on carefully constructed instruments to identify these curves. In our model, agents, acting as

[2] Under our framework without explicit nominal rigidity, if firms have complete information, money would be neutral. But firms with incomplete information is not allowed to explored the *entirety* of the demand curve. This point is discussed further below

econometricians, might not accurately identify the curve in real-time. Additionally, agents' actions, in reality, become data sources for other agents, leading to an endogenous feedback loop.

We argue that this mechanism explains why inflation sometimes re-emerges and persists while remaining muted at other times. Revisiting Lucas (1972) and Woodford (2001) notion that monetary non-neutrality could largely stems from expectations, we show that rational information frictions, even without explicit nominal rigidity (e.g., Calvo or menu costs), match business cycle facts and elucidate the underlying mechanisms behind de-anchored, volatile economic activities and boom-bust phenomena. When we match business cycle moments with both time-dependent pricing friction and information friction, we found that the micro-level price change frequency, with which the implied aggregate slope in a standard model is one to two order of magnitude larger than empirical estimates, matches and explains the empirical puzzles very well.

**Details of the Model** Our baseline model integrates realistic information frictions into the neoclassical core of the New Keynesian model. In our model, agents receive signals about the economy's underlying state and learn from inflation and output. Learning from inflation and output allows us to characterize a new source of the Phillips curve — inflation and output is connected because price setters' belief about aggregate economy depends on observed inflation and output. Another feature of the model is that it incorporates of multiple aggregate shocks – demand and supply shocks – simultaneously. This results in novel economic dynamics, as agents mistakenly attribute economic fluctuations to counterfactual shocks and respond inappropriately. As such, the effect of different shocks on inflation is different — the slope of the Phillips curve is shock dependent. The model also predicts that when inflation is not informative about underlying source of aggregate fluctuation, inflation disconnects from real variables. When inflation is informative about the origins of aggregate fluctuation, inflation de-anchored and became self-fullfilling.

**Policy Implications** When central bank is *slightly* less aggressive in contaminated de-anchored inflation, inflation would jumps by a large degree, and would result in volatile, boom-bust response of inflation. This is because monetary policy decreases the strategic complementary not only in pricing, but also the information externality stemming from using other firms' action as a signal. When inflation is high, firms utilizes less private information since inflation is deemed to be more informative. However, firms failed to internalize the fact that inflation is only information *if* firms response to their own private information. This channel is similar to the ones highlighted by Vives

(2017) in the micro theory literature. Slightly less aggressive monetary policy triggers a boom-bust oscillating behavior of inflation, known as "confounding dynamics" (Rondina and Walker (2017)).

**Empirical Evidence** We use a structural VAR design incorporating key macroeconomic variables and expectations from the Survey of Professional Forecasters (SPF). Our findings reveal that inflation responded more strongly to a demand shock (excess bond premium shock) before 1990 than after. Additionally, output nowcast errors (differences between actual and perceived output) were more pronounced before 1990, indicating greater awareness of aggregate output post-1990. Conversely, inflation nowcast errors were more pronounced after 1990, showing greater inflation awareness pre-1990. This evidence supports our hypothesis: if agents focus more on output, inflation responds less; if they focus more on inflation, inflation rises endogenously.

**Related Literature** Our paper contributes to three related literatures. In terms of the foundation of monetary non-neutrality, the Phillips curve, and aggregate supply, we build on the literature that explores the impact of information friction on business cycle dynamics (Lucas (1972), Woodford (2001), Mankiw and Reis (2001), Maćkowiak and Wiederholt (2009), Hellwig and Venkateswaran (2009), Angeletos and La'o (2010), Flynn et al. (2023)). Our baseline model without explicit nominal rigidity is particularly close to Lucas (1972) in terms of learning from endogenous objects and is also similar to Woodford (2001) in assuming implicit nominal rigidity[3]. However, our model differs from Lucas (1972) by using the skeleton of the New Keynesian model[4], and is capable of generating large and persistent monetary non-neutrality, addressing the critique that limited the broader application of Lucas (1972). Our model also differs from Woodford (2001) in that firms learn *endogenously* from output and inflation, allowing us to discuss a novel foundation for the shock dependent Phillips curve and its consequences. Finally, unlike these two paper that solely study money supply shocks, we study multiple aggregate shocks jointly and can speak to the disconnection between inflation and real variables, as the Phillips curve are shock-dependent.

Our paper also relates to the empirical literature on flattened Phillips curve, inflation disconnect, and periodic de-anchored inflation. Del Negro et al. (2020) and Hazell et al. (2022) have documented the flattening of the Phillips curve since 1990s, while the Phillips curve is estimated to have steepend after the COVID-19 pandemic. Angeletos et al. (2020) found that the shocks that explain most of

[3]i.e., firms set prices within a period and are not allowed to explore the entirety of the demand curve

[4]That is, the standard New Keynesian model without Calvo friction, thus having output and prices as separate endogenous signals, with the central bank implementing a Taylor rule.

the real variables' variation in business cycle is essentially non-inflationary, while the shock that drives most of inflation does not cause movement in consumption, GDP and other real variables. Our model jointly explained these facts, by incorporating the fact that agents learn endogenously from output and prices, in an environment where aggregate economy can be driven by both demand and supply shocks.

Finally, the mechanism of our model is related to the theory literature on information friction and externality (Morris and Shin (2002), Angeletos and Pavan (2007), Vives (2017), Pavan et al. (2022)). The setup of our model is closest to Vives (2017), but with agents setting a pricing schedule and observing prices and output, rather than setting a supply schedule and observe prices alone. We also extend the analysis of such models to general equilibrium.

In terms of the solution techniques, models with dynamic, dispersed and endogenous information are difficult to solve since it has to "forecast the forecast of others" Townsend (1983). We leverage the recent development in using frequency domain solution to solve such a class of model. Using tools developed by Han et al. (2022), Adams (2019), Huo and Takayama (2023) and Rondina and Walker (2017). We solve the dynamic model in frequency space.

**Outline** In the second section, we introduce a simple model to illustrate the intuition. In the third section, we quantify the impact of the channels using a dynamic models with richer details. Section four talks about survey evidence that support our findings. Section five concludes.

## 2 Simple Model

In this section, we lay out a simple New Keynesian model without explicit nominal rigidity. In a full fledged micro-founded model, firm and household's information sets are interacted, with firms learning from their own demand, prices sold, and prices of competitors, etc.. Instead, in this section, only firms are subject to information frictions. This is meant to deliver intuitions in the simplest way. In the full dynamic model, we relax these assumptions.

In the model, households are infinitely lived, they consume goods and supply labor. The government provides transfers and issues bonds. Its budget constraint is respected. The central bank controls interest rate.

There are a continuum of firms. Each firm produces a differentiated variety. Firms are subject to i.i.d. idiosyncratic and aggregate productivity shocks. Additionally, the economy subjects to i.i.d aggregate demand shocks, *e.g.,* discount rate shocks and monetary policy shocks. In this simple model, we assume households have full information and firms do not know the underlying shocks and have incomplete information. In particular, firms receive signals about aggregate output and inflation. The timeline is the following: Nature draws aggregate shocks and idiosyncratic shocks. In the morning, firms set a pricing schedule[5] based on their observed signals such as prices and aggregate demand, in the afternoon. In the afternoon, firms hire labor and produce. Households make labor supply and consumption-saving decisions. Labor market and good market clear.

In the afternoon, given the pricing schedule and full information about the underlying aggregate shocks, the representative household solves the following problem,

$$\max_{\{C_t,N_t,B_t,P_t\}} E_t \sum^{\infty}(\Pi^k_{s=0}\beta_{t+s-1})(\frac{1}{1-\sigma}C^{1-\sigma}_{t+k}+\frac{1}{1+\eta}N^{1+\eta}_{t+k})$$

where $\beta_t$ is the discount factor between $t$ and $t+1$. The first-order condition after log-linearization gives us the Euler equation,

$$c_t = c_{t+1} - i_t + \pi_{t+1} + \beta_t$$

All lower case letters represents log deviation from steady state, i.e., $c_t$ is log changes of consumption, $p_t$ is the log changes of price index, $\pi_t$ is log changes of inflation and $i_t$ is the log-deviation of interest rate. $\beta_t$ is the log-linearized i.i.d. distributed aggregate discount rate shock, with mean zero and variance $\sigma^2_d$.

The final consumption good is consisted of a continuum of differential goods aggregated with CES, i.e., $C_t = \int_0^1 C_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj$. The demand curve for each good writes $C_{jt} = (\frac{P_{jt}}{P_t})^{-\varepsilon}C_t$. Where $P_t$ is the price index defined as $P_t = (\int_0^1 P_{jt}^{1-\varepsilon}dj)^{\frac{1}{1-\varepsilon}}$.

Each variety is produced by a monopolistic firm with the production function $Y_{it} = A_{it}N_{it}^{1-\alpha}$. Firms are subject to aggregate and idiosyncratic productivity shocks, $a_{it} = a_t + \eta^{\epsilon}_{it}$. With $a_t$ being the aggregate log deviation of TFP, $\eta^{\epsilon}_{it}$ follows the normal distribution with mean zero and variance $\sigma^2_{\epsilon}$. The aggregate productivity shock $a_t$ is normally distribution i.i.d shock with mean zero and variance $\sigma^2_s$. We interpret the productivity shocks to be *any exogenous shocks that directly lead firms to change prices.* This encompasses TFP shocks, cost-push shocks and markup shocks that are commonly used in the literature.

Firms have incomplete information about aggregate shocks. In particular, firms receives two signals about endogenous variables: noisy aggregate output $\tilde{y}_{i,t}$ and inflation $\tilde{\pi}_{i,t}$, where the noise

[5] The use of this pricing schedule is similar to the set up of Vives (2017), which micro-founds the rational expectation notation of Grossman and Stiglitz (1980).

in both signals are i.i.d, with mean 0 and variances $\sigma_y^2$ and $\sigma_\pi^2$. The information contained in the signals has to be consistent with the equilibrium $y_t$ and $\pi_t$ as imposed by rational expectations equilibrium. Notice that we do not provide micro-foundation for these two signals here. As we show in other sections, natural candidates for endogenous signals are wage and the intercept of the demand curve. Firms know their own productivity $a_{it}$, we assume that they do not realize the informational value of $a_{it}$. Again, this assumption is to ease exposition, and will be fully relaxes in the full model. Firms know all the underlying shocks in the next period. The firm $i$'s information set is given by $\mathcal{I}_{it}^s = \{\beta_{t-1}, a_{t-1}\} \cup \{\tilde{y}_{it}, \tilde{\pi}_{it}\}$

Firm $i$ maximizes profit only conditional on its information set,

$$\max_{P_{it},Y_{it},N_{it}} E_{it}\left(\frac{\Lambda_t}{P_t}(P_{it}Y_{it} - W_t N_{it})\right)$$
$$\text{s.t.} \quad Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t; Y_{it} = A_{it} N_{it}^{1-\alpha}$$

Log-linearization of the first-order condition writes,

$$p_{it} = E_{it}\Big(\varphi(p_t + \frac{1}{\varepsilon} y_t) + (1-\varphi)(w_t - \frac{1}{1-\alpha} a_{it})\Big)$$

$\varphi = \frac{\alpha}{1-\alpha}/(\frac{\alpha}{1-\alpha} + 1/\varepsilon)$. Labor market clears in the second stage. Firms form expectations about nominal wage when setting prices. The effect of decreasing return to scale is summarized by parameter $\varphi$. Higher $\alpha$ implies higher $\varphi$ and thus larger strategic complementarity. When $\alpha \to 0$, the optimal pricing rule degenerates to $p_{it} = E_{it}(w_t) - a_{it}$. Aggregate supply shocks influence prices through $a_{it}$. Firms know the good and labor markets have to clear, plugging these condition into the pricing rule. And assume that the economy was in its steady state at $t-1$, $p_{t-1} = 0$.

$$\pi_t = \bar{E}_t \pi_t + \psi_y \bar{E}_t y_t - \psi_a(\eta \bar{E}_t a_t + a_t)$$

where $\psi_y = \frac{\varphi}{\varepsilon} + (1-\varphi)(\sigma + \frac{\eta}{1-\alpha})$, $\psi_a = \frac{1-\varphi}{1-\alpha}$. It is easy to show that both $\psi_y$ and $\psi_a$ are decreasing in $\alpha$. $\psi_y$ denotes the income effect of expected output. $\psi_a$ denotes the effect of the supply shocks. Notice that the supply shocks contain both the actual aggregate supply shock and the expected one which is originated from firms' expectation of the real wage. The monetary authority sets the nominal interest rate according to Taylor rule. After log-linearizing the monetary policy rule,

$$i_t = \phi \pi_t$$

To summarize, the simple model boils down to the system of equations:

$$y_t = y_{t+1} - \frac{1}{\sigma}(\phi\pi_t - \pi_{t+1} - \beta_t) \tag{1}$$

$$\pi_t = \bar{E}_t\pi_t + \psi_y\bar{E}_t y_t - \eta\psi_a\bar{E}_t a_t - \psi_a a_t \tag{2}$$

Firms uses two signals $\tilde{y}_{i,t} = y_t + \epsilon_i^y$ and $\tilde{\pi}_{i,t} = \pi_t + \epsilon_i^\pi$ to learn about the state of the economy. $\sigma_y$ and $\sigma_\pi$ are the standard deviation of the noises. The state of the economy is subject to two aggregate shocks, one encapsulate demand and another supply, $\beta_t$ and $a_t$, with standard deviation $\sigma_d$ and $\sigma_a$.

