Information Asymmetry and Monetary Non-Neutrality: A Sequential Search Approach *

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Abstract

This paper develops a model of monetary non-neutrality driven by information asymmetry between consumers and firms about nominal marginal costs in a sequential search framework. With only consumer-side frictions, this approach is distinguished from the standard one that relies on firm-side pricing frictions. Consumers' value of search is determined by their information about the price index, and firm's elasticity of demand depends on the perceived relative price. The passthrough of aggregate shocks to prices is therefore incomplete. The key mechanism is that, following a monetary shock, consumers attribute some of the resulting price changes to firm-specific adverse shocks, inducing them to search for alternatives. To dissuade search, firms compress the markup and limit the passthrough of the shock. I further show that the output gap is proportional to the nowcast error of price index in the Phillips curve. Despite its parsimonious nature, the calibrated dynamic general equilibrium model can generate substantial monetary non-neutrality. Consistent with the mechanism, higher inflation is associated empirically with more active consumer search.

Keywords: Monetary non-neutrality, Phillips curve, Search, Information frictions, Information asymmetry, Passthrough, Higher-order beliefs

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"This paper presents a theory that justifies price stickiness, namely, that firms, fearing to upset their customers, attribute a cost to price changes."

— Rotemberg (1982)

Monetary policy is known to have large real effects on the economy in the short run. Both output and inflation decline following an unexpected increase in interest rate. This pattern has been repeatedly uncovered in the empirical literature.[1] Most of existing theories explain this phenomenon by focusing on frictions on the firm side. Models of price stickiness posit that price adjustments are infrequent due to either exogenous factors (Taylor, 1980; Calvo, 1983) or fixed costs (Mankiw, 1985; Golosov and Lucas, 2007). Another theory which dates at least back to Phelps (1969) and Lucas (1972) suggests that firms set prices based on incomplete information about aggregate shocks. Reis (2006) and Alvarez et al. (2016) further argue that costs of acquiring and processing information contribute to price rigidity.

Growing evidence suggests that consumers are subject to more severe frictions than firms. For instance, consumers have misperception about inflation (Binetti et al., 2024; Candia et al., 2023) and pay particular attention to salient prices (Kumar et al., 2015; D'Acunto et al., 2021). Moreover, Kaplan and Menzio (2015) and Kaplan et al. (2019) document a large price dispersion for an identical product even within the same market and week. This indicates some friction that hinders consumers from finding the cheapest one. These evidence motivates two central questions: Can we micro-found monetary non-neutrality using only the consumer-side frictions? How do these consumer-side frictions affect the transmission of monetary policy?

To address these questions, I develop a new monetary model that places consumer-side frictions at the center. Consumers have two frictions: (i) information friction about firms' nominal marginal costs and (ii) search frictions on the good market. On the other hand, firms have full information about the model economy and set the prices flexibly. The main mechanism operates as follows. Following a positive monetary shock, the nominal wage increases. Firms tend to increase prices. However, shoppers with incomplete information about the nominal wage attribute much of this price increase to firm-specific adverse shocks. Shoppers who are initially indifferent between purchasing

[1] Christiano et al. (1999) identify this effect using timing restrictions in VAR. Recently, high-frequency identification approach helps resolve the endogeneity bias in the VAR approach and confirms this finding ((Gertler and Karadi, 2015); (Bauer and Swanson, 2023)). Hazell et al. (2022) estimate the slope of Phillips curve to be very flat using cross-state variation in price indices. Ramey (2016) provides a great summary of this literature.

and searching are now incentivized to seek outside options. To dissuade search, firms compress markups, thereby limiting the passthrough of the monetary shock.

I start with a static model to show the core mechanism. The model features shoppers, firms, and a monetary authority that sets the nominal wage. A monetary shock is modeled as a shock to the nominal wage. The shopper's problem follows the sequential search literature (Wolinsky, 1986; Anderson and Renault, 1999). Specifically, shoppers search sequentially and randomly without commitment. Shoppers must incur a search cost to visit a firm and learn both the price and the associated match utility of its good. I extend this framework in two ways. First, firms are heterogeneous in productivity. Second, shoppers have incomplete information about the nominal wage. These two features leads to shoppers' rational confusion between the aggregate (nominal wage) and idiosyncratic (firm productivity) components of the cost. In particular, when observing a price, shoppers who believe the nominal wage is low will interpret the price as indicative of encountering a firm with low productivity and vice versa.

Each shopper receives a signal about the nominal wage before shopping. They construct their own perceived price distributions using posterior belief of the nominal wage and the knowledge imposed by the rational expectations equilibrium. The value of an additional search depends crucially on the perceived price distribution. I show that the optimal search strategy maps shopper's signal to a unique threshold. The optimal search strategy is then characterized by a threshold rule: if the value of a good exceeds the threshold, the shopper proceeds with the purchase. Otherwise, the shopper continues shopping. Firm's demand is derived from the aggregation of individual shopper's optimal search decisions. Firms then select the optimal pricing strategy that maps the idiosyncratic productivity and the nominal wage to its price.

I start with the characterization of the full-information rational expectations equilibrium. First, I prove the existence of a unique equilibrium in which shoppers search actively. Then, I show the first main result of this paper: under the full information about the nominal wage, the monetary policy is neutral. This is the exact result. The key takeaway of this result is that the search friction alone is not sufficient to generate monetary non-neutrality. We need the interplay of two frictions.

To characterize the incomplete-information rational expectations equilibrium, I rely on first-order approximations. I show that the threshold decreases with shopper's expectation of the price index. The intuition is simple: shoppers are more inclined to make purchases when they expect higher price index and therefore, worse outside options. Next, I show that the elasticity of demand increases in the perceived relative price, defined as the ratio between the actual firm's price and the average expectation of the price index. The average expectation is typically dampened due to the information friction. As a result, firms behave as if they are competing against other firms setting lower prices. In response, firms lower their prices, which compresses the markup and limits the passthrough of the shock. The key takeaway is that firms respond to shopper' expectation of the price index rather than the actual one.

Next, I characterize the aggregate total passthrough, i.e., the passthrough from the shock to the price index. It is composed of two fundamental passthroughs as seen in the literature (Amiti et al., 2019): own-cost passthrough and cross-price passthrough. I then present the main theorem of this paper that shows (i) the aggregate total passthrough is generically incomplete and (ii) high-productivity firms contribute more to the incompleteness and (iii) the aggregate total passthrough decreases with both frictions, with one friction amplifies the effect of the other.

The aggregate total passthrough brings the intuition we developed before for the individual firms to the aggregate. To develop intuition, consider an extreme case where shoppers have no information about the nominal wage. In this scenario, a nominal wage shock resembles an idiosyncratic shock to firms, and the aggregate total passthrough reduces to the aggregate own-cost passthrough. When shoppers possess some information, firms start to respond to other firms' prices perceived by shoppers. The aggregate response of firms can be understood as occurring in iterative rounds. In each round, firms incorporate part of the additional change in the price index from the previous round into their prices, with the passthrough attenuated by shoppers' information frictions. Iterating ad infinitum, the aggregate total passthrough ultimately represents the infinite sum of these rounds of cross-price passthroughs, with each successive round increasingly attenuated, leading to the incomplete passthrough.

In the last section of the static model, I characterize the Phillips curve. It has following properties. First, it is static and does not depend on the future inflation expectation. In particular, the output gap is proportional to the nowcast error of price index. Second, unlike the modern Phillips curve literature that emphasizes the role of firm's expectation in the determination of the slope, this model provides a mechanism that household expectations can influence firms' pricing decision. Moreover, the slope of Phillips curve can be decomposed into two parts, each associated with a friction. The slope decreases in both frictions.

Then, I incorporate the main mechanism from the static model into a dynamic general equilibrium model. In the model, monetary policy sets the nominal interest rate. Shoppers search period by period, and in each period, they receive a signal about the inflation and learn about the shock over time. I present a three-equation system that describes the joint dynamics of aggregate consumption, inflation and interest rate.

I then calibrate the model parameters to match the moments from the literature and my own empirical finding. In particular, the aggregate own-cost passthrough is a sufficient statistic that summarizes the "deep" parameters related to the search friction. I take its value from Amiti et al. (2019) and Gopinath et al. (2011). Next, I estimate the impulse responses of inflation and inflation expectation following a main inflation shock (Angeletos et al. (2020)). I calibrate the information friction such that the time needed for the nowcast error shrinking to zero in the model is similar to its empirical counterpart.