### 2.1 What Determines Belief?

Upon seeing the signal, how would a firm form its expectations? The economy is subject to an aggregate demand shock $\beta_t$, an aggregate supply shock $a_t$, and idiosyncratic noises in the signals $\epsilon_i^y$ and $\epsilon_i^\pi$. As in the classic supply and demand relations, in equilibrium, demand shocks give rise to a positive co-movement of output and inflation, while supply shocks result in a negative co-movement. If a firm sees that output has increased while inflation reacts relatively modestly, it might think that it is a combination of positive demand and supply shocks. If the firm sees an increase in inflation with a modest change in output, it might think that the economy is hit by a positive demand shock and a negative supply shock. This is captured by propositions in this section.

Let the economy-wide output response to a unit demand and supply shock be $\chi^{yd}$ and $\chi^{ys}$, respectively; let the inflation response to a unit demand and supply shock be $\chi^{\pi d}$ and $\chi^{\pi s}$. Then, the firm's signal can be represented by the following equation.

$$\underbrace{\begin{pmatrix} \tilde{y}_{i,t} \\ \tilde{\pi}_{i,t} \end{pmatrix}}_{x_{i,t}} = \underbrace{\begin{pmatrix} \sigma_d\chi^{yd} & \sigma_s\chi^{ys} & \sigma_y & 0 \\ \sigma_d\chi^{\pi d} & \sigma_s\chi^{\pi s} & 0 & \sigma_\pi \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} \eta_t^d \\ \eta_t^s \\ \eta_t^y \\ \eta_t^\pi \end{pmatrix}}_{s_{it}}$$

Where $s_{i,t}$ is the underlying vectors of shocks, garbled through the matrix $B$, and eventually received by agent as $x_{i,t}$, the vector containing their observed output and inflation. Observe that if the matrix $B$ has full rank, then the fully rational firms would invert the matrix, and will be able to learn about the underlying shocks of the economy perfectly. If $B$ is not full rank, firms will have incomplete understanding of the economy, despite being fully rational. However, understanding the structure of the economy, they would optimally combine signals, and form their belief about the economy perfectly.

**Proposition 1 (Projection)** *Firms average belief about the aggregate state of the economy, captured by their belief about the aggregate demand and supply shocks, is formed according to:*

$$\begin{pmatrix} \bar{E}_t \beta_t \\ \bar{E}_t a_t \end{pmatrix} = \underbrace{ZB'(BB')^{-1}B}_{\Omega} \begin{pmatrix} \beta_t \\ a_t \end{pmatrix}$$

*Where the off-diagonal 2-by-2 matrix $\Omega$ captures the confusion of firms. $Z$ is the matrix that captures the equilibrium relations between aggregate shocks and the expectation of interest.*[6]

The $\Omega$ matrix might resemble Ordinary Least Squares (OLS) regression or the Kalman filter. In a static setting, the $\Omega$ matrix is exactly equivalent to the Kalman filter, OLS regression, and multivariate Bayesian updating. Despite using the optimal updating rule, because firms receive fewer signals than shocks, they cannot perfectly infer the underlying shocks and may incorrectly attribute an innovation in demand shocks to supply shocks. This is captured by the off-diagonal elements of the matrix. Indeed, if the $(1,2)$ entry of $\Omega$ is positive, it means that when a supply shock occurs, firms might mistakenly believe that it is partly a *positive* demand shock ($a_t$ positively affects $\bar{E}_t \beta_t$). Similarly, if the $(1,2)$ entry is negative, firms might mistakenly believe that a positive supply shock is actually a *negative* demand shock. We will discuss the consequences of such confusion. Intuitively, if firms believe that economic fluctuations are partly due to positive demand and partly to positive supply factors, they would not increase their prices as much due to the opposite co-movement of output and inflation from these two types of shocks. However, if firms believe that the economy is hit by positive demand shocks and/or negative supply shocks, they would unambiguously increase their prices, potentially giving rise to self-fulfilling inflation. How do we know when firms would confuse a positive demand shock with a negative supply shock?

[6] In this case, it is an identity matrix with extra two columns of zeros.

**Proposition 2 (Confusion)** *Denote $C$ as the $(1,2)$ entry of $\Omega$,*[7] *it captures how firms confuse a supply shock as demand shock.*

$$C \propto \frac{1}{\sigma_y^2}\chi^{yd}\chi^{ys} + \frac{1}{\sigma_\pi^2}\chi^{\pi d}\chi^{\pi s}$$

*Where $\chi^{yd}$ is the output $y$ response to a unit demand shock. And $\sigma_y$, $\sigma_\pi$ are noises in firms' output and inflation signals.*

From the proposition, we can see that the sign of confusion is determined by the strength of the output response and the inflation response, weighted by the output and inflation signals. Despite being equilibrium objects, standard economic intuition would predict a positive supply shock to increase output and decrease prices, $\chi^{ys} > 0$, $\chi^{\pi s} < 0$, whereas a positive demand shock would increase both output and prices, $\chi^{yd} > 0$, $\chi^{\pi d} > 0$. Therefore, conditional on signal precision, if inflation reacts aggressively to supply and demand shocks, $\chi^{\pi d}\chi^{\pi s} < 0$ would dominate, and firms would confuse a positive supply shock with a negative demand shock, leading them to further increase prices. If the output response is stronger, $\chi^{yd}\chi^{ys} > 0$ would dominate, and firms would not want to increase their prices by much. On the other hand, if firms pay more attention to inflation, or $\sigma_\pi$ is small, the negative term dominates, and firms would again confuse a positive supply shock with a negative demand shock.

Intuitively, this proposition tells us that when inflation becomes informative to household decisions, either by having a large response or by firms paying more attention to it, it triggers negative confusion due to the negative co-movement between output and inflation for supply shocks. Firms with a negative confusion would want to increase prices, regardless of the origins of economy fluctuation, since firm would want to increase prices with both positive demand and negative supply shocks. In the next section, we will investigate the consequences of such confusion.

## 2.2 Shock Dependent Phillips Curve

Despite the fact that the economy does not feature explicit nominal rigidity, there is a structural relations between output and inflation due to the information channel. Intuitively, when a firm observe an increase in output, she would adjust her expectation on aggregate inflation. Moreover, when a firm perceive the economy to experience a greater turbulence, she would respond more in terms of her pricing decision.

[7] By the structure of this projection matrix, the other off-diagonal entry $(2,1)$ are proportional to $C$. Under this information structure, $(1,2)$ entry and $(2,1)$ entry are exactly identical.

**Proposition 3 (Informational Phillips Curve)** *The* ***signal extraction consistent*** *relation between inflation and output gap (when defined) is:*

$$\pi_t = \kappa_y y_t + \kappa_a a_t \tag{3}$$

*The Phillips curve is generically defined, except when firms have full information.*

To understand this relations better, let us revisit the firm's pricing decision.

$$\pi_t = \bar{E}_t \pi_t + \psi_y \bar{E}_t y_t - \eta \psi_a \bar{E}_t a_t - \psi_a a_t$$

Firms' pricing decisions rely on expectations of output, inflation, and aggregate supply shocks. They also depend on their own supply shocks $a_t$. With endogenous and incomplete information, aggregate output affects aggregate inflation even though firms do not condition their prices on their quantities. Output $y_t$ appears through the information channel — firms learn from output $y_t$ and inflation $\pi_t$. When observing a given level of output and inflation, they update their beliefs about $\bar{E}_t \pi_t$, $\bar{E}_t y_t$, and $\bar{E}_t a_t$. More concretely, this can be seen from the following equation:

$$\bar{E} y_t = \omega_{\tilde{y}}^{y} y_t + \omega_{\tilde{\pi}}^{y} \pi_t$$

In this example, $\omega_{\tilde{y}}^{y}$ denotes the impact of seeing the output signal $\tilde{y}$ on firms' belief about output $y$. $\omega_{\tilde{\pi}}^{y}$ denotes the impact of seeing the inflation signal $\tilde{\pi}$ on firms' belief about output $y$.

More concretely, when firms imperfectly see a high level of output, they might be confused about the origin of the aggregate shocks impacting the economy, leading them to adjust their inflation expectations differently. Moreover, the higher the observed output, the larger the underlying shock, and firms will increase their prices more.

**Shock Dependent Effect on Inflation** Another notable feature of the Phillips curve is its departure from the standard model, where there is only one slope of the Phillips curve regardless of shocks. Here, $\kappa_a$ corresponds to the additional effect of supply shocks on $\pi_t$ not through $y_t$; we call this the slope of supply shocks. When a supply shock occurs, firms' marginal costs are concretely affected, and this is not subject to expectation frictions. Therefore, it is natural that firms price differently in response to demand shocks, which propagate through the expectation of marginal cost, versus supply shocks that have concrete impact on firms' marginal costs. In the standard New Keynesian model, supply shocks like TFP shocks are not able to generate realistic scale of inflation, precisely because expectation of future output are so large that it offsets a big chunk of the decrease in marginal cost today. Later, we will see that this feature of our model allows us to observe a large differential effect of different shocks on inflation.[8]

**Phillips Curve Without Supply Shocks** Before characterizing the equilibrium Phillips curves, it is worthwhile to study a simplified version of it, where demand shocks is the only source of aggregate fluctuation in the economy, i.e., $\sigma_s = 0$. This version of the model is similar to the model of Lucas (1972), in which firms are hit by idiosyncratic and aggregate demand shocks, and have to learn from demand of their own good. The simplified model is even closer to Woodford (2001): similar to firms in Woodford (2001), firms in our model can not adjust their prices *conditional* on the demand curve. In this sense, our model shares the same implicitly assumed nominal rigidity as in Woodford (2001). *However*, firms in the model of Woodford (2001) has to set prices based on private information about innovation of nominal GDP, an *exogenous* shock generated by monetary authorities. In our model, firms learn *endogenously* from noisy signals of inflation and output. It is the possibility of observing endogenous inflation and output that makes the Phillips curve representation possible — again, the Phillips curve representation reflects how agents learn from output and inflation, and how the resulting belief in turns affect their pricing decision.

Defined $\hat{\kappa}, \hat{\kappa}_a$ as the equilibrium slope of the Phillips curve when there is no supply shock, i.e., $\sigma_s = 0$. Then:

**Proposition 4 (Phillips Curve Without Supply Shocks)** *If expectation of income does not increase wages by too much, i.e., $\psi_y < \bar{\psi_y}$:*
*(1) When $\sigma_\pi > 0$: $\hat{\kappa} > 0$ and $\hat{\kappa}_a < -\psi_a$.*
*(2) When $\sigma_\pi = 0$: $\hat{\kappa} = 0, \hat{\kappa}_a = 0$.*

This proposition tells us that in terms of inflation responses, there are shock dependent responses. In general, the size of supply shock's slope is lower bounded by $\psi_a$ whereas the slope of the demand shock can be quite small. The intuition is again, the fact that supply shocks affect marginal cost concretely, whereas demand shocks affect firms' pricing decision by changing firms' expectations on output and aggregate inflation. Figure 1 shows the equilibrium slope of output and supply, $\hat{\kappa}$ and $\hat{\kappa_a}$, against the variance of the noise component in the inflation signal. First, note that the slope of output is within the range of 0.05 to 0.09 while the slope of supply ranges

[8] This formalization of the Phillips curve is quite general—if agents additionally see and internalize additional signals about aggregate shocks such as demand shock $\beta_t$ and supply shock $a_t$, it will also show up in an alternative version of equation 3, $\pi_t = \kappa_y y_t + \kappa_a a_t + \kappa_\beta \beta_t$. We study this simple example to simplify exposition.

from 0.2 to 0.9. The slope of supply is one order of magnitude larger than the slope of output, consistent with the proposition. Second, when agents are more attentive to inflation, i.e., variance of inflation noise is small, slope of both curves are steepened by a large magnitude. This is due to the strategic complementary in firms' pricing decision: if the observed inflation is higher, a firm would want to increase its prices, and this in terms lead to higher observed inflation. In summary, when firms observed endogenous signal about output and inflation, supply shocks would be much more inflationary than demand shocks. When firms are attentive to inflation, slopes of the both curves will steepen. In this exercise, we are varying attention to inflation as it is the most exogenous source to vary degree of confusion $C$. However, the lesson shall hold for more general changes that makes agents confusion about a positive demand and a negative supply shock.

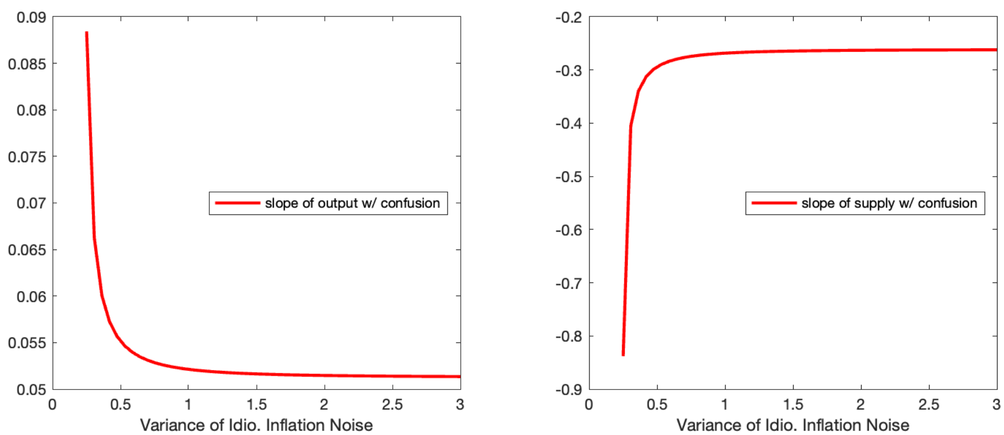


Figure 1: This figure plots the equilibrium slope of the Phillips curve against the variance of the noise in the inflation signal, when the variance of the supply shocks is 0, i.e., $\sigma_s = 0$. The variance of the aggregate demand shock is normalized to 1. This simple example uses calibration from Galí (2015). The variances of the demand shocks, as well as the noises of the signals, are taken from the quantitative model in the next section.

Now, let us look at the model featuring supply shocks. We use the model without supply shock as the benchmark. Defined $\kappa^*, \kappa_a^*$ as the equilibrium slope of the Phillips curve when there is supply shock, i.e., $\sigma_s > 0$. Then:

**Proposition 5 (Flattened and Steepened Phillips Curve)** *Restricting attention to unique equilibrium with* $\kappa > 0, \kappa_a < 0$ *and if expectation of income does not increase wages by too much* $\psi_y < \bar{\psi}_y$*:*

*(1) Confusing positive demand with positive supply shocks (i.e.,* $C > 0$*) flattens the Phillips curve*

*(i.e., $\hat{\kappa} > \kappa^*$).*

*(2) Confusing positive demand with negative supply shocks (i.e., $C < \bar{C}(\kappa_a) < 0$) steepens the Phillips curve (i.e., $\hat{\kappa} < \kappa^*$).* [9]

This proposition states that when the economy features a supply shock, and agents confuse a positive demand shock with a positive supply shock, the response of inflation is further dampened when comparing to the benchmark without supply shock. This can be seen clearly from the following figure.

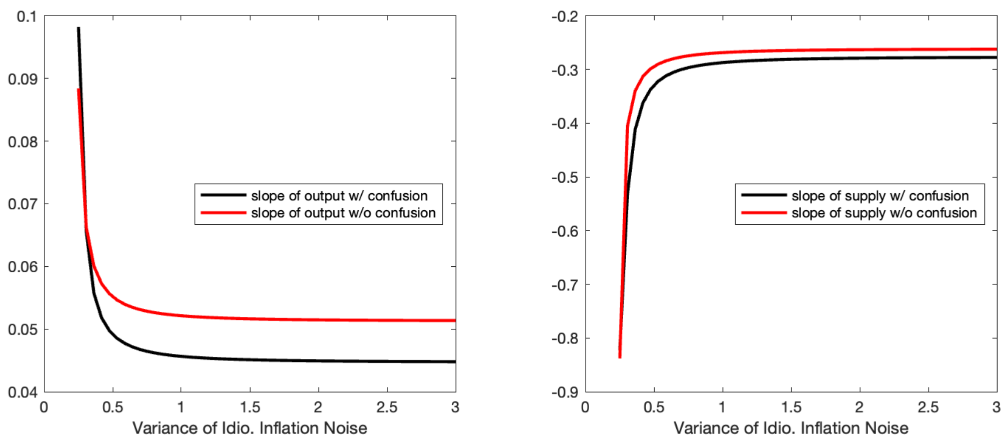


Figure 2: This figure plots the equilibrium slope of the Phillips curve against the variance of the noise in the inflation signal. The variance of the demand shock is normalized to 1. The red line represents the version of the model where the variance of the supply shock is 0. The black line represents the model with both supply and demand shocks. This simple example uses calibration from Galí (2015). The variances of the supply and demand shocks, as well as the noises of the signals, are taken from the quantitative model in the next section.

The left panel of Figure 2 shows how the slope of the output Phillips curve varies when the attention to inflation changes. Proposition 2 tells us that when the attention to inflation decreases, or equivalently, the variance of noise in the inflation signal increases, agents would confuse a positive demand shock with a positive supply shock. When this happens, Proposition 5 states that confusion flattens the Phillips curve. This can be seen by the fact that the red line, representing the case where there is no confusion, is on top of the black line when the variance is large. Again, we are varying attention to inflation as it is the most exogenous source to vary degree of confusion $C$.

[9] See definition of $\bar{\psi}_y$ and $\bar{C}(\kappa_a)$ in the appendix, also see discussions about multiple equilibria in the appendix.

But the lesson shall hold for more general changes that makes agents confusion about a positive demand and a negative supply shock.

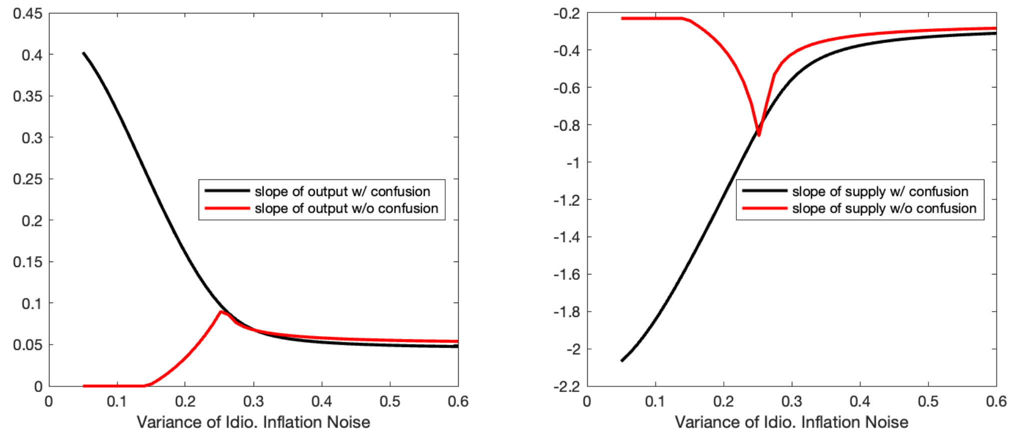


Figure 3: This figure plots the equilibrium slope of the Phillips curve against the variance of the noise in the inflation signal, similar to figure 1. However, this figure zooms into the region where the variance of the noise in the inflation signal is between 0.05 and 0.6 for better visibility.

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On the other hand, figure 3 shows what would happen when attention to inflation continues to increase and when agents confuse a positive demand shock with a negative supply shock. Figure 2 is similar to figure 1, but look at a different range of the variance. It shows that when the variance of noise in the inflation signal decreases further, the slope of output increases dramatically. The right panel shows how the slope of supply factor in the aggregate supply curve changes with attention to inflation. Proposition 2 tells us that when the attention to inflation increases, or equivalently, the variance of noise in the inflation signal decreases, agents would confuse a positive demand shock with a *negative* supply shock. When this happens, Proposition 5 states that confusion steepens the Phillips curve further. As we can see, when agents are attentive to inflation, they respond to the observed high inflation by raising prices, since raising prices is the optimal choice regardless of whether the economy is experiencing a positive demand shock or a negative supply shock. But this quickly becomes self-fulfilling, and hence triggers a large increase in the slope of the Phillips curve.

Readers might be curious to know why the red line starts at zero. Indeed, one might imagine that agents who are hyper-attentive to inflation will trigger a large self-fulfilling inflation. However, as indicated by the second point of proposition 4, *holding other information parameters fixed*, when agents nearly perfectly observe inflation, they do not internalize the fact that inflation is only informative if agents react to their private information. The precise signal they observe *crowds out* other signals. That is, they do not internalize the information externality of their actions. In the extreme, when agents observe inflation perfectly, they only trust the inflation signal and do not react to other information. But when no agents in the economy react to other information, inflation itself has no information content.[10]

In the next section, we investigate such channels in actions by studying a full quantitative model.

# 3 Quantitative Model

The economy is populated by entrepreneurs, each owns a firm that produce a differentiated good. Entrepreneurs maximize the following:

$$\max_{\{C_{l,t},N_{l,t},B_{l,t},P_{l,t}\}} E_{l,t}\sum^{\infty}(\Pi_{s=0}^{k}\beta_{l,t+s-1})(\frac{1}{1-\sigma}C_{l,t+k}^{1-\sigma}+\frac{1}{1+\eta}N_{l,t+k}^{1+\eta})$$

where $\beta_{l,t}$ is the discount factor between $t$ and $t+1$. Entrepreneurs' production function is the following:

$$Y_{l,t} = A_{l,t}N_{l,t}^{1-\alpha}$$

Entrepreneurs face the following budget constraint:

$$B_{l,t} + \int_{\mathcal{B}_{l,t}} P_{j,t}C_{j,t}dj = R_{t-1}B_{l,t} + P_{l,t}Y_{l,t} + T_{l,t}$$

$T_{l,t}$ is government transfer that undoes any heterogeneity in wealth. This is to simplify our analysis by not considering endogenous distribution of wealth due to *individual* information friction and uninsurable income risk, although uninsurable risk stemming from information friction will be an interesting direction for future research. We assume that entrepreneurs are not actively learning from the transfers. Agents does not consume goods from all other entrepreneurs. Instead, in each period, they consume a fixed measure $b$ of random subset of goods available in the economy similar to the setting of Lorenzoni (2009). Consumption for entrepreneurs is a Dixit-Stigliz aggregation of the following form:

$$C_{l,t} = (b^{-\frac{1}{\epsilon}}\int_{\mathcal{B}_{l,t}} C_{l,j,t}^{\frac{\epsilon-1}{\epsilon}}dj)^{\frac{\epsilon}{\epsilon-1}}$$

[10]The x-axis stops at 0.05 for presentation purpose, however, the black line also starts at 0.

where $C_{l,j,t}$ denotes the good that $l$ purchased from entrepreneur $j$. Entrepreneur $l$ only purchases good from a subset $B_{l,t}$ of all entrepreneurs. For each entrepreneur, the relevant price index for consumption is:

$$\bar{P}_{l,t} = (\frac{1}{b}\int_{\mathcal{B}_{l,t}} P_{j,t}^{1-\epsilon} dj)^{\frac{1}{1-\epsilon}}$$

Note that the consumption price index is different from the price that the entrepreneur sells to others, $P_{l,t}$. The demand for good $j$ for entrepreneur $l$ is thereby:

$$C_{j,l,t} = \frac{1}{b}(\frac{P_{j,t}}{\bar{P}_{l,t}})^{-\epsilon} C_{l,t}$$

On the flip side, demand for entrepreneur $l$'s good is:

$$Y_{l,t} = \int_{\mathcal{C}_{l,t}} \frac{1}{b}(\frac{P_{l,t}}{\bar{P}_{j,t}})^{-\epsilon} C_{j,t} dj$$

Where $C_{l,t}$ is the customers whose baskets are matched to entrepreneur $l$'s good. From here, the aggregate price index in the economy is:

$$P_t = (\int_{[0,1]} \bar{P}_{l,t}^{1-\epsilon} dj)^{\frac{1}{1-\epsilon}}$$

For entrepreneur's pricing problem, we assume that there is no explicit nominal rigidity in pricing. We derive entrepreneurs' optimal decision rules here:

$$\begin{aligned}
\lambda_t &= \beta E_{l,t} \lambda_{t+1} \frac{1}{R_t} \beta_{l,t} \\
\frac{1}{C_{l,t}} &= \lambda_t \int_{\mathcal{B}_{l,t}} \frac{1}{b}(\frac{P_{j,t}}{\bar{P}_{l,t}})^{-\epsilon} dj = \lambda \bar{P}_{l,t} \\
0 &= E_{l,t}[\lambda_t P_{l,t} Y_{l,t} - (\frac{\epsilon}{\epsilon - 1} \frac{1}{1-\alpha} \frac{(Y_{l,t}/A_{l,t})^{\frac{\eta+\alpha}{1-\alpha}}}{A_{l,t}} Y_{l,t})]
\end{aligned}$$

The first and second equations shows entrepreneurs' tradeoff between consuming today or tomorrow. The third equation balances the marginal revenue that entreprenueurs get from changing prices, and disutility of working from producing goods. From now on, we denote lower letter as log-deviation from steady state. Market clearings imply: $y_t = \int_0^1 y_{l,t} di$, $\int_0^1 b_{l,t} di = 0$, $p_t = \int_0^1 p_{l,t} di$ and $\pi_t = p_t - p_{t-1}$.

Central bank sets interest rate to contaminate inflation, and to close the output gap. They use the standard Taylor rule. We assume that the central bank receives the same signals about aggregate demand and supply shock as the entrepreneurs, and set interest rate conditional on the true realization of $y_t$ and $\pi_t$[11]. In log-linear form:

$$r_t = \rho r_{t-1} + \phi \pi_t + (1-\rho)\phi_y (y_t - E_t^{cb} y_t^*)$$

There is no reason to assume that the central bank has perfect information about the potential output, $Y_t^*$ of the economy[12]. Therefore, the potential output of the economy enters into the monetary policy rule with central bank's expectation.