Despite the model's parsimonious nature, the calibrated model can generate substantial monetary non-neutrality comparable to a standard New-Keynesian (NK) model with Calvo parameter equal to 0.7. That is, 70% firms cannot adjust prices in each period. The new insight in the dynamic model is that since the output response depends on the gap between the actual inflation and inflation nowcast, rapid learning can close this gap before the shock fully dissipates, leading to endogenously reduction in the persistence of the output response.

Finally, I present the empirical evidence that supports my mechanism. I use the 2006–2019 NielsenIQ Consumer Panel data, which includes approximately 55,000 households annually, with each household participating in the panel for an average of 30 quarters. The search behavior observed in the data demonstrates path-dependence and a "one-stop" shopping pattern. To capture actual search efforts, I first calculate the spending in a given product category that does not occur in the store the consumer visits most frequently. I then derive the non-routine share of total spending by summing this spending across all product categories. The average non-routine share of spending is about 35%. I find that one standard deviation (51 bp) increase in unanticipated inflation for food and drinks leads to a 26.5 bp increase in the non-routine share of spending. The response is highly statistically significant. The magnitude of response is modest (about a half) and I discuss several reasons that could potentially bias the estimate downward.

I conclude the empirical section by documenting the secular decline in both the non-routine share of spending and the average shopping time. I argue that this trend aligns with the increasing market concentration and price uniformity in the retail industry. Furthermore, I connect this empirical pattern to the observed decline in the secular slope of the Phillips curve, highlighting the potential macroeconomic implications of these structural changes in consumer behavior and market dynamics.

Related Literature – This paper contributes to three strands of literature. The first studies the price stickiness and monetary non-neutrality by focusing on the consumer-side mechanisms. The prominent paper in this literature is Matějka (2015). He shows that firms set discrete prices as consumers "hate" price fluctuations, which increases their costs of attention. My paper has the similar flavor in the sense that the price stickiness originates from the strategic interaction between consumers and firms. However, my paper has different mechanisms and investigates these mechanisms in general equilibrium. Gabaix and Graeber (2024) and Rebelo et al. (2024) propose behavioral theories of price stickiness. My paper is within the realm of rational expectations.

Second, it adds to the research on the role of search frictions in explaining monetary non-neutrality. This literature is divided into two main streams based on different search frameworks. The first stream follows Burdett and Judd (1983) where firms adopt a mixed pricing strategy where a range of prices is optimal when market has both shoppers and non-shoppers. Head et al. (2012) show that price stickiness can result from this strategy. As nominal price increases, profit can still be maximized despite a fall in real price. Similarly, Burdett and Menzio (2018) incorporate same mechanism into a menu-cost model, where a broader range of optimal prices leads to larger price adjustments.

The second stream leverages the sequential search framework. Benabou (1988) shows that when monopolistic competition arises from costly consumer search, the inaction region in a menu-cost model expands with increasing search costs. More recently, Sara-Zaror (2024) document that price dispersion for identical goods varies with inflation levels. Gaballo and Paciello (2021) show a model where consumers are motivated to leave the monopolistic firm and find lower prices in a separate market where firms have perfect competition, when the inflation rises. However, my paper does not rely on the separation of markets, and firms are heterogeneous in productivity and compete monopolistically in a unified market. This paper advances this second stream by integrating search models a là Wolinsky (1986) and Anderson and Renault (1999) with a heterogeneous firm block, a monetary general equilibrium framework, and incomplete information on the consumer side.

Third, this paper contributes to the literature on the role of consumer-side information frictions in explaining monetary non-neutrality. Bénabou and Gertner (1993), L'Huillier (2020) and L'Huillier and Zame (2022) focus on the role of individual prices consumers observe during the search as revealing the information about aggregate shocks. Especially, L'Huillier (2020) consider a signaling game between a monopolistic firm and consumers. The price of one firm is rigid if we select the pooling equilibrium. That price becomes more volatile in the separating equilibrium than the full-information case. I shut down the signaling effect and my mechanism does not rely on the equilibrium selection. This paper also broadly contributes to the literature on information frictions and the transmission of monetary policy. For example, Angeletos and La'O (2013) model aggregate demand fluctuations driven by sentiment in beliefs. Venkateswaran (2014) examines an incomplete-information version of the Diamond–Mortensen–Pissarides model. Angeletos and Lian (2018) show that incomplete information games among consumers can mitigate the forward guidance puzzle.

Outline − The rest of the paper is organized as follows. Section 1 presents a static model and establishes the main results on the passthrough. Section 2 presents the dynamic model and the calibration results. Section 3 shows empirical evidences. The last section concludes. Appendix A contains some of the proofs omitted from the text. Appendix B contains the proof of the dynamic model and additional calibration details. Appendix C contains details of empirical setup and additional empirical evidence.

1 Static Model

I develop a macroeconomic model with (i) information asymmetry between shoppers and firms about marginal costs and (ii) search frictions on the goods market. I proceed in two steps. First, I present a static model to explain the core mechanism. Second, I present a full-fledged dynamic general equilibrium framework in the next section.

I first consider static partial equilibrium model in which there are firms, shoppers and a monetary authority and then close the model in the general equilibrium.

Notation − I use lower case to denote $\log Y$ for any variable Y, i.e., $y = \log Y$ and lower case with hat to denote log-deviation from the steady-state value, i.e., $\hat{y} = \log Y - \log \bar{Y}$.

1.1 States, Strategies and Distributions

At the beginning of each period, the Nature draws the aggregate state w from a given distribution Φ_w, idiosyncratic states for each firm a_k from Φ_a, idiosyncratic noisy signals for each shopper x_i from Φ_x about the aggregate state w. $\Phi_w, \Phi_x, \Phi_a, G$ are the common knowledge for all agents. Each firm produces a good and selects the optimal pricing strategy that maps its state $\{a_k, w\}$ to the price of its good, i.e., $p_k = p^*(a_k, w)$, where $p : \mathcal{R}^2 \to \mathcal{R}$. The optimal pricing strategy p^* monotonically decreases in a_k and increases in w. Variation in a_k across firms induces a price distribution $F(p|w)$, which has a density distribution $f(p|w)$. Let $p^{*-1}(p, w)$ denote the inverse mapping from p_k to a_k given w. It is straightfoward to show that p^{*-1} also monotonically decreases in a_k and increases in w. The price distribution conditional on w is given by,

$$F(p|w) = 1 - \Phi_a(p^{*-1}(p, w)) \tag{1}$$

In the rational expectations equilibrium, shoppers know the equilibrium pricing strategy p^*. Conditional on knowing p^* and w, the shoppers are able to derive the price distribution. However, they have incomplete information about w. Their posterior belief about w is denoted by $H(w|x)$. The shopper's perceived price distribution conditional on x, $f(p|x)$, is given by,

$$f(p|x) = \int f(p|w)h(w|x)dw \tag{2}$$

The above equation has two implications. First, when $x = w$, the perceived and objective price distributions coincide. Second, if $h(w|x_i)$ first-order stochastically dominates (FOSD) $h(w|x_j)$, then $f(p|x_i)$ FOSD $f(p|x_j)$. Intuitively, when the shopper places more probability weights on low w, since prices increase in w, the perceived price distribution is stochastically smaller.

Moreover, the shopper forms the expected idiosyncratic state of firm k after observing the price p_k,

$$E(a_k|x) = \int p^{*-1}(p_k, w)h(w|x)dw \tag{3}$$

Similarly, it is obvious to show that $E(a_k|x_i) < E(a_k|x_j)$ when $h(w|x_i)$ FOSD $h(w|x_j)$. The shopper who believes the nominal wage is low will perceive the firm's a_k as lower than its actual value.

Finally, shoppers search sequentially and have free recall. Each shopper's search strategy depends on the perceived price distributions $F(p|x)$. The optimal search strategy can therefore be represented as a function that maps x to an optimal utility level, $v_i = v^*(x)$, where $v^* : \mathcal{R} \to \mathcal{R}$.

This strategy then determines the demand allocation across firms on the aggregate. In the rational expectations equilibrium, firms know the equilibrium search strategy v^* and select optimal pricing strategy p^* that maximizes the profit.