One innovation of this model is to realistically consider multiple aggregate shocks simultaneously. Agents might wrongly attribute origins of the economy fluctuation to different fundamental shocks due to the classic econometric identification problem of supply and demand curves. Such confusion will either dampen or amplify the response of inflation and output. Broadly, aggregate shocks in the economy can be characterized into two categories: demand shocks and supply shocks. Example of demand shocks includes financial deleveraging shocks[13], investment specific technology shocks, TFP news shocks, monetary policy shocks, etc. Example of supply shocks includes oil shocks, supply chain disruption, transitory TFP shocks, or any types of shocks that directly impact marginal costs. To cleanly discuss these two types of shocks into the the economy, the model incorporate two shocks, discount rate shock and TFP shock that represents demand shocks and supply shocks respectively. The two shocks follows an AR(1) law of motion: $\ln \beta_{l,t} = \rho_d \ln \beta_{l,t-1} + \epsilon_t^d + \epsilon_{l,t}^d$ and $\ln A_{l,t} = \rho_a \ln A_t + \epsilon_t^a + \epsilon_{l,t}^a$.

With $\epsilon_t^d$ being the true innovation of aggregate demand shock innovation idiosyncratic demand shock. $\epsilon_t^a$ is innovation of aggregate supply shock and $\epsilon_{l,t}^a$ is innovation of idiosyncratic supply shock. Entrepreneurs information about the underlying shocks that change the economy will be discussed below.

**Solution concept** We log-linearize the system, and solve a linearized system of equation. Each entrepreneur will exogenous learn about the aggregate economy from strength of their productivity, $A_{l,t}$, and their own demand shock $\beta_{l,t}$. They will also learn from the quantity and price of goods they buy and sell. Eventually, demand of their own good, prices they observed from the basket, and

[11]We could introduce noises in central bank's observed output and inflation, this will effectively make the central bank confused too, and help us matching the data by introducing more degree of freedoms. However, to highlight channels, we will leave out this consideration

[12]Defined as the first best outcome of the economy absent of frictions.

[13]See Guerrieri and Lorenzoni (2017)

potential news coverage about the output and inflation are a linear combination of aggregate prices, output and idiosyncratic shocks[14]. By rotating the linear combination of the signals[15], we denote the output signal entrepreneurs observe as $y_{l,t}$ and inflation signal as $\pi_{l,t}$ with $\tilde{y}_{l,t} = y_t + \xi^d_{l,t}$ and $\tilde{\pi}_{l,t} = \pi_t + \xi^\pi_{l,t}$, where $\xi^y_{l,t}$ and $\xi^\pi_{l,t}$ are results of random consumption basket, and potentially source of news such as releases from statistics agency. All shocks, including noises in signals are assumed to be independent and normally distributed, with mean zero and standard deviation specified in the calibration table. The solution concept of the model is incomplete information rational expectation: similar to the seminal work of Grossman and Stiglitz (1980) that pioneered rational expectation, entrepreneurs observe signals and make their decisions that ultimately constitute equilibrium outcomes. The interpretation similar to Vives (2017): entrepreneurs plan their pricing and demand schedule as a function of aggregate price, output, idiosyncratic and aggregate shocks.

To summarize, every entrepreneur learn from their own supply factor, and will also learn from prices, quantity and news about aggregate demand shock. The central bank receives the same exogenous signals about aggregate demand and supply shock as the entrepreneurs. We do not assume that entrepreneurs learn from the interest rate. There are two reasons for this choice: first of all, information effect of the monetary policy is not the focus of this paper. Previous work such as Lorenzoni (2009) used noises in central bank's observed inflation, or other added noises to make interest rate less informative for agents. For the purpose of highlighting the channels, it is not worthy to introduce more noise to the system with our goal of cleanly delivering the message. Secondly, there is an ongoing debated over whether households and firms are learning from interest rates — the controversial "Fed information effect" Bauer and Swanson (2023). It is not settled yet about whether agents are learning from monetary policy, and we will leave discussion about central bank's persuasion and information effect in an endogenous information environment for future work.

[14] See Lorenzoni (2009) for a detailed discussion.

[15] From entrepreneurs' random consumption basket $\bar{p}_{l,t} = p_t + \xi^p_{l,t}$. From demand of entrepreneurs' output, $y_{l,t} = y_t + \epsilon(p_{l,t} - p_t) + \xi^d_{l,t}$. With $\xi^p_{l,t}$ and $\xi^d_{l,t}$ being the realizations of the sampling shocks. Note that it is informational equivalent to linearly combine these two signals, together with the fact that entrepreneurs know the price they set $p_{l,t}$, and write it as $\tilde{y}_{l,t} = y_t + \xi^y_{l,t}$ and $\tilde{p}_{l,t} = p_t + \xi^p_{l,t}$.

### 3.1 Calibration and Results

The model features dispersed, heterogeneous and endogenous information. Information received by firms, and consequently their expectation, are partially determined by the expectation of other firms. Therefore, agents has to "forecast the forecasts of others", as pointed out by Townsend (1983). This class of model features interest dynamics, such as those observed by Keynes, where financial market features "waves of optimism and pessimism". But as a cost, the problem becomes very difficult to solve. Many paper has shown such models in general, has no analytically solution (Huo and Takayama (2015), Kasa (2000)). We build on the recent development of utilizing frequency domain methods to solve this model (Han et al. (2022), Adams (2019)).

| Parameter | Description | Value | Source |
|---|---|---|---|
| $\gamma$ | risk aversion | 1 | - |
| $\eta$ | inverse Frisch elasticity | 1 | - |
| $\alpha$ | decreasing return | 0.33 | - |
| $\epsilon$ | good elasticity of sub. | 6 | s.s. 20% markup |
| $\rho$ | Taylor rule persistent | 0.75 | Smet and Wouters (2007) |
| $\phi_\pi$ | Taylor rule inflation | 1.678 | Melosi (2017) |
| $\phi_y$ | Taylor rule output gap | 0.673 | Melosi (2017) |
| $\sigma_a$ | aggregate TFP shock s.d. | 0.014 | Kahn and Thomas (2006) |
| $\sigma_{ai}$ | idio. TFP shock s.d. | 0.35 | Decker et. al. 2020 |
| $\sigma_m$ | Demand shock s.d. | 0.0368 | Melosi (2017) |
| $\sigma_{di}$ | relative demand shock s.d. | 0.349 | Melosi (2017) |
| $\rho_a$ | TFP shock persistence | 0.859 | Prescott et al. (2005) |
| $\rho_m$ | Demand shock persistence | 0.804 | Melosi (2017) |
| $\sigma_y$ | disagreement of perceived output | 0.0908 | McClure et al. (2023) |
| $\sigma_\pi$ | disagreement of perceived inflation | 0.0755 | McClure et al. (2023) |

The table shows the calibration of this model. Standard parameters, such as risk aversion, Frisch elasticity and decreasing return to scale, are taken from textbook calibration. We calibrate the elasticity of substitution between goods to match 20% markup in steady state. Given that the only paper that uses Bayesian method to estimate a monetary model with dispered information is

Melosi (2017), we use estimates from Melosi (2017) to calibrate moments that are not possible to get from micro level evidence. These parameters include Taylor rule coefficients, variance of aggregate shocks, and idiosyncratic demand shock. To calibrate the idiosyncratic TFP shock, we look at the recent estimate from Decker et al. (2020) that uses US census data to estimate dispersion of TFP. Recently. McClure et al. (2024) has ask price setter in a firm, about their perceived current inflation and output (measured by unemployment). We use these estimates to calibrate the signal to noise ratio for inflation and output signals.

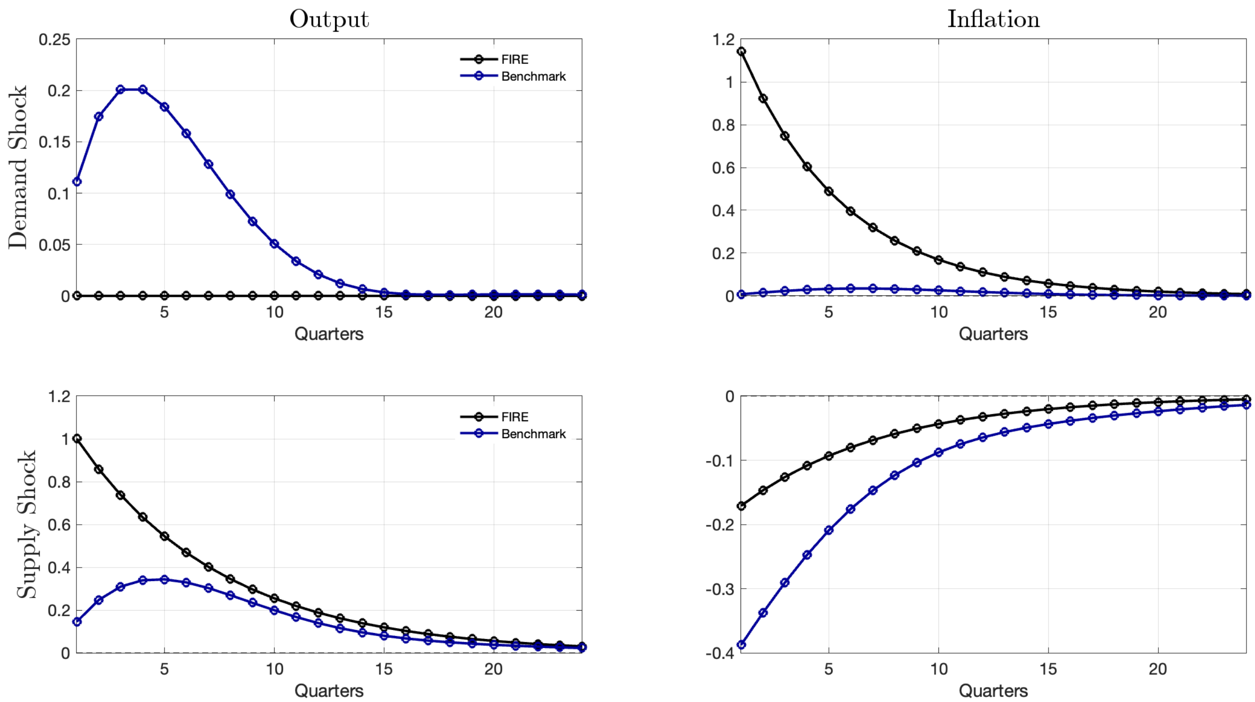


Figure 4: First and second row plots impulse responses of inflation and output, for full information and disperse information models, to demand shock (discount factor) and supply shock, respectively. Black curve represents the response of a model with full information rational expectation, The navy curve represents the response of disperse information model with benchmark calibration.

**Monetary Non-neutrality** Figure 4 shows responses of the model to demand and supply shocks. In figure 4, the black curve shows the response of the model with *full information rational expectation* (FIRE). The first panel shows the response of both the full information model, represented in black, and the calibrated model, represented in blue, to a 1% demand shock. The second row shows the responses of the models to a 1% supply shock. Note that when the full information economy is hit by a demand shock, inflation response aggressively, while output does not move. This is the famous *monetary neutrality* result (Lucas (1972). With complete information and without explicit pricing friction such as Calvo or menu cost, demand shock such as monetary policy shock does not affect output. With incomplete information model, output responses significantly while inflation increases slightly.

**Non-inflationary Demand Shock, Business Cycle Decomposition** Apart from the flattening and steepening of the Phillips curve, our model address one empirical puzzle: inflation rarely response to inflation when the economy is hit by a demand shock, while inflation response much more to a supply shock Angeletos et al. (2020). This model matches the variance decomposition of the shock that drives most of the business cycle. Angeletos et al. (2020) shows that the shock that drives most of the business cycle are almost completely disconnected with variation in inflation, suggesting that most of the business cycle variation is *driven by demand shock*. The main shock that drives most of the variation in business cycle explains 73.71% of the variation in unemployment and 58.51% in output, in business cycle frequency. In our model, the demand shock, by definition disconnect from variation in supply shock, explains 62.56% of the output variations in business cycle frequency.

In terms of inflation, Angeletos et al. (2020) shows that the shock that drives most of the output and unemployment variation in business cycle causes very slight change in inflation. It is puzzling from the perspective of the New Keynesian model: unless the Phillips curve is exceedingly flat, demand shock should cause a large response in inflation. In our benchmark model, demand shock explains 10.03% of inflation in this economy, while Angeletos et al. (2020) find that the main business cycle shock explains 6.96% of variation in inflation in business cycles.

Overall, our model matches business cycle puzzles that are otherwise difficult to explain with the New Keynesian paradigm.

**Strongly inflationary supply shock** Another puzzling fact about the business cycle fact is the following: demand shocks are no-inflationary and there are still sizeable inflation variation in the economy. One explanation is a very flat Phillips curve. However, if the Phillips curve is exceedingly flat, inflation will not be varying much. To accommodate this puzzle, previous research, such as Smets and Wouters (2007) uses "cost push" or "markup" shock that affects prices without entering into the output gap. But these notions exist only because we need to use them to match data in quantitative model: conceptually, if a shock changes a firm's marginal cost, it should enter into prices via the slope of the Phillips curve. Taking the estimate of Hazell et al. (2022) and Del Negro et al. (2020) as an example, to generate 1% changes in inflation, the desired markup has to increase by 575%, while the cost push shock has to increase by 500% (L'Huillier and Phelan (2023)). In our model, a demand shock that generates 1% increase in output only causes roughly 0.18% inflation at peak. While a supply shock that generate 1% in output causes roughly 1.2% decrease in inflation. Our model effectively matches the empirically desired feature without adding mechanical shocks.

**Mechanism** Why does this model perform well in matching the empirical facts? To see the channel cleanly, let us look at dynamic response of the average beliefs to different shocks. Figure 5 shows the agents' average belief of the underlying shocks, nowcast error of output, and nowcast error of inflation. One the first panel, we show the response of beliefs when a demand shock happens. When the economy is hit by a positive demand shock, agents partially attributes it to a positive aggregate supply shock (first item in the first row) partially attributes it to a positive demand shock (second item in the first row), and partially to idiosyncratic shocks. As a result, they do not increase their price by much. Their perceived output is also lower than the complete information counterpart. The inertia in increasing inflation creates room for demand shock to have large real effect.

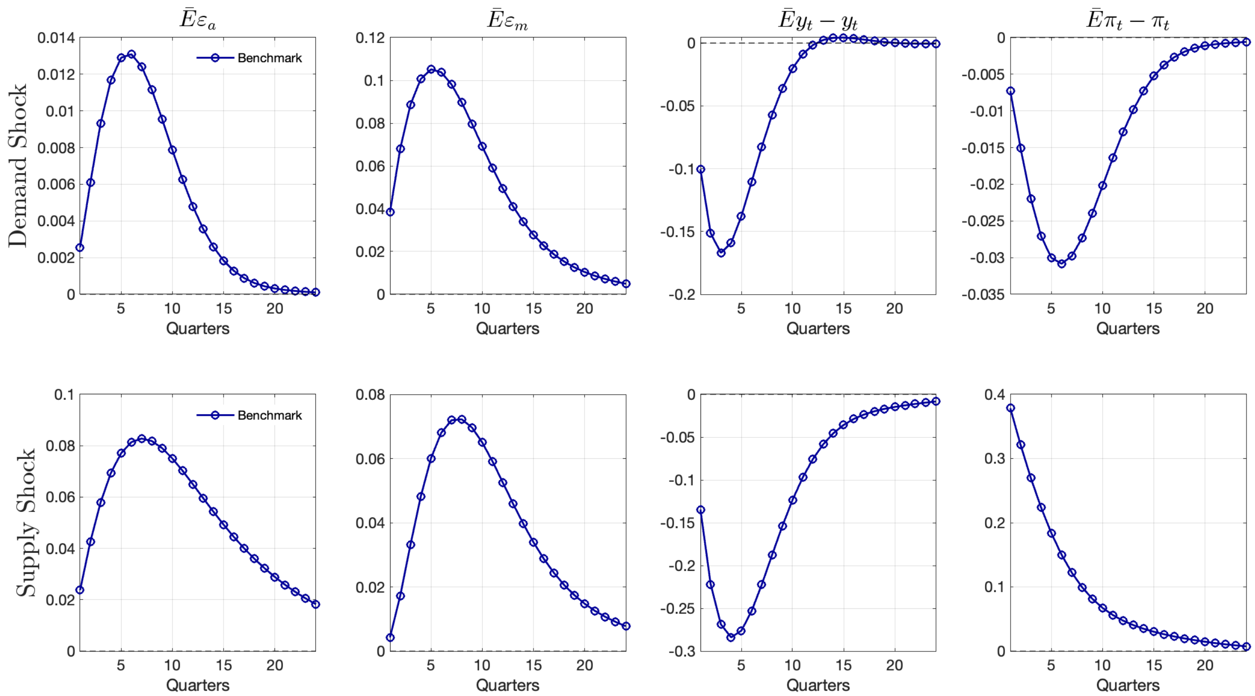


Figure 5: First and second row plots impulse responses of beliefs about underlying shocks and endogenous variables, for the disperse information model, to demand shock (discount factor) and supply shock, respectively. The navy curve represents the response of disperse information model with benchmark calibration.

Similarly, when a supply shock happens, agents partially attributes it to a positive supply shock (figure 5 first figure in the second row), partially attributes it to a positive demand shock (figure 5 second figure in the second row), and partially to idiosyncratic shocks. However, unlike the case of a demand shock, firms marginal costs are concretely changed by the supply shock, using the pricing rule similar to section 2, they would lower their price by $\psi_a a_t$, with $\psi_a$ being defined in section 2. But given that the perceived output $y_t$ and price level $p_t$ in the economy is dampened, firm would not increase their price by much. Consequently, we see a much larger inflation response in the dispersed information model.

**Salient Events and De-anchored inflation** Next, we turn to the situation where agents are more attentive to inflation, due to some exogenous events such as the OPEC oil crisis, collapse of the Bretton Wood, Covid-19 pandemic, the Ukraine energy crisis, and so on. If these events drove up people's concern about inflation, agents would pay more attention to inflation. Recent survey and randomized control trial evidence supported this view Weber et al. (2023). The RCT evidence show that survey respondents' perception about inflation are much more precise when inflation is high. We mimic the RCT evidence on attention to inflation under high inflation regime, by decrease the noise in the signal of inflation by 2.5 times. Figure 6 plots the response of the economy when salient event happens, against the benchmark dispersed information economy. Again, first row shows the response of output and inflation to a demand shock, and the second row shows the response of output and inflation to a supply shock. The yellow curve represents the model with higher inflation attention. When a demand shock happens, inflation reacts close to twice as much as the benchmark case. If we normalize the demand shock such that it induces a 1% increase in output for the benchmark economy, inflation react from a peak of roughly 0.2% to roughly 0.35%. Additionally, inflation remains high even after 4 years (16 quarters).

Similarly, when a supply shock happens, inflation response much more than the benchmark case. Normalizing the output response to 1% in the benchmark case, inflation's peak increase from 1.2% to 1.7% when comparing the benchmark economy to the salient economy. Unlike the case of demand shock, where the benchmark and salient economy has similar response, supply shock induces a much larger response in output – output responses over 40% more.

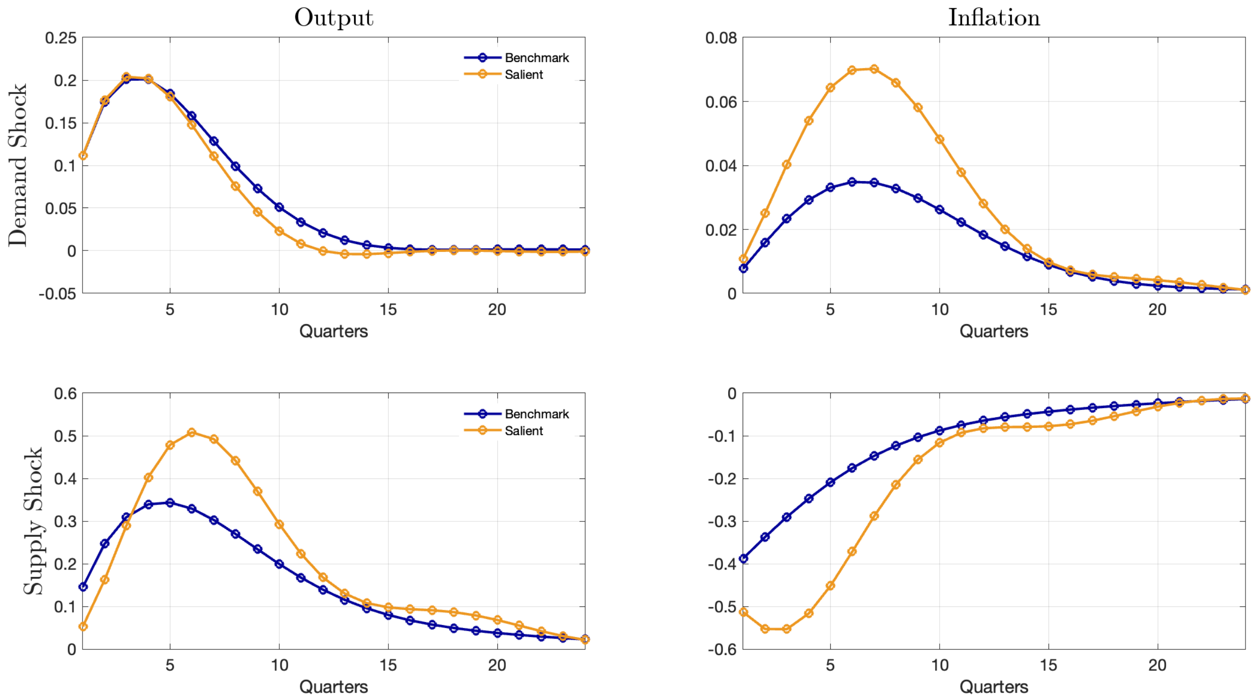


Figure 6: First and second row plots impulse responses of inflation and output, for two disperse information models, to demand shock (discount factor) and supply shock (TFP),respectively. The navy curve represents the response of disperse information model with benchmark calibration. The yellow curve plots the disperse information model with a more precised inflation.

**Mechanism for De-anchoring** Now, let us decompose the channels of this phenomenon. Again, we zero in the dynamic response of belief when demand and supply shocks happen. Figure 7 plots the evolution of beliefs when demand and supply shocks happen. And the yellow curve represents the response of the salient economy. First, when we look at the response of a demand shock, as opposed to the benchmark economy where agents are confused about positive demand shock and positive supply shock, they are now confusing a positive demand shock with a *negative* supply shock. According to the channel highlighted in the simple model, this leads to a much larger response in inflation. When looking at the response of supply shocks, the pattern is similar: agents confused a positive supply shock with a *negative* demand shock. From their pricing rule, we know that the household would want to increase their price in any cases.In terms, this increase the inflation observed by other agents, and they themselves would want to increase the price by even more. Eventually, this confusion leads to the de-anchoring of inflation.

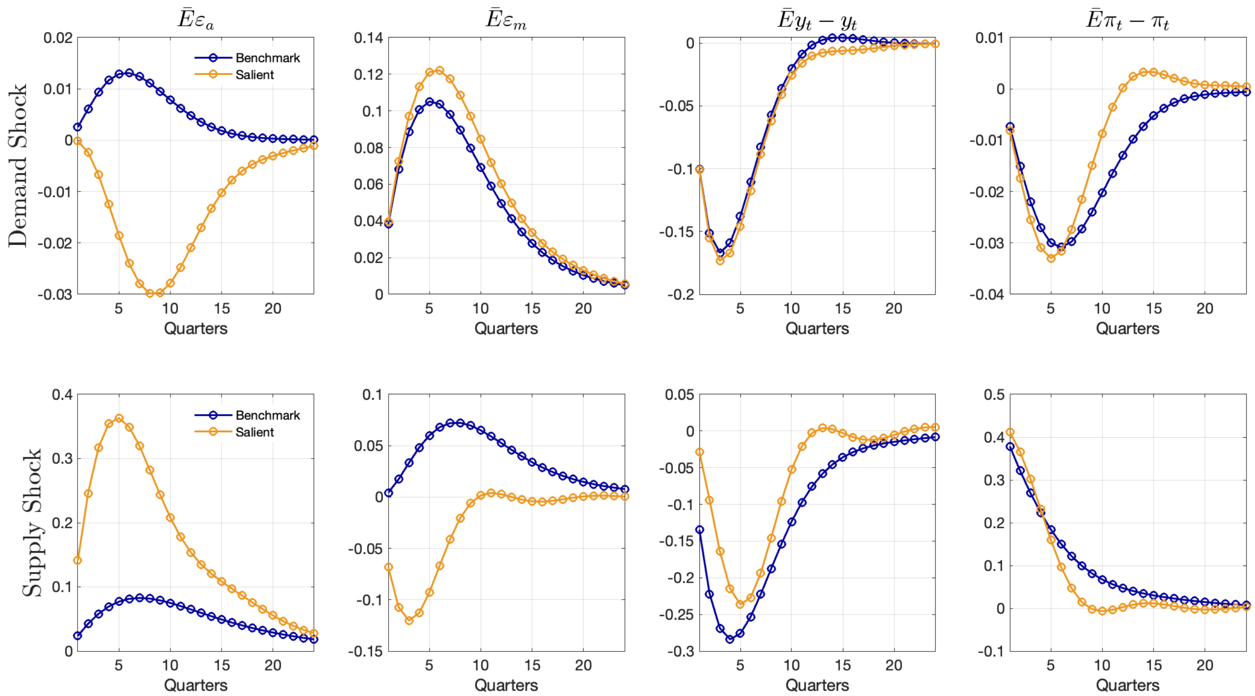


Figure 7: First and second row plots impulse responses of beliefs about underlying shocks and endogenous variables, for two disperse information models, to demand shock (discount factor) and supply shock (TFP), respectively. The navy curve represents the response of disperse information model with benchmark calibration. The yellow curve plots the disperse information model with a more precised inflation.