1.2 Setup

Now, I state the model. Time is discrete and infinite $t \in \mathbf{N}$. The timeline is as follows. Within a period, the monetary authority first sets the nominal wage. Shoppers are endowed with cash and form a conditional distribution about prices. Firms post prices and shoppers search sequentially. A fraction of shoppers make the purchase in a given round and the remaining keep searching. The period ends until all shoppers make the purchase. I denote the round of search $r = 1, 2, 3, ...$ within a period. All rounds of search happen within one period.

Firm $-$ The economy is populated with a unit mass of firms indexed $k \in [0, 1]$, each of which produces a differentiated product using the following production technology,

$$Y_k = A_k L_k \tag{4}$$

where L_k is the amount of labor employed. A_k is the firm's productivity, which is i.i.d. across firms. Specifically, I assume that $\log A_k$ draw from the normal distribution $\mathcal{N}(0, \sigma_a^2)$. The marginal cost is $\frac{W}{A_k}$, where W is the nominal wage. The nominal wage represents the average nominal marginal cost. Labor is supplied outside this economy in the partial economy.

Monetary Authority $-$ The monetary authority directly controls the nominal wage W. It draws w from the normal distribution $\mathcal{N}(W, \sigma_w^2)$. Monetary shock is defined as $\hat{w} = w - W$.

Shopper $-$ The economy is populated with a continuum of shoppers indexed by $i \in [0, 1]$. At the beginning of the period, each shopper is endowed with cash X_i. It follows,

$$X_i = W \exp\left(\sigma_x \varepsilon_{xi} - \frac{\sigma_x^2}{2}\right) \tag{5}$$

where ε_{xi} is i.i.d. across shoppers and it follows $\varepsilon_{xi} \sim \mathcal{N}(0, 1)$. It is also independent of productivity shocks and monetary shocks. The shopper treats X_i as signal about w. The expected (log) nominal wage is then given by,

$$E(w|x_i) = \theta_x x_i + (1 - \theta_z - \theta_s)W \tag{6}$$

where $\theta = \frac{\sigma_x^{-2}}{\sigma_w^{-2}+\sigma_x^{-2}}$. Let $H(w|x_i)$ denote the posterior belief about the nominal wage with density $h(w|x_i)$. Based on Bayes' rule, it follows $\mathcal{N}(E(w|x_i), (\sigma_w^{-2} + \sigma_z^{-2})^{-1})$. Then, shoppers construct their perceived price distribution according to (2).

The shopper consume one good.[2] The utility that shopper i gains from consuming good k is given by,

$$\log \frac{Z_i}{P_k} + \frac{1}{\lambda}\epsilon_{ik} \tag{7}$$

where $\epsilon_{ik} \sim G$ is match utility between shopper i and good k. G is twice continuously differentiable and its density function is g. It captures idiosyncratic consumer preferences for certain goods over others. ϵ_{ik} are i.i.d across firms and shoppers.[3] Also, following the literature, I assume that G is log-concave. Note that some commonly used distribution functions are log-concave, e.g., normal distribution, uniform distribution, Gumbel distribution.[4] The parameter λ controls the relative importance between the two types of utility. A larger λ implies that the shopper places greater value on utility from consumption relative to match utility. Relative prices are less important in the purchase decision. Since X_i only affects the level of utility and does not change the relative utilities across goods, I define the normalized utility as follows,

$$y_{ik} = -p_k + \frac{1}{\lambda}\epsilon_{ik} \tag{8}$$

Shoppers search sequentially and randomly following Wolinsky (1986) and Anderson and Renault (1999). By incurring a search cost $\kappa > 0$, the shopper can visit a firm to learn both its price and the associated match utility. Shoppers have free recall, meaning there are no additional costs for purchasing goods from firms they have previously visited. The shopper continues to search if the expected value of searching is larger than making the purchase at the firm that provides the maximum value until now. According to (8), the distribution of value of drawing a random firm y

[2] In Section 1.5, I extend the model by allowing shoppers to get access to a bunch of goods by incurring one search cost.

[3] The price distribution is not degenerated if the random utility term is match-specific. If it is only shopper-specific, it does not matter for the search strategy. Firms compete only on price. The price distribution is degenerated to a single optimal price. If it is only good-specific, the price distribution will be exogenous.

[4] See Bagnoli and Bergstrom (2005) for a broad discussion of log-concavity that do and do not satisfy this condition. The assumption of log-concavity ensures that the hazard rate $\frac{g(x)}{1-G(x)}$ is monotonically increasing.

is a convolution of the perceived price distribution and the distribution of match utility,

$$\psi(y|x) = \int \lambda g(\lambda(y+p))f(p|x)dp \tag{9}$$

I assume that $h(w|x_i)$ remains fixed throughout all rounds of search. In particular, I impose the following restriction on shoppers' information sets.

Assumption 1. *Shoppers do not learn about the nominal wage from individual prices they observe during the search.*

This assumption is plausible if the idiosyncratic variations, contributed by productivity shocks, in prices are way larger than the aggregate variations induced by the nominal wage. Even when a shopper consistently encounters firms charging high prices, she attributes this to bad luck rather than an increased nominal wage. I make this assumption for tractability. If shoppers' information sets depend on the whole search history, it leads to exploding states. The model becomes intractable and this complexity is not relevant to the main mechanism of the model as I will show below. In the dynamic model, I assume that shoppers receive a signal about the current change in the inflation before searching as an alternative way to incorporate learning from shopping.

I now state the shopper's problem. The shopper undertakes sequential search with perfect recall and without being restricted by any plan made before setting out to search. I refer to the latter as search without commitment.[5] Let v_{ir} denote the maximum value of previously visited firms in rth round. In particular, after sampling the first firm, $v_{i1} = y_{i1}$. I define v_{ir} for $r > 1$ as follows,

$$v_{ir} = \max\{v_{ir-1}, y_{ir}\} \tag{10}$$

In the rth round of the sequential search, the state of the shopper is v_{ir}. The shopper has the option to stop searching and accept v_{ir} or continue searching. The value function for the shopper, $U : \mathcal{R} \rightarrow \mathcal{R}$, in each state $v \in \mathcal{R}$, satisfies,

$$U(v|x) = \max\left\{v, -\kappa + U(v|x)\int_{-\infty}^{v}\psi(y|x)dy + \int_{v}^{\infty}U(y|x)\psi(y|x)dy\right\} \tag{11}$$

[5]If the shopper formulated her search plan prior to search and she committed to that plan, then she would take into account the expected total search costs of sampling, and she would stop with a lower quality match if she were unlucky and happened to sample a sequence of firms for which she ill-suited. In the case of a shopper doing sequential search without commitment, she ignores past fixed costs of search as sunk. Burdett and Judd (1983) considers a search problem with commitment and homogeneous firms.

The maximum represents that the shopper can either receive the maximum value v until this round and stop searching more, or continue searching by incurring a search cost κ and drawing a random good. If the value of that good is lower than v, which occurs with probability $\int_{-\infty}^{v_{ir}} \psi(y|x_i)\, dy$, the shopper will retain the value $U(v)$, since she has free recall. Otherwise, she will obtain a higher value from the newly drawn good and update v according to (11). The value function U is stationary only when Assumption 1 holds. Otherwise, with the information sets expanding over the rounds of search, the value function U should be indexed by the search round.

The shopper's problem is solved in two steps. First, she finds the U function that solves the functional equation (11). Second, she keeps sampling firms until v first exceeds the expression to the right of the comma in (11).

Partial Equilibrium – The equilibrium concept is Perfect Bayesian Nash equilibrium (PBNE). Since productivity is assumed to have unbounded support, any positive price is an on-equilibrium price. The regulations on the off-equilibrium belief is not strictly needed in this model.[6] Formally, I define the equilibrium as follows:

Definition 1 (Equilibrium). *A Perfect Bayesian Nash equilibrium is a triplet of allocation, prices, and beliefs such that*

1. *Firms choose the optimal pricing strategy p^* to maximize profits given the optimal search strategy.*

2. *Shoppers search without commitment. They do not update beliefs after observing prices during the search. Conditional on the information sets, they combine the optimal pricing strategy p^* and other exogenous distributions to compute $U(v|x)$ for each state v. Shoppers' optimal search strategy is then determined by the stopping rule as shown in (11).*

3. *Nominal wage is chosen exogenously.*

4. *Goods market clears.*

In addition, I define the full-information equilibrium in which shoppers know the nominal wage. It serves a natural benchmark to analyze the incomplete-information equilibrium.

[6]In the standard search literature, consumers know the firms' marginal cost and there is no correlated cost shocks. Consumers, therefore, are able to detect the off-equilibrium prices.

Definition 2 (Full-Information Equilibrium)**.** *A full-information equilibrium is the equilibrium defined above, except that shoppers know w.*

1.3 Equilibrium Characterization

I now characterize the equilibrium. I proceed in three steps. First, I characterize the search strategy and the pricing strategy. Second, I show the existence and properties of full-information equilibrium. Third, I show the monetary neutrality under full information.

Characterization of the Search Strategy – I first characterize the search strategy. The shopper needs to first find the U function and then decide when to stop searching. The solution to the shopper's problem is presented in the following proposition,

Proposition 1. *Under Assumption 1, the optimal search strategy follows a threshold rule. The threshold is denoted $v^*(x)$. If $v < v^*(x)$, the shopper keeps searching; otherwise, she stops and makes the purchase. The threshold is unique for each x. It is determined by,*

$$v^*(x) = -\frac{\kappa}{1 - \Psi(v^*(x)|x)} + \frac{\int_{v^*(x)}^{\infty} y\psi(y|x)dy}{1 - \Psi(v^*(x)|x)} \tag{12}$$

where $\Psi(p|x_i)$ is the perceived distribution of the value of a random draw.

Proof. See Appendix **??** ■

The optimal search strategy is simple. The shopper keeps sampling firms until the target value $v^*(x)$ is reached. Indeed, in Appendix A, I show that the value function U is given by,

$$U(v|x) = \max\{v, v^*(x)\} \tag{13}$$

This implies that, regardless of the current state v, the value of conducting an additional search is constant, equal to the threshold v^*. When $v < v^*$, the value function is always v^*, indicating that the shopper opts to continue searching. Conversely, when $v \geq v^*$, the value function equals the state v, implying that the shopper accepts v.

This search strategy is optimal due to two assumptions. First, shoppers cannot commit to a plan prior to search. To understand the intuition, consider a shopper who draws a long sequence of goods that offer consistently low values. The shopper will keep searching even when the total search costs paid in this period is already very large. Indeed, there is a measure zero of shoppers who search forever and pay infinitely large search costs. This is apparently not optimal from the ex ante view. Second, the standard threshold rule may fail without Assumption 1. Rothschild (1974) shows that if shoppers do not know the price distribution, they may buy at high price because they infer from prices that the average price can be even higher. On the other hand, they may continue searching at a low price. Bénabou and Gertner (1993), L'Huillier (2020), Gaballo and Paciello (2021) focus on the role of individual prices in revealing information about aggregate shocks. Their analysis is thereby often restricted to two-firm case. [7]

The result extends the standard threshold result in the literature (Weitzman, 1979 and Wolinsky, 1986) by incorporating both endogenous price distribution and incomplete information. In their analysis, firms are homogeneous and shoppers have correct belief about the optimal price. Here, firms are heterogeneous and shoppers have incomplete information about the price distribution. Proposition 1 shows that under Assumption 1 ,the optimal search strategy still follows the threshold rule. The threshold depends on the shopper's information set x.

To make clear how two distributions affect the threshold, the following Corollary shows an alternative way to solve the threshold.

Corollary 1. *The threshold $v^*(x)$ is given by,*

$$\int \int_{\lambda(v^*(x)+p)}^{\infty} \big(\frac{1}{\lambda}\epsilon - p - v^*(x)\big) g(\epsilon) d\epsilon f(p|x) dp = \kappa \tag{14}$$

where $f(p|x)$ is the perceived price distribution.

The left-hand side represents the expected additional benefit of this search. To see this, consider a shopper who has the state $v^*(x_i)$. From Proposition 1, she is indifferent between sampling another firm and stopping searching. Suppose she samples another firm k, she will prefer the new good if $\frac{1}{\lambda}\epsilon_{ik} - p_k > v^*(x_i)$. Since the shopper can return without cost, the additional utility obtained in this case is $\max\{\frac{1}{\lambda}\epsilon_{ik} - p_k - v^*(x_i), 0\}$. The threshold is achieved when the expected additional utility is equal to the search cost.

Aggregation − I now show the aggregation of optimal search decisions. In particular, I show the expenditure allocation across firms and the resulting profits. According to the optimal search strategy, the shopper only purchases the good k if $\frac{1}{\lambda}\epsilon_{ik} - p_k > v^*(x)$. The shopper's probability of purchasing from firm k in each round is $1 - G\big(\lambda(v^*(x) + p_k)\big)$. Since learning from shopping is prohibited, this probability is the same for all rounds. The probability of any shopper purchasing

[7] Janssen et al. (2017) investigates the non-reservation property in a search equilibrium.

from any firm in each search given w is given by,

$$\rho = \int\int \big(1 - G\big(\lambda(v^*(x) + p)\big)\big) d\Phi_x(x) f(p|w) dp \tag{15}$$

Suppose the mass of shoppers who visit any firm in the first round is one. A fraction ρ of these shoppers settle with the firms they visit in the first round. The remaining $1-\rho$ shoppers search in the second round, a further $(1-\rho)^2$ search in the third round, and so on. It is straightforward to show that the expected number of searches of each shopper is ρ^{-1}. Firms are atomistic and take ρ as given when setting prices. Furthermore, the total expenditure spent in firm k after all rounds of search is given by,

$$\omega_k = \frac{1}{\rho}\int X\big(1 - G\big(\lambda(v^*(x) + p_k)\big)\big) d\Phi_x(x) \tag{16}$$

The dispersion in the cash in hand affects the expenditure through the dispersion of thresholds. The inequality also matters through the covariance between the cash in hand and the threshold. The more cash in hand is positively correlated with higher expectation of the nominal wage. The profit for firm k is given by,

$$\pi_k = \frac{1}{\rho}\int X\big(1 - G\big(\lambda(v^*(x) + p_k)\big)\big) d\Phi_x(x) \frac{1}{P_k}(P_k - \frac{W}{A_k}) \tag{17}$$

The demand is derived by dividing the total expenditure spent in firm k by P_k. The profit is thus the total demand times the profit per sale.

Characterization of the Pricing Strategy – Monopolistic firms compete on prices. They maximize the profit in (17) with respect to its price. The following proposition presents the optimal pricing strategy,

Proposition 2. *Let μ_k denote the markup and e_k denote the elasticity of demand Firm charges a markup over the marginal cost,*

$$P_k = \mu_k \frac{W}{A_k}; \quad \mu_k = \frac{e_k}{e_k - 1} \tag{18}$$

The elasticity of demand e_k is determined by,

$$e_k = \lambda \frac{\int X g\big(\lambda(v^*(x) + p_k)\big) d\Phi_x(x)}{\int X\big(1 - G\big(\lambda(v^*(x) + p_k)\big)\big) d\Phi_x(x)} + 1 \tag{19}$$

Proof. See Appendix A. ■

The optimal price is a markup times the marginal cost. There are two factors that contribute to the firm's market power. One is the relative importance of the utility of a match with the utility of consumption, λ. Larger λ implies higher elasticity.[8] Another factor is that search friction naturally gives rise to monopoly power (Diamond, 1971). In particular, the effect of search friction is represented by a hazard function, where the density g represents the marginal shoppers who are indifferent between making purchase and continuing searching. The survival function $1 - G$ indicates that the adjustment of the price will affect the profit obtained from all infra-marginal shoppers. The ratio between these two captures the trade-off that setting a higher price motivates shoppers to search, while extracting more profit from the infra-marginal shoppers.