In summary, this experiment tells us that in normal time, agents confuse positive demand shock with a positive supply shock. We explains why demand shocks are non-inflationary, while supply shocks are much more inflationary, resolving the issue with inflation's response when the Phillips curve is believed to be flat. When salient event happens, and agents are more attentive to inflation, it confuses agents in an opposite direction. But now, raising prices is a best response regardless. Belief becomes self-fulling and leads to a de-anchored inflation dynamics.

## 3.2 Managing De-anchored Inflation

What should the central bank do when de-anchored inflation happens? In our context where the de-anchored inflation is a product of expectation, the central bank plays a crucial role in managing strategy and information complementarity arising from endogenously learning. We will first use our simple model to illustrate the role of the central bank. From the simple model in section 2, we have:

$$y_t = y_{t+1} - \frac{1}{\sigma}(\phi\pi_t - \pi_{t+1} - \beta_t)$$
$$\pi_t = \bar{E}_t\pi_t + \psi_y\bar{E}_t y_t - \eta\psi_a\bar{E}_t a_t - \psi_a a_t$$

If we substitute the Euler equation into firm's pricing rule, and making the same assumption that the underlying shock will be revealed in the next period:

$$\pi_t = (1 - \psi_y\frac{\phi}{\sigma})\bar{E}_t\pi_t + \chi_a\bar{E}_t a_t + \chi_\beta\bar{E}_t\beta_t - \psi_a a_t$$

where $\chi_a$ and $\chi_\beta$ summarizes the impact of average belief of aggregate shocks coming from average expectation of $y_{t+1}$, $\pi_{t+1}$ and $\beta_t$ to ease exposition.

We see that $1 - \psi_y\frac{\phi}{\sigma}$ decreases in $\phi$. If $\phi$ is small, the pricing rule is strongly complement in the belief about inflation. If agents see an increase in inflation, they will want to increase it further. Adding on top of the literature on "beauty contest" game with exogenous information (Morris and Shin (2002), Angeletos and Huo (2021)) is the fact that: apart from complementary coming from fundamental complementarity, there is also information complementarity in this model: even if the pricing rule is independent of belief about aggregate prices, the fact the one's action becomes another's signal already exhibit a large degree of complementarity. These forces leads to a strong degree of strategic complementarity, and lead to the de-anchoring of inflation.

However, If $\phi$ is large, the forces of strategic complementarity decreases. Conceptually, if agents is not sure if there is any forces in contaminating inflation, it is easy to see inflation being self-fulfilling. However, if agents belief that there are strong forcess in the economy that will stop high inflation from happening, they would not be worried about high inflation scenarios, and will consequently not have to increase their price in response.

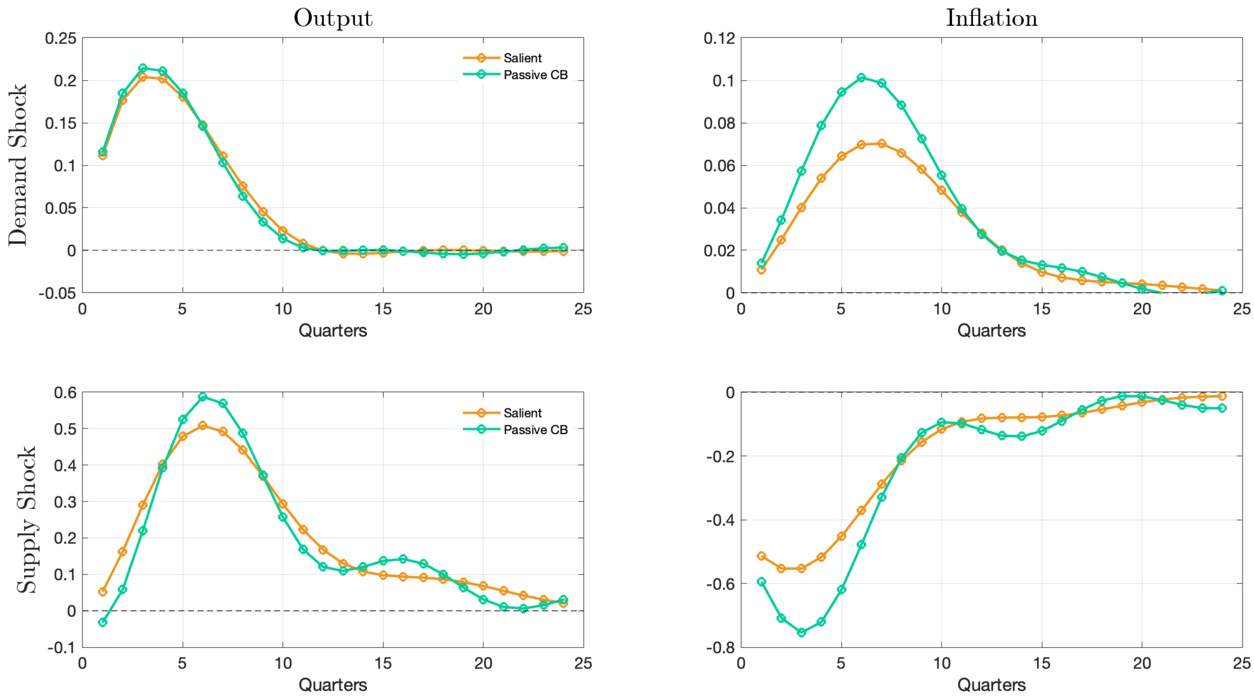


Figure 8: First and second row plots impulse responses of beliefs about underlying shocks and endogenous variables, for two disperse information models, to demand shock (discount factor) and supply shock (TFP), respectively. The navy curve represents the response of disperse information model with benchmark calibration. The yellow curve plots the disperse information model with a more precised inflation.

Figure 8 put this logic in action: here, we perturb the Taylor rule coefficient slightly when the agents are already attentive to inflation, set it from 1.678 to 1.478, a decrease of 0.2. That is, the central bank reacts slightly more accommodative to inflation. With a small decrease in the coefficient, inflation de-anchored much more: in response to a 1% increase in output led by a demand shock, inflation jumps to 0.5% instead of 0.35%. Compared to the benchmark economy, inflation response increases from 0.2% to 0.5% percent.

Similarly for supply shock, if we normalize the size of supply shock such that it induces a 1% increase in GDP, inflation response increases from 1.2% to 1.7% between the benchmark and salient economy. With an accomodative central bank, inflation increases to almost 2.4%, twice as much as the already large response from the benchmark economy.

**Confounding Dynamics: Failure of Information Aggregation** Another interesting fact about this economy is the fact that supply shock induces an oscillating behaving in inflation and output: output and inflation features high volatility when facing a supply shock. This phenomena is called confounding dynamics (Rondina and Walker (2017)) in the literature: technically, it means that there are inside roots in the system. Economically, it implies that agents rationally learn more from inflation since inflation is perceived to be more information, and under-response to their private signals. However, they do not internalize the fact that information content in the aggregate inflation is in fact a product of agents' own private signals. In the extreme, if nobody responses to their private information about marginal cost, the aggregation inflation will also contain no information about the aggregate supply shock. Since agents pay a lot of information to inflation, they can not be systemically under-react or over-react to inflation. Eventually, beliefs and the actual outcomes have to fluctuate, and creates a volatile response. Figure 9 shows this mechanism in action

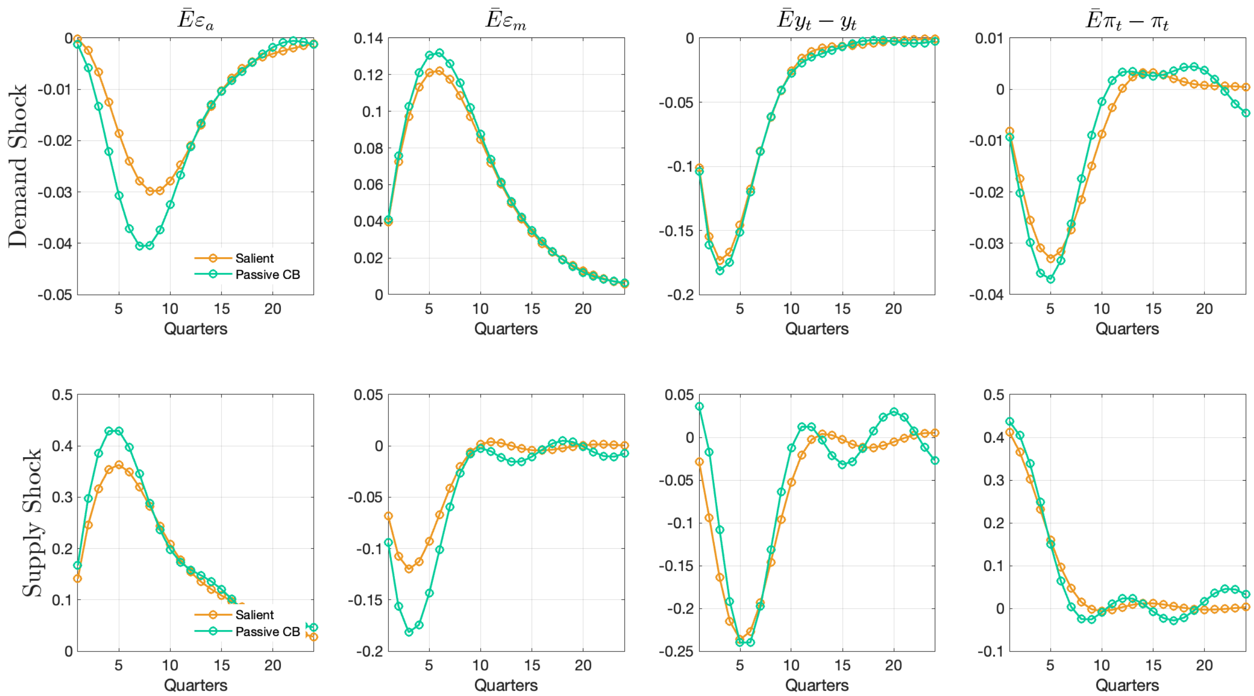


Figure 9: First and second row plots impulse responses of beliefs about underlying shocks and endogenous variables, for two disperse information models, to demand shock (discount factor) and supply shock (TFP), respectively. The navy curve represents the response of disperse information model with benchmark calibration. The yellow curve plots the disperse information model with a more precised inflation.

**Hawkish Response to Inflation** As our simple illustration pointed out, if the central bank response more aggressive to inflation. It alleviates some of the de-anchoring by limiting the role of coordination coming from both fundamental and informational complementarity. Figure 10 examplifies this insight. Here, we change the responsiveness to inflation from 1.678 to 1.878, an equal size change as before, of 0.2. From figure, we can see that inflation response less to both demand and supply shocks, when comparing to the salient economy. However, we see that there is a large degree of asymmetry for passive or aggressive response to inflation. The discussion of optimal policy in this economy is left for a future paper.

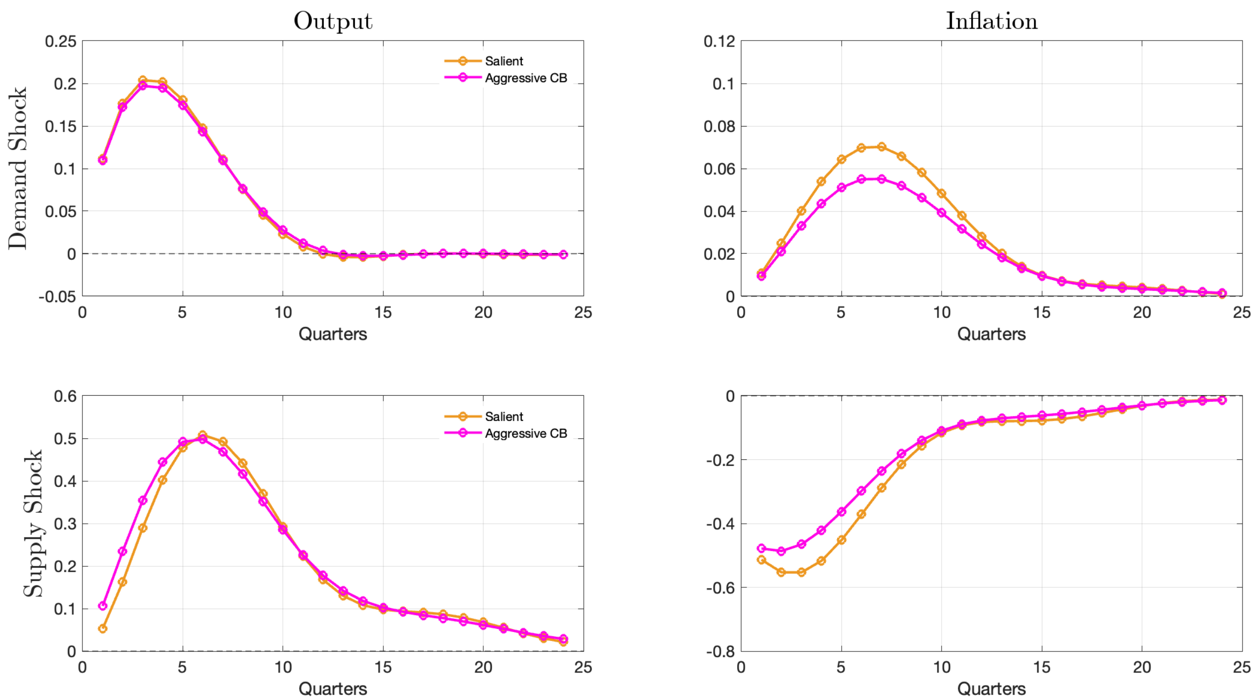


Figure 10: First and second row plots impulse responses of beliefs about underlying shocks and endogenous variables, for two disperse information models, to demand shock (discount factor) and supply shock (TFP), respectively. The navy curve represents the response of disperse information model with benchmark calibration. The yellow curve plots the disperse information model with a more precised inflation.