Characterization of the Full-Information Equilibrium In the full-information equilibrium, shoppers know the nominal wage. Let $v^*(w)$ denote the value of threshold in this equilibrium. Similar to Corollary 1, it is given by,

$$\int \int_{\lambda(v^*(w)+p)}^{\infty} \big(\frac{1}{\lambda}\epsilon - p - v^*(w)\big) g(\epsilon) d\epsilon f(p|w) dp = \kappa \tag{20}$$

where $f(p|w)$ is the actual price distribution. It is easy to show that firm's profit is given by,

$$\pi_k = \frac{1}{\rho}\Big(1 - G\big(\lambda(v^*(w) + p_k)\big)\Big)\frac{W}{P_k}(P_k - \frac{W}{A_k}) \tag{21}$$

where $\rho = \int \big(1 - G(\lambda(v^*(w) + p))\big) f(p|w) dp$. The difference between (21) and (17) is two-fold. First, the distribution of thresholds is reduced to a single value $v^*(w)$. Second, the inequality in cash in hand is no longer correlated with the threshold. I can similarly define the expenditure allocation in the full-information case,

$$\omega_k = \frac{1}{\bar{\rho}}\Big(1 - G\big(\lambda(v^*(w) + p_k)\big)\Big) \tag{22}$$

The first-order condition of firm's problem results in,

$$P_k = \frac{e_k}{e_k - 1}\frac{W}{A_k} \tag{23}$$

$$e_k = \lambda \frac{g\big(\lambda(v^*(w) + p_k)\big)}{1 - G\big(\lambda(v^*(w) + p_k)\big)} + 1 \tag{24}$$

Since G is log-concave, the elasticity of demand is increasing in firm's own price. The decrease in the density on the right tail is dominated the decrease in $1 - G$. The intuition is that the incentive

[8] Anderson et al. (1987) shows that without search frictions, if G is Gumbel distribution, the demand system is exactly CES and the elasticity of substitution is $\lambda + 1$.

to extract more profit from infra-marginal shoppers overwhelms the concern of losing marginal consumers. Therefore, high-productivity firms will set lower price and higher markup. Moreover, higher threshold implies pickier shoppers, which drives up the elasticity.

Now, I present an important property of $v^*(w)$. It is a crucial step in proving the existence of the equilibrium. This property is also useful to understand the effect of search frictions on other equilibrium objects I will discuss in Section 1.5.

Lemma 1. *The full-information threshold $v^*(w)$ decreases with the search cost, κ.*

Proof. See Appendix A. ■

The increase in the threshold $v^*(w)$ increases elasticity for all firms. I prove that the decrease in price is less than the increase in the threshold for any firm. Therefore, the left-hand side of (20) is strictly deceasing in $v^*(w)$. I now establish the existence of the full-information steady-state equilibrium.

Theorem 1 (Existence of the Full-Information Steady-State Equilibrium)**.** *There exists a unique full-information steady-state equilibrium in which consumers search actively.*

Proof. See Appendix A. ■

This theorem is proved in two steps. First, the elasticity of demand increases in price. It implies that higher price induces higher elasticity and lower markup. Therefore, the individual prices are uniquely determined by (23) and (24). Second, from Lemma 1, the threshold $v^*(w)$ is uniquely determined by (20) for the given price distribution that is derived from optimal pricing strategy. The equilibrium is then the fixed point of the reservation value and the price distribution.

Note that there always exist equilibria in which firms charge sufficiently high prices and shoppers do not search. However, there is only one equilibrium in which shoppers search actively. From now on, I will call the full-information equilibrium in which $w = \bar{w}$ the full-information steady state.

Now, I present the first main result about monetary non-neutrality.

Theorem 2. *In the full information equilibrium, monetary policy is neutral.*

Proof. See Appendix A. ■

This theorem establishes that the passthrough from the monetary shock to the price index is complete under full information. It indicates that search friction alone is not sufficient to generate monetary non-neutrality. This theorem is the exact result. To understand this theorem, consider a scenario where the monetary authority raises the nominal wage by $b\%$. If firms respond by increasing prices by $b\%$, then their values decrease by $b\%$ according to (8). The key step in the proof is that the threshold $v^*(x)$ also decreases by $b\%$. This implies that search decisions remain unchanged, as the relative positioning of the threshold and value distribution is preserved. Consequently, expenditure allocation remains the same as before, validating the guess that firms respond by increasing prices by $b\%$.

1.4 Approximate Optimal Strategies

Both the threshold shown in Proposition 1 and the pricing strategy shown in Proposition (2) are highly non-linear. Following the literature, I consider the case in which the monetary authority draws monetary shocks from a distribution with small standard deviation, i.e., $\sigma_w \to 0$. At the same time, I keep variance ratio $\frac{\sigma_x^2}{\sigma_w^2}$ fixed, which correspond to fixed values of θ. The fixed ratio preserve the same level of information frictions about the nominal wage. An implication is that $\sigma_x \to 0$. The existence of the full-information equilibrium allows us to approximate the non-linear strategies to the first order around the full-information steady state.

To proceed, first note that the posterior belief about the nominal wage, $H(w|x)$, follows $\mathcal{N}(E(w|x), (\sigma_w^{-2} + \sigma_z^{-2})^{-1})$. As all $\sigma_w, \sigma_x \to 0$, $H(w|x)$ collapses to a Dirac function centered at $E(w|x)$. Then, I approximate the perceived price distribution $f(p|x)$ to the first order,

$$f(p|x) = \int f(p|w)h(w|x)dw = f(p|E(w|x)) \tag{25}$$

On the first order, only the expectation of the nominal wage matters for the perceived price distribution.

Second, I define the passthrough from the monetary shock $\hat{w}$ to prices in equilibrium. The first-order approximation to the optimal pricing strategy p^* is given by,

$$p^*(a_k, w) = p^*(a_k, \bar{w}) + p_w^*(a_k, \bar{w})\hat{w} \tag{26}$$

I call $\varphi_k = p_w^*(a_k, W)$ the total passthrough, as it reflects the sum of two passthroughs which I will show in Section 1.5. Shoppers know the total passthrough as they know p^*. The total passthrough is determined in the equilibrium. The shopper's expected price conditional on x is given by,

$$E(p_k|x) = \bar{p}_k + \varphi_k E(\hat{w}|x) \tag{27}$$

Finally, unlike the standard demand system where an aggregate demand function is available and then the price index is naturally defined. In the model with search frictions, there is no obvious way to define a price index. However, the standard theory (Hulten, 1973, Hulten, 1978) offers a simple non-parametric formula for the change in price index. Under the assumptions of homotheticity and preference stability[9], the log change in the price index is the expenditure share-weighted, as measured in the base period, log changes in all the prices. Apparently, both conditions are satisfied here, I define the expenditure share-weighted log change in the price index.

$$\hat{p} = \Phi \hat{w} \tag{28}$$

where $\Phi = \int \varphi_k \bar{\omega}_k dk$ and $\bar{\omega}_k$ is the expenditure share in the full-information steady state. The aggregate effect of a monetary shock on price index is measured by the aggregate total passthrough Φ. Furthermore, the average expectation of the price index is $\bar{E}\hat{p} = \theta \hat{p}$. The response of $\bar{E}p$ is dampened compared to the actual change in price index since the signals about the shock are noisy.

Now I state the results on the approximation of the non-linear equilibrium. In particular, the following proposition presents a first-order approximation to the threshold in (14) and the elasticity in (19).

Proposition 3. *Fix the variance ratio* $\frac{\sigma_z^2}{\sigma_w^2}, \frac{\sigma_s^2}{\sigma_w^2}$. *To the first order as* $\sigma_w \to 0$,
[Part 1] *the threshold* $v^*(x)$ *is given by,*

$$v^*(x) = v^*(\bar{w}) - E(\hat{p}|x) \tag{29}$$

where Φ *is the aggregate total passthrough.*
[Part 2] *The elasticity of demand is given by,*

$$e_k = \lambda \frac{g\big(\lambda(v^*(w) + \bar{p}_k + \hat{p}_k - \bar{E}\hat{p})\big)}{1 - G\big(\lambda(v^*(w) + \bar{p}_k + \hat{p}_k - \bar{E}\hat{p})\big)} + 1 \tag{30}$$

where $\bar{E}\hat{p} = \theta\hat{p}$. *In addition,* e_k *increases in* $\hat{p}_k - \bar{E}\hat{p}$.

Proof. See Appendix A. ■

This proposition shows that the threshold under incomplete information is equal to the threshold under full-information steady state minus the expected change in the price index. In particular, higher signal about the nominal wage implies a lower threshold. To understand the intuition,

[9]Homotheticity requires that income effects are uniform, meaning that the income elasticity of demand must equal one for each good. Stability requires that consumers adjust their spending only in response to changes in income and relative prices.

consider a shopper who has a higher expectation of the price index. She is more likely to purchase from the current firm since the expected value of a random draw is lower. As a result, the threshold is reduced. Moreover, notice that $E(\hat{p}|x) = \Phi E(\hat{w}|x)$. The aggregate total passthrough, Φ, governs the passthrough from the expected monetary shock to the threshold for shoppers with any information sets. The aggregate total passthrough encapsulates all the changes in the price distribution following the shock. To see the intuition, suppose the distribution of passthrough is a singleton, i.e., $\varphi_k = \varphi_0$, then $\Phi = \varphi_0$. An increase in the perceived nominal wage would shift every point in the distribution to the right by the same amount. This is the case in the full-information equilibrium. On the other hand, if the distribution of passthrough is not degenerate, the shape of the price distribution also changes after the shock. Importantly, the aggregate passthrough depends on the covariance between the distribution of expenditure shares and the distribution of passthrough. In particular, If firms with higher average expenditure shares also pass through more of the increase in nominal costs to prices, it decreases the option value of search, thereby lowering the threshold. We can achieve the average change in the threshold compared to the benchmark $v^*(\bar{w})$ by integrating on both sides of (29),

$$\int v^*(x)dx = v^*(\bar{w}) - \theta\hat{p} \tag{31}$$

Shoppers, on average, believe the increase in the price index is not as large as the actual one, which motivates them to search for outside options more than in the full-information case. This will have significant impact on the elasticity of demand.

The second part of the proposition provides a simple characterization of the elasticity of demand. On the first order, only the average expectation of the price index is retained. The distribution of the thresholds and covariance between cash in hand and thresholds are second-order. The proposition shows that the elasticity depends on perceived relative price. Shoppers, on average, believe the relative price is larger than it actually is. As a result, firms behave as if they are competing with others that set prices lower than the actual levels, which induces them to reduce their own prices in response. This leads to compressed markups and incomplete passthroughs. This is the key driver of the main results presented in Section 1.5.

1.5 Characterization of Passthroughs in General Equilibrium

I now characterize passthroughs in the general equilibrium. I present the main finding: the aggregate passthrough of a money supply shock to the price index is generically incomplete. The slope of Phillips curve is flat if the information friction is large.

The total passthrough is composed of the own-cost passthrough and the cross-price passthrough. Decomposing the total passthrough into these two elements is crucial to understand the intuition behind results. Following Amiti et al. (2019), I define both passthroughs using markup elasticities. I present the following Lemma,

Lemma 2. *The price responds to both firm's own cost shocks and competitors' prices,*

$$\hat{p}_k = \gamma_k \hat{w} + \xi_k \hat{p} \tag{32}$$

where γ_k is own-cost passthrough and ξ_k is cross-price passthrough.

$$\gamma_k = (1 - \frac{d\mu_k}{dp_k}\Big|_{\hat{w}=0})^{-1}; \xi_k = \frac{d\mu_k}{dp}\Big|_{\hat{w}=0}\gamma_k \tag{33}$$

The total passthrough of each firm is given by,

$$\varphi_k = \gamma_k + \Phi\xi_k \tag{34}$$

The aggregate total passthrough Φ is given by,

$$\Phi = \frac{\Gamma}{1 - \Xi} \tag{35}$$

where $\Gamma = \int \gamma_k \bar{\omega}_k dk$, $\Xi = \int \xi_k \bar{\omega}_k dk$.

Proof. See Appendix A. ■

The lemma links passthroughs with the elasticity of demand through the markup elasticities. As a result, passthroughs also depend on perceived relative prices as presented in Proposition 3. Plugging the definition of the total passthrough, the lemma then derives the total passthrough for each firm in equilibrium. Integrating on both sides, the aggregate total passthrough is obtained. Since the passthroughs are defined at the steady state, the own-cost passthrough, γ, does not depend on the information friction θ. Firms pass part of their own cost shocks to prices in all cases.

I now state our main results on passthroughs under incomplete information. To push the results on monotone comparative statics as far as possible, I assume that the distribution of the match utility, G, follows the Gumbel distribution.

Theorem 3 (Total Passthrough under Incomplete Information)**.** *Under incomplete information about the nominal wage, the aggregate total passthrough has following properties,*

1. ***[Incompleteness]*** $\Phi < 1$
2. ***[Composition]*** φ_k *decreases in productivity;* $\bar{\omega}_k$ *increases in productivity.*
3. ***[MCS on*** κ***]*** Φ *decreases in search friction* κ *given* θ*. Limits:* $\lim_{\kappa\to\infty} \Phi = 0$; $\lim_{\kappa\to 0} \Phi = 1$

   [MCS on θ***]*** Φ *increases in information friction* θ *given* κ*. Limits:* $\lim_{\theta\to 1} \Phi = 1$; $\lim_{\theta\to 0} \Phi = \Gamma$

Proof. See Appendix A. ■

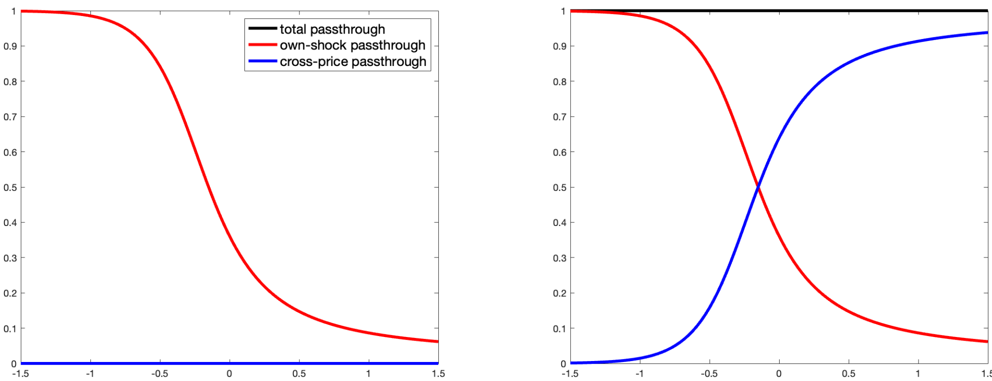
This is the main theorem of this paper. It establishes that the total passthrough is generically incomplete if there exists any information friction. The full-information case is a knife-edge case. I first explain the result on incompleteness. The key equation to understand the intuition of this result is:

$$1 - (\gamma_k + \xi_k) = (1 - \gamma_k)(1 - \theta) > 0 \tag{36}$$

It shows that the difference between the sum of the passthroughs, $\gamma_k + \xi_k$, and one can be decomposed into two terms. The first term captures the role of the search friction. The own-cost passthrough is not related to the information friction. The second term captures the role of information asymmetry about the nominal wage between consumers and firms. They are both positive. Figure 1 illustrates the three passthroughs. The right panel shows that under full information, the decrease in the own-shock passthrough over productivity is exactly offset by the increase in cross-price passthrough. The resulting total passthrough is always one for firms of any productivity. This result corresponds to Theorem 2 in which I prove the exact result. Notably, Amiti et al. (2019) shows that the total passthrough is one for broad preferences including nested CES and first-order Kimball demand family as well as for the broad homothetic families of demand considered in Matsuyama and Ushchev (2017). The theorem complements their results and emphasizes that the complete information about the price index is also essential. In fact, in all these commonly used demand system, the complete knowledge of prices set by individual firms is often assumed.

On the other hand, in the absence of any information as shown in the left panel, the cross-price passthrough is zero for any firm. Then, the total passthrough equals the own-cost passthrough. The intuition is that when shoppers do not perceive any change in the price index, firms find it

Figure 1: Distribution of three types of passthroughs under incomplete and complete information



Notes: The figure plots distributions of passthroughs based on one calibration of the model. The red line represents own-cost passthrough. The blue line represents cross-price passthrough. The black line represents total passthrough. The left and right panels show passthroughs in the incomplete and complete information cases, respectively.

optimal not to pass any of the actual change in the price index onto their own prices, in order to dissuade marginal shoppers from search. In the intermediate case when information is incomplete, the increase in cross-price passthrough is still not sufficient to offset the decrease in own-shock passthrough, leading to incomplete total passthroughs.