# 4 Empirics

To investigate the role of information friction in macroeconomic dynamics, we employ a structural VAR strategy similar to Primiceri (2006). This paper is one of the most influential papers in showing that the Phillips curve, or the co-movement between inflation and unemployment had a structural shift before and after 1990s. We add on the evidence by incorporating agents' perceived inflation and unemployment, measured by nowcast error of inflation and unemployment, into the SVAR, aiming to uncover the role of nowcast error in this dynamics.

The SVAR utilize an 10-variable VAR model encompassing: civilian unemployment rate; CBO's natural unemployment estimate; core inflation via the Personal Consumption Expenditures price index (excluding food and energy); GDP deflator-based inflation; per-capita real GDP; per-capita hours worked; wage inflation from average hourly earnings of production and nonsupervisory employees; labor share of GDP; Excess bond premium constructed by Gilchrist and Zakrajšek (2012); inflation nowcast error of SPF measured by active inflation minus average preceived inflation; unemployment nowcast error of SPF measured by active inflation minus average unemployment inflation. This model not only covers broad economic variables like GDP and labor share but also focuses on wage dynamics through the less volatile PNSE wage inflation series, alongside core PCE inflation to reflect underlying inflationary trends.

Following Primiceri (2006), this analysis spans two distinct periods—1964:II to 1989:IV and 1989:I to 2019:III—to explore shifts in these variables' interrelations. The segmentation at 1990 balances between earlier macroeconomic shifts and the stabilization of inflation in the mid-1990s, offering comparably sized samples for robust analysis. Employing Bayesian methods with a Minnesota prior for estimation, the VAR's 4-lag structure accommodates the data's quarterly frequency, with prior tightness determined by Giannone et al. (2015)'s data-driven approach. Using a Cholesky identification approach and order EBP as the last variable, we look at the response of innovation to EBP orthogonal to toher variables in the system. As argument by Primiceri (2006), Gilchrist and Zakrajšek (2012), the idea is to get a shock primarily shifting demand component of the economy. And innovation to EBP has been show to exhibit such property.

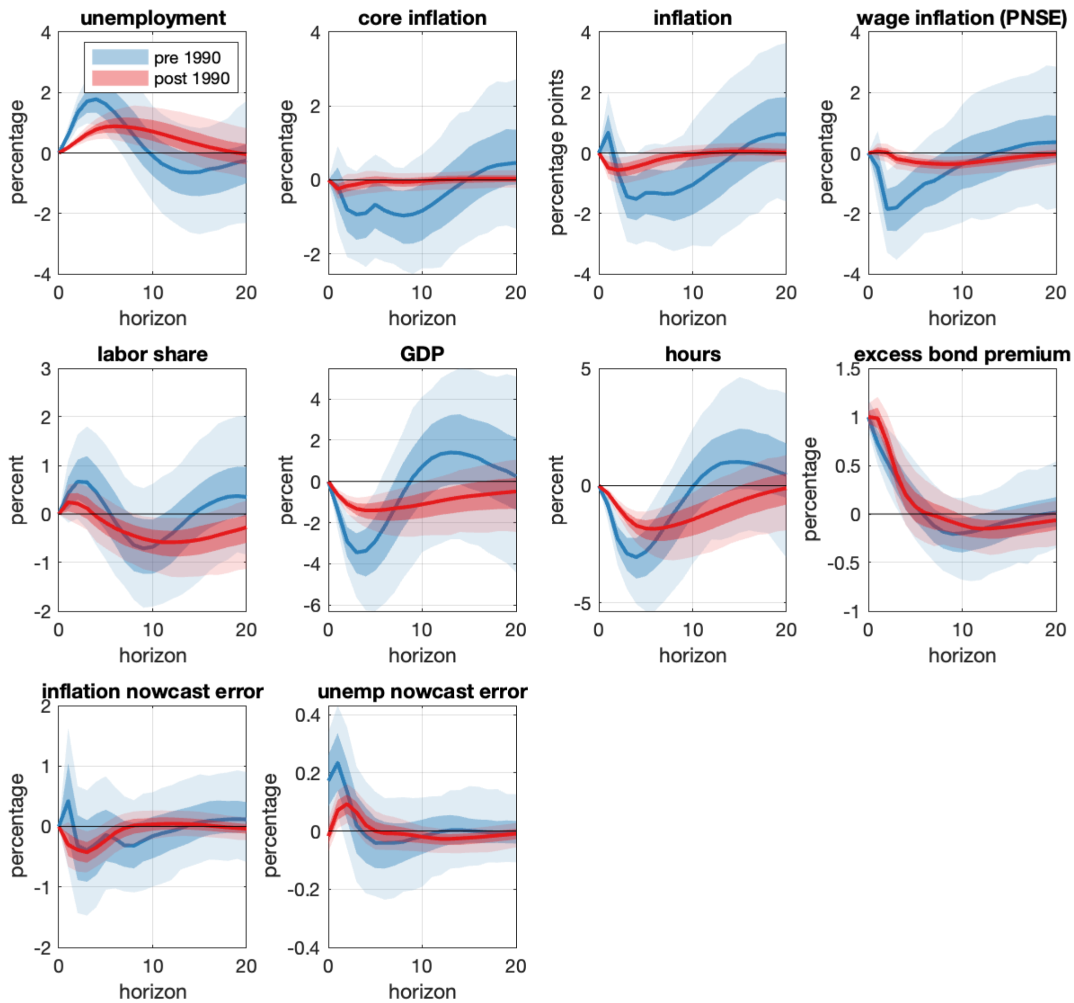


Figure 11: Impulse response to a demand shock (EBP) shock for two sample period. Pre-1990 sample starts from 1973 and ends at 1989. Post-1990 sample starts from 1990 and ends at the third quarter of 2019. Cholesky identification with shaded area correspond to 68 and 95 credible regions. Inflation nowcast error is defined as the difference between actual inflation and average nowcast inflation from Survey of Professional Forecasters (SPF). Unemployment nowcast error is defined as the difference between actual unemployment rate and average nowcast unemployment rate from Survey of Professional Forecasters (SPF).

There are two phenomena worth noting here: in response to the shock, inflation was much more active in the pre 1990 sample. In responding to this negative demand shock, core inflation decreased by 1 percent and wage inflation decreased by 2 percent at peak. In the post 1990 sample, however, there is barely any movement in inflation and unemployment. There is another salient feature: unemployment nowcast error jumps for the pre 1990 sample, and remain two times as large as the post 1990 sample at peak. It shows that professional forecasters' belief on unemployment under-responded. However, inflation nowcast error is statistically insignificant — professional forecasters knows current inflation, but largely gets unemployment wrong. Post 1990, however, exhibit a different pattern: inflation nowcast error is significantly negative, while unemployment nowcast error is significantly positive — professional forecasters think inflation is not moving, and thinks that unemployment is lower than it actually is.

In sum: pre 1990, professional forecasters have been largely getting inflation correct, but perceived unemployment incorrectly. Post 1990, however, they perceived both inflation and unemployment wrongly. This evidence immediately rejects full information rational expectation, under which we should see a zero response in nowcast errors. Literature on macroeconomic expectation has establish that, unsurprisingly, professional forecasters are closest to full information expectation, compared to firms and households. Therefore, the evidence here might be a conservative estimate on the actual information friction among agents in the economy.

## 5 Conclusion

In conclusion, our exploration of inflation dynamics and the Phillips curve within a framework that incorporates firms' expectation frictions and endogenous learning offers an alternative perspective on business cycle puzzles. We highlight that the Phillips curve's slope is not only a fundamental object, but varies based on information friction stemming from endogenous learning. By allowing for multiple types of shocks—demand and supply—and recognizing the limits of firms' knowledge about the economy, we account for the observed disconnect between inflation and real variables during certain periods.

Our findings suggest that the traditional view of the Phillips curve may oversimplify the complex interplay between inflation, output, and agents' expectations. The episodic steepening of the curve during periods of economic turmoil, such as the 1970s oil shocks or the COVID-19 pandemic, underscores the role of expectation-driven dynamics. The model also sheds light on the phenomenon of de-anchored inflation, where firms and households rely heavily on observable inflation signals, potentially leading to self-fulfilling inflationary spirals.

The policy implications of our model underscores the importance of central bank's reaction to inflation, in different period of time. When agents are not learning much from inflation, it is not as harmful to react accommodatingly to inflation. When agents are learning more from inflation it is particularly important for the central bank to react aggressively to inflation. Discussion about optimal policy is left for future research.

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# A Details of the Simple Model

In this section, we study a slightly more general model: we allow shocks to be an AR(1) process, with demand shocks having a persistent of $\rho_d$ and supply shocks $\rho_a$. Since household understands that firms know all the underlying shocks in the next period, the next-period inflation and output should be determined by the full-information rational expectations equilibrium (FIRE). Therefore, household's Euler equation rewrites,

$$\begin{aligned} y_t &= -\phi\pi_t + n_s a_t + n_d \varepsilon_t^m \\ n_s &= \Phi^{sy}\rho_s + \Phi^{s\pi}\rho_s;\ n_d = \Phi^{d\pi}\rho_d + 1 \end{aligned} \tag{3}$$

where $\Phi^{sy}$, $\Phi^{s\pi}$, $\Phi^{d\pi}$, $\Phi^{dy}$ are the impulse response of the system under full information. For example, $\Phi^{sy}$ is the FIRE impulse response of output to a supply shock. $\Phi^{dy} = 0$ and is omitted in the equation. In particular, $\Phi^{d\pi} = \frac{1}{\phi - \rho_d}$, $\Phi^{sy} = \frac{\psi_a}{\psi_y}(1+\eta)$, $\Phi^{s\pi} = -\frac{\sigma\psi_a(1-\rho_a)}{\psi_y(\phi-\rho_a)}(1+\eta)$. When shocks are transitory, $n_s = 0, n_d = 1$. Monetary policy and discount rate shocks only differ in terms of variances. Combining equations (2) and (3), we can solve for the equilibrium response of output and inflation to aggregate demand and supply shocks $\chi^{yd}, \chi^{ys}, \chi^{\pi d}, \chi^{\pi s}$ with two unknowns $\kappa$ and $\kappa_a$,

$$\chi^{yd} = \frac{n_d}{1+\kappa\phi}, \chi^{\pi d} = \frac{n_d\kappa}{1+\kappa\phi}, \chi^{ys} = \frac{n_s - \kappa_a\phi}{1+\kappa\phi}, \chi^{\pi s} = \frac{n_s\kappa + \kappa_a}{1+\kappa\phi}$$

Signals are projected to underlying shocks,

$$\begin{pmatrix} x_{it}^1 \\ x_{it}^2 \end{pmatrix} = \underbrace{\begin{pmatrix} \sigma_d\chi^{yd} & \sigma_s\chi^{ys} & \sigma_y & 0 \\ \sigma_d\chi^{\pi d} & \sigma_s\chi^{\pi s} & 0 & \sigma_\pi \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} \eta_t^d \\ \eta_t^s \\ \eta_t^y \\ \eta_t^\pi \end{pmatrix}}_{s_{it}}$$

$B$ is the projection matrix. Using the optimal steady state filter Han et al. (2022), the expectation of output and inflation will be projected to these two signals,

$$\begin{pmatrix} E_{it}y_t \\ E_{it}\pi_t \\ E_{it}a_t \end{pmatrix} = \underbrace{ZB'(BB')}_{\Omega} \begin{pmatrix} x_{it}^1 \\ x_{it}^2 \end{pmatrix}$$

Where $Z$ is the mapping between underlying cost shocks to prices:

$$Z = \begin{pmatrix} \sigma_d \chi^{yd} & \sigma_s \chi^{ys} & 0 & 0 \\ \sigma_d \chi^{\pi d} & \sigma_s \chi^{\pi s} & 0 & 0 \end{pmatrix}$$

In simple form, we can obtain

$$E_{it} u_t = \omega_{uy} x_{it}^1 + \omega_{u\pi} x_{it}^2$$

where $u = y, \pi, a$. Taking average over all firms,

$$\bar{E}_t u_t = \omega_{uy} y_t + \omega_{u\pi} \pi_t$$

Plug them into equation (1) and recall our guess in equation (2),

$$\kappa = \frac{\omega_{\pi y} + \psi_y \omega_{yy} - \psi_a \eta \omega_{ay}}{1 - \omega_{\pi\pi} - \psi_y \omega_{y\pi} + \psi_a \eta \omega_{a\pi}} \tag{4}$$

$$\kappa_a = -\frac{\psi_a}{1 - \omega_{\pi\pi} - \psi_y \omega_{y\pi} + \psi_a \eta \omega_{a\pi}} \tag{5}$$

Remember $\omega's$ are the functions of $\kappa, \kappa_a$. We thus have two equations for two unknowns.