Moreover, the individual total passthrough is defined as $\varphi_k = \gamma_k + \Phi\xi_k$. The effect of strategic complementarity is further dampened by the aggregate total passthrough as $\Phi < 1$. This induces a even smaller individual total passthrough. Intuitively, the fact that shoppers understand that aggregate total passthrough is incomplete reduces the expected price index even more. Firms then reduce prices in response, resulting in even lower passthrough. Therefore, the incompleteness of the total passthroughs is amplified by the firms' incentive of setting prices close to the price index, which is a form of real rigidity (Klenow and Willis, 2016).

I leave the discussion of composition and comparative statics results to the later section. Here, I focus on clarify the main insight behind the incompleteness result.

Main Insight: Strategic Complementarity Attenuation – In Proposition 3, we have discussed that the strategic incentives of firms are dampened by the information friction on the shopper side. In this section, I "dig deeper" to show how such mechanism for the individual firms relates to the attenuation on the strategic complementarity in pricing on the *aggregate*. For this purpose, I will borrow heavily from game theory as in Morris and Shin (2002), Woodford (2003), and Angeletos and Lian (2023). The following proposition presents a way to transform the above economy into a beauty contest game.

Proposition 4. *The model economy can be expressed as a beauty contest game as follows,*

$$\hat{p} = \Gamma\hat{w} + (1-\Gamma)\bar{E}\hat{p} \tag{37}$$

where $\bar{E}\hat{p}$ represents shoppers' average expectation of the change in price index. Under the rational expectations equilibrium, the change in price index can be express as the infinite sum of higher-order beliefs of the monetary shock $\hat{w}$,

$$\hat{p} = \Gamma\sum_{h=0}^{\infty}(1-\Gamma)^h \bar{E}^h\hat{w} \tag{38}$$

where $\bar{E}^h\hat{w} = \bar{E}(\bar{E}^{h-1}\hat{w})$, and $\bar{E}^0\hat{w} = \hat{w}$. Using the fact that $\bar{E}^h\hat{w} = \theta^h\hat{w}$, the aggregate total passthrough is given by,

$$\Phi = \Gamma\sum_{h=0}^{\infty}(1-\Gamma)^h\theta^h \tag{39}$$

Proof. See Appendix A. ■

The proposition frames the model economy as a beauty contest game, where firms exhibit strategic complementarity in response to *shoppers'* beliefs about the price index. This is distinguished from the standard beauty contest game between firms (Woodford, 2003), where strategic complementarity is driven by *firms'* beliefs about the price index. Therefore, to write the game into the infinite sum of higher-order beliefs as in (38), the requirement on rationality is different. In particular, we need (i) firms are rational and (ii) shoppers know that firms are rational and (iii) the common knowledge of rationality among shoppers: the first iteration requires that shoppers know that firms and other shoppers are rational, the second iteration requires that shoppers know that others know they are rational and firms are rational, and so on. In Appendix A, I show which rationality is required in each step of derivation. The rational expectations equilibrium is a "super" concept that includes all these rationality.

To understand the intuition, we can interpret this infinite sum as the aggregate response of firms unfolding in the notional time within one period. In the first round, consider no firm has yet responded to the monetary shock. The shock therefore acts as an idiosyncratic shock to each firm. The resulting price increase is thus $\Gamma\hat{w}$. In the second round, it is the common knowledge among firms that the price index has increased by this amount. Then, due to the strategic complementarity to other firms' price increases, they pass additional portion of the shoppers' believed increase in the price index from the first round to prices, i.e., $(1-\Gamma)\bar{E}\Gamma\hat{w}$. Iterate forward, in the $h+1$th round, the additional passthrough is $(1-\Gamma)^h\bar{E}^h\hat{w}$. Notice that the above intuition leverages another layer of rationality: it is common knowledge among firms that firms know shoppers possess a common knowledge of rationality.[10]

The information friction attenuates the strategic complementarity in pricing. It attenuates more the high-order strategic complementarity, since the higher-order beliefs of shoppers are more anchored to the prior. For θ close to zero (meaning a sufficiently large departure from common knowledge of the shock), the aggregate total passthrough is arbitrarily close to the own-cost passthrough. But as θ increases (meaning a higher degree of common knowledge of the shock), the higher-order strategic complementarity rises rapidly and thus increases overall passthrough. Therefore, by varying θ between 0 and 1, we can thus span all the values between the aggregate own-cost passthrough and the full-information outcome.

Price, Elasticity and Passthrough – I now connect the results so far to understand the relationship between price, elasticity and passthrough. Suppose the nominal wage increases by 1%. The aggregate total passthrough in equilibrium is 0.4. Thus, firms on average raise their prices by 0.4%. If a firm deviates and instead raises its price by 1%, according to Proposition 3, its elasticity will increase due to both the information friction and the incomplete aggregate total passthrough. This would reduce its markup. More specifically, the increase in the elasticity can be understood as follows: the increase in the marginal shoppers g outweighs the increase in the infra-marginal shoppers $1-G$. In this situation, the firm is more concerned about losing marginal shoppers to other firms than extracting additional profits from infra-marginal shoppers. This refrains the firm from passing fully through the shock to its price. This interpretation echos the message in Rotemberg (1982), which is that firms, fearing to upset consumers, limit the passthrough.

On the other hand, suppose the nominal wage decreases by 1%. If a firm reduces its price by 1%, its elasticity will decrease, resulting in a higher markup. In particular, since the firm will attract more shoppers as shoppers do not believe the price index has decreased by 1%, the firm is incentivized to extract more profits from infra-marginal shoppers. This again refrains the firm

[10]The intuition is also related to the literature on level-k thinking (García-Schmidt and Woodford, 2019 and Farhi and Werning, 2019) where the rounds of thinking $k \to \infty$.

from fully decreasing price in response to the shock. In this case, the appropriate interpretation is that firms limit the passthrough to gain more profits from infra-marginal consumers.

As a final remark, one may think that, if passthroughs from the shock to prices are complete, while shoppers are motivated to search more in each round, mass of shoppers still increases in subsequent rounds of shopping, since the total mass of shoppers is fixed. Firms may end up with same demand as in the steady state under some conditions.[11] However, this argument overlooks the fact that firms treat ρ as exogenously given. Since firms cannot collude on prices or impose penalties for deviations, each individual firm has a strong incentive to deviate from the strategy of fully passing the shock through to prices. It thus cannot be a Perfect Bayesian Nash Equilibrium.

Results on Individual Passthroughs – In this section, I complete the picture by showing the properties of three types of individual total passthroughs and their limit results.

I first present results on cross-sectional markups and passthroughs within an equilibrium. I define passthroughs on primitives: $\varphi_k = \varphi(a_k)$, $\gamma_k = \gamma(a_k)$ and $\xi_k = \xi(a_k)$.

Proposition 5. *In a given equilibrium, markup $\mu(a)$ and cross-price passthrough $\xi(a)$ increases in productivity; own-cost passthrough $\gamma(a)$ and total passthrough $\varphi(a)$ decreases in productivity. Also, the following limit results hold:*

1. $\lim_{a\to\infty} \mu(a) = \infty$; $\lim_{a\to\infty} \varphi(a) = \Phi\theta$; $\lim_{a\to\infty} \gamma(a) = 0$; $\lim_{a\to\infty} \xi(a) = \theta$
2. $\lim_{a\to-\infty} \mu(a) = \frac{\lambda+1}{\lambda}$; $\lim_{a\to-\infty} \varphi(a) = 1$; $\lim_{a\to-\infty} \gamma(a) = 1$; $\lim_{a\to-\infty} \xi(a) = 0$

Proof. See Appendix A. ■

The proposition shows that high-productivity firms set lower prices, attract more shoppers, and choose higher markups. Similar to oligopolistic CES models, the lower bound of markups is determined by the degree of substitutability between goods, while the upper bound approaches infinity. These firms also have lower own-cost passthrough and higher cross-price passthrough. Since the information friction dampens only the cross-price passthrough, this leads to a more pronounced decline in cross-price passthrough and, consequently, total passthrough. As a result, high-productivity firms contribute a lot more to the incompleteness of the aggregate total passthrough. Proposition 5 also aligns with with empirical evidence highlighted in the literature: (i) more productive firms charge higher markups (Amiti et al., 2014); (ii) more productive firms pass through less exchange rate shocks (Amiti et al., 2019).