### A.1 Proposition 2:

**Proof.** Using the information structure and the optimal projection matrix $\Omega$,

$$\begin{pmatrix} x_{it}^1 \\ x_{it}^2 \end{pmatrix} = \underbrace{\begin{pmatrix} \sigma_d \chi^{yd} & \sigma_s \chi^{ys} & \sigma_y & 0 \\ \sigma_d \chi^{\pi d} & \sigma_s \chi^{\pi s} & 0 & \sigma_\pi \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} \eta_t^d \\ \eta_t^s \\ \eta_t^y \\ \eta_t^\pi \end{pmatrix}}_{s_{it}}$$

$$\begin{pmatrix} \bar{E}_t a_t \\ \bar{E}_t \beta_t \end{pmatrix} = \underbrace{Z B' (B B')^{-1} B}_{\Omega} \begin{pmatrix} a_t \\ \beta_t \end{pmatrix}$$

We arrives at the following expression for $C_d$ ((1, 2) entry of $\Omega$):

$$C_d = \frac{\sigma_a \sigma_d \left( \sigma_y^2 \chi^{\pi \mathrm{d}} \chi^{\pi \mathrm{s}} + \sigma_\pi^2 \chi^{\mathrm{yd}} \chi^{\mathrm{ys}} \right)}{\sigma_d^2 \left( \sigma_a^2 \left( \chi^{\pi \mathrm{d}} \chi^{\mathrm{ys}} - \chi^{\pi \mathrm{s}} \chi^{\mathrm{yd}} \right)^2 + \sigma_y^2 \left( \chi^{\pi \mathrm{d}} \right)^2 \right) + \sigma_\pi^2 \left( \sigma_a^2 \left( \chi^{\mathrm{ys}} \right)^2 + \sigma_d^2 \left( \chi^{\mathrm{yd}} \right)^2 + \sigma_y^2 \right) + \sigma_a^2 \sigma_y^2 \left( \chi^{\pi \mathrm{s}} \right)^2}$$

$$C_s = \frac{\sigma_a \sigma_d \left( \sigma_y^2 \chi^{\pi \mathrm{d}} \chi^{\pi \mathrm{s}} + \sigma_\pi^2 \chi^{\mathrm{yd}} \chi^{\mathrm{ys}} \right)}{\sigma_d^2 \left( \sigma_a^2 \left( \chi^{\pi \mathrm{d}} \chi^{\mathrm{ys}} - \chi^{\pi \mathrm{s}} \chi^{\mathrm{yd}} \right)^2 + \sigma_y^2 \left( \chi^{\pi \mathrm{d}} \right)^2 \right) + \sigma_\pi^2 \left( \sigma_a^2 \left( \chi^{\mathrm{ys}} \right)^2 + \sigma_d^2 \left( \chi^{\mathrm{yd}} \right)^2 + \sigma_y^2 \right) + \sigma_a^2 \sigma_y^2 \left( \chi^{\pi \mathrm{s}} \right)^2}$$

Therefore, $C_d$ and $C_s$ ((2,1) entry of $\Omega$) are proportional to each other, and are proportional to $\frac{1}{\sigma_y^2}\chi^{yd}\chi^{ys} + \frac{1}{\sigma_\pi^2}\chi^{\pi d}\chi^{\pi s}$. ■

Plugging in our information Phillips curve with transitory shocks, we see that:

$$C_d \propto k_a \sigma_a \left(k\sigma_y^2 - \phi\sigma_\pi^2\right)$$

## A.2 Proposition 4

**Proof.**

We normalize the standard deviation of demand shocks to 1, i.e., $\sigma_d = 1$. When $\sigma_a = 0$, $\hat{\kappa}$ and $\kappa_a$ are the solutions to the following system:

$$\kappa(1+\kappa\phi)^2\sigma_\pi^2\sigma_y^2 - \psi_y(\sigma_\pi^2 + \kappa^2\sigma_y^2) = 0$$
$$\kappa_a + \psi_a \frac{(1+\kappa\phi)^2\sigma_\pi^2\sigma_y^2 + (\sigma_\pi^2 + \kappa^2\sigma_y^2)}{(1+\kappa\phi)^2\sigma_\pi^2\sigma_y^2 + (\sigma_\pi^2 - \kappa\sigma_y^2\psi_y)} = 0$$

Note that the solution to the system can be obtained by solving the first equation to get $\kappa$, and then solving the second equation using the obtained $\kappa_a$. Re-write the first equation:

$$(1+\kappa\phi)^2\sigma_\pi^2\sigma_y^2 = \frac{\psi_y\sigma_\pi^2}{\kappa} + \sigma_y^2\kappa$$

Solution to the first equation is determined by the intersection between its LHS and RHS. Since the RHS is negative for every $\kappa < 0$, and the LHS is non-negative for every $\kappa < 0$, any equilibrium shall have $\hat{\kappa} > 0$.

Rearranging the second equation:

$$\kappa_a = -\psi_a \frac{(1+\kappa\phi)^2\sigma_\pi^2\sigma_y^2 + (\sigma_\pi^2 + \kappa^2\sigma_y^2)}{(1+\kappa\phi)^2\sigma_\pi^2\sigma_y^2 + (\sigma_\pi^2 - \kappa\sigma_y^2\psi_y)} = 0$$
$$\kappa_a = -\psi_a(1 + \frac{\kappa\sigma_y^2(\kappa+\psi_y)}{(1+\kappa\phi)^2\sigma_\pi^2\sigma_y^2 + (\sigma_\pi^2 - \kappa\sigma_y^2\psi_y)}) = 0$$

Notice that if we look at the solution to $(1+\kappa\phi)^2\sigma_\pi^2\sigma_y^2 + (\sigma_\pi^2 - \kappa\sigma_y^2\psi_y) = 0$:

$$\kappa = \frac{\frac{\psi_y}{\sigma_\pi^2} - \sqrt{\frac{\psi_y\left(\sigma_d^2\psi_y - 4\sigma_\pi^2\phi\right)}{\sigma_\pi^4} - \frac{4\phi^2}{\sigma_y^2}} - 2\phi}{2\phi^2} \text{ if } \psi_y > 4\sigma_\pi^2\phi \wedge \sigma_y > 2\sigma_\pi^2\phi\sqrt{\frac{1}{\psi_y\left(\psi_y - 4\sigma_\pi^2\phi\right)}}$$
$$\kappa = \frac{\frac{\psi_y}{\sigma_\pi^2} + \sqrt{\frac{\psi_y(\psi_y - 4\sigma_\pi^2\phi)}{\sigma_\pi^4} - \frac{4\phi^2}{\sigma_y^2}} - 2\phi}{2\phi^2} \text{ if } \psi_y > 4\sigma_\pi^2\phi \wedge \sigma_y > 2\sigma_\pi^2\phi\sqrt{\frac{1}{\psi_y\left(\psi_y - 4\sigma_\pi^2\phi\right)}}$$

With the assumption that $\psi_y < \phi\sigma_\pi^2$, the solution does not exist. Therefore, $\kappa_a < -\psi_a$ when $\kappa_a$ is defined.

■

## A.3 Proposition 5

**Proof.** We normalize the standard deviation of demand shocks to 1, i.e., $\sigma_d = 1$. When $\sigma_a > 0$, $\kappa^*$ and $\kappa_a^*$ are the solutions to the following system:

$$\kappa(1+\kappa\phi)^2\sigma_\pi^2\sigma_y^2 - \psi_y(\sigma_\pi^2+\kappa^2\sigma_y^2) = \sigma_a^2\kappa_a^2\Big(\psi_y - \phi(1+\kappa\phi)\gamma_\pi - (\kappa\sigma_\pi^2 - \phi\sigma_\pi^2)((1+\kappa\phi)\eta\frac{\psi_a}{\kappa_a} + \phi\psi_y)\Big)$$

$$\kappa_a + \psi_a\frac{(1+\kappa\phi)^2\sigma_\pi^2\sigma_y^2 + (\sigma_\pi^2+\kappa^2\sigma_y^2) + \kappa_a^2\sigma_a^2(1+\phi^2\sigma_\pi^2+\sigma_y^2)}{(1+\kappa\phi)^2\sigma_\pi^2\sigma_y^2 + (\sigma_\pi^2 - \kappa\sigma_y^2\psi_y) + \kappa_a\sigma_a^2\big(\kappa_a\phi^2\sigma_\pi^2 + \eta\psi_a + \sigma_y^2(\eta(1+\kappa\phi)\psi_a + \kappa_a\phi\psi_y)\big)} = 0$$

First, note that if we define the nominator of entry (1,1) of $\Omega$ to be $P_a$, the nominator of entry (1,2) of $\Omega$ to be $C$, the nominator of entry (2,2) of $\Omega$ to be $P_d$,

$$C = k_a\sigma_a\left(k\sigma_y^2 - \phi\sigma_\pi^2\right)$$
$$P_d = k_a^2\sigma_a^2 + k^2\sigma_y^2 + \sigma_\pi^2$$
$$P_a = k_a^2\sigma_a^2\left(\sigma_\pi^2\phi^2 + \sigma_y^2 + 1\right)$$

Then after rearranging the first equation in the system:

$$\kappa(1+\kappa\phi)^2\sigma_\pi^2\sigma_y^2 - \psi_y(\sigma_\pi^2+\kappa^2\sigma_y^2) = -(\eta(1+\kappa\phi)\psi_a\sigma_a + \kappa_a\sigma_a\phi\psi_y)C + \kappa_a^2\sigma_a^2\Big(\psi_y - \phi(1+\kappa\phi)\sigma_\pi^2\Big)$$

Simplifies:

$$(1+\kappa\phi)^2\sigma_\pi^2\sigma_y^2 = \frac{\psi_y\sigma_\pi^2 - \overbrace{\Big((\eta\psi_a\sigma_a + \kappa_a\sigma_a\phi\psi_y)C + \kappa_a^2\sigma_a^2(\phi\sigma_\pi^2 - \psi_y)\Big)}^{\Delta_1}}{\kappa} + \psi_y\sigma_y^2\kappa - \underbrace{\Big(\eta\psi_a\sigma_a\phi C + \phi^2\sigma_\pi^2\Big)}_{\Delta_2}$$

For an equilibrium to exist, the quadratic equation on the LHS has to cross with the function on the RHS. To compare $\hat{\kappa}$ and $\kappa^*$, that is, the output Phillips curve slope with or without confusion, we are comparing the solution to the system with or without $\Delta_1 =$ and $\Delta_2 = 0$. Define:

$$\hat{\text{RHS}} = \frac{\psi_y\sigma_\pi^2}{\kappa} + \psi_y\sigma_y^2\kappa$$

$$\text{RHS}^* = \frac{\psi_y\sigma_\pi^2 - \Big((\eta\psi_a\sigma_a + \kappa_a\sigma_a\phi\psi_y)C + \kappa_a^2\sigma_a^2(\phi\sigma_\pi^2 - \psi_y)\Big)}{\kappa} + \psi_y\sigma_y^2\kappa - \Big(\eta\psi_a\sigma_a\phi C + \phi^2\sigma_\pi^2\Big)$$

**Flattened Phillips Curve** With $C > 0$, the Phillips curve will be flattened. When the income effect on wages is not too large, $\psi_y < \phi\sigma_\pi^2 \equiv \bar{\psi}_y$, $\Delta_1$ and $\Delta_2$ are both positive when $C > 0$. Using the property of the two functions, we know that the interactions have to be such that $\hat{\kappa} > \kappa^*$.

Moreover, $\kappa^*$ is positive. When $\psi_y\sigma_\pi^2 - \Big((\eta\psi_a\sigma_a + \kappa_a\sigma_a\phi\psi_y)C + \kappa_a^2\sigma_a^2(\phi\sigma_\pi^2 - \psi_y)\Big) > 0$ it is obvious that $\kappa^* > 0$. When $\psi_y\sigma_\pi^2 - \Big((\eta\psi_a\sigma_a + \kappa_a\sigma_a\phi\psi_y)C + \kappa_a^2\sigma_a^2(\phi\sigma_\pi^2 - \psi_y)\Big) < 0$, interaction between RHS and LHS can only occur with $\kappa < 0$, but this implies that $C < 0$.

**Steepened Phillips Curve** When $C < 0$ and is sufficiently negative in the sense that:

$$C < \min\{-\frac{\kappa_a^2 \sigma_a^2(\phi\sigma_\pi^2 - \psi_y)}{\eta\psi_a\sigma_a + \kappa_a\sigma_a\phi\psi_y}, -\frac{\phi^2\sigma_\pi^2}{\eta\psi_a\sigma_a\phi}\} \equiv \bar{C}(\kappa_a)$$

$\Delta_1$ and $\Delta_2$ are both negative. Using the property of the two functions, we know that the interactions have to be such that $\hat{\kappa} < \kappa^*$.

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