[11] The conditions are that firms have same productivity and shoppers do not back out money supply from price.

Second, to understand the results of comparative statics of the aggregate total passthroughs in Theorem 3, I establish that, all else equal, (i) the total passthrough is larger when the search cost is smaller for any given productivity and (ii) the total passthrough is larger when information friction is smaller for any given productivity.

Proposition 6. *Let equilibrium passthrough distributions $\gamma(a;\kappa), \xi(a;\kappa), \varphi(a;\kappa)$ for given κ. Given $\sigma_z > 0$, for $\kappa_2 > \kappa_1$,*

1. *Own-cost passthrough: $\forall a, \gamma(a;\kappa_2) < \gamma(a;\kappa_1)$; $\lim_{\kappa\to\infty} \gamma(a) = 0$; $\lim_{\kappa\to 0} \gamma(a) = 1$*
2. *Cross-price passthrough: $\forall a, \xi(a;\kappa_2) > \xi(a;\kappa_1)$; $\lim_{\kappa\to\infty} \xi(a) = \theta$; $\lim_{\kappa\to 0} \xi(a) = 0$*
3. *Total passthrough: $\forall a, \varphi(a;\kappa_2) < \varphi(a;\kappa_1)$; $\lim_{\kappa\to\infty} \varphi(a) = 0$; $\lim_{\kappa\to 0} \varphi(a) = 1$*

Proof. See Appendix A. ■

This proposition establishes that, all else equal, total passthrough is lower in an economy with higher search costs for any given productivity. Therefore, the search friction can amplify the effect of the information asymmetry on total passthroughs. As we know from Lemma 1, higher search cost lowers the thresholds, which results in lower elasticity of demand. Higher markups are more sensitive to the change in prices, leading to lower own-cost passthrough. Based on (36), lower own-cost passthrough implies larger gap between the sum of two fundamental passthroughs and one, leading to lower total passthrough. In addition, the limit results implies that the total passthrough can vary from zero to one, implying the possibility of large degree of monetary non-neutrality.

Proposition 7. *Let equilibrium passthrough distributions $\gamma(a;\theta), \xi(a;\theta), \varphi(a;\theta)$ for given θ. For $\theta_2 > \theta_1$,*

1. *Own-cost passthrough: $\forall a, \gamma(a;\theta_2) = \gamma(a;\theta_1)$*
2. *Cross-price passthrough: $\forall a, \xi(a;\theta_2) > \xi(a;\theta_1)$; $\lim_{\theta\to 1} \xi(a) = 1 - \gamma(a)$; $\lim_{\theta\to 0} \xi(a) = 0$*
3. *Total passthrough: $\forall a, \varphi(a;\theta_2) > \varphi(a;\theta_1)$; $\lim_{\theta\to 1} \varphi(a) = 1$; $\lim_{\theta\to 0} \varphi(a) = \gamma(a)$*

Proof. See Appendix A. ■

This proposition highlights that, all else equal, the total passthrough increases at any given productivity as information becomes more precise. Intuitively, if shoppers are more aware of changes in price index, firms will be more responsive to changes in competitors' prices. Higher cross-price passthrough pushes up the total passthrough. In contrast, the own-cost passthrough is irrelevant to the information frictions. In the dynamic model, shoppers will learn the shock over time. This proposition shows that if learning leads to the common knowledge of the shock, the monetary policy will be neutral in the long run.

Taken together, Theorem 3 shows that the passthrough from changes in the nominal wage to price index is generically incomplete. The incompleteness is mostly contributed by high-productivity firms. The aggregate passthrough is lower when there is more search frictions and information frictions. Proposition 4 shows that the main insight lies on the attenuation effect of the information friction on strategic complementarity. Propositions 5-7 yield more disaggregated predictions of three types of passthroughs for individual firms and additional limit results.

1.6 General Equilibrium Model

In this section, I close the above partial equilibrium model in the general equilibrium. The timeline is as follows. The period is divided into morning and afternoon. In the morning, the monetary authority sets the nominal GDP, and shopper make decisions about labor supply and the cash they will spend on shopping in the afternoon. In the afternoon, firms post prices, and shoppers are constrained by the amount of cash allocated to them in the morning. Unlike in the partial equilibrium model, the cash in hand X_i is now determined endogenously.

Notably, there is no ex-ante securities market in which shoppers could trade contingent claims that are paid off conditional on final choices of goods and the number of searches. For example, a security might provide positive returns if a shopper experiences a long sequence of unfavorable draws or if the final choice only slightly exceeds the threshold.[12]

I state the shopper's problem in the morning. The shopper maximizes the expected value that she will obtain in the afternoon, net of the disutility associated with labor supply.

$$\begin{aligned} &\max_{X_i, L_i} E_i \left(\log \frac{X_i}{P_k} + \frac{1}{\lambda} \epsilon_{ik} \right) - L_i \\ \text{s.t.} \quad & X_i = W_i L_i + \Pi_i \end{aligned}$$

where X_i is the consumption expenditure used for shopping in the afternoon. P_k and ϵ_{ik} are the price and the match utility of the firm that the shopper accepts in the search process. Since the

[12]Mongey and Waugh (2024) show that the demand allocations in a standard discrete-choice model without search frictions can be different when the market is complete.

final choice of the firm is random, the shopper takes expectation based on her own information when making the decision in the morning. L_i is the labor effort, W_i is shopper i' specific nominal wage, and Π_i is part of firms' total nominal profit that is accrued to the shopper.

Due to the log utility, the cash X_i only shifts the level of the value and does not affect the search decisions. Therefore, the shopper's problem in the morning and afternoon can be separated. It also implies that shoppers' choices on labor supply and consumption expenditure are time-consistent. In other words, the shopper will not work and consume more, if she obtains a good with high match utility and low price. The reason is that under log utility, the real income effect just offsets the substitution effect of buying cheaper goods.[13]

The first-order conditions imply $X_i = W_i$. Also, by definition, $p_i + c_i = x_i$. Aggregation gives,

$$\hat{p} + \hat{c} = \hat{w} \tag{40}$$

I assume that shoppers only learn the aggregate nominal wage W from W_i, not from Π_i to maintain the information structure as before. W_i draws from the distribution $\mathcal{N}(0, \sigma_x^2)$ same as in (5). In the afternoon, shoppers search sequentially. This part of the model is the same as in the partial equilibrium model.

Aggregate Supply Shocks The framework can also incorporate the aggregate supply shocks. Here, we consider aggregate supply shocks as aggregate productivity shocks. In particular, the firm productivity has two components,

$$\log A_k = \log A + \sigma_a \varepsilon_{ak} \tag{41}$$

where A is the aggregate productivity shock. Let $a = \log A$. It draws from $\mathcal{N}(0, \sigma_A^2)$. I only consider one aggregate shock a time. Specifically, when analyzing aggregate supply shocks, the nominal wage is held constant and is known to all agents. The shopper also receives noisy signal about a.

$$s_{ai} = a + \sigma_{as} \varepsilon_{ai} \tag{42}$$

where ε_{ai} is i.i.d across shoppers and it follows $\varepsilon_{ai} \sim \mathcal{N}(0, 1)$.

[13]This point is also hinted in Mongey and Waugh (2024). They show that under log utility, the demand allocation is the same for both complete and incomplete markets.

Since both the nominal wage and the aggregate productivity affect the prices only through the marginal costs, the optimal pricing strategy is homogeneous of degree zero in $\{W, A\}$, which implies,

$$p^*(a_k, w, a) = p^*(a_k, w - a) \tag{43}$$

Any positive change in aggregate productivity acts equivalently to a proportional decrease in the nominal wage. Therefore, I can similarly define $\hat{p}_k = -\varphi_k \hat{a}$ and aggregate total passthrough Φ. The value of Φ is different from the one in the case of monetary shock because shoppers receive different signals about the aggregate shock.

Phillips Curve – In this section, I present the Phillips curve. The following proposition shows how two consumer-side frictions influence the slope of Phillips curve.

Proposition 8. *The Phillips curve is given by,*

$$\hat{p} = \frac{\Gamma}{1-\Gamma}\frac{1}{1-\theta}(\hat{c} - \hat{a}) \tag{44}$$

where θ is the average information friction about the particular shock of interest.

Proof. See Appendix A. ■

This proposition shows that the slope of Phillips curve is composed by two parts. First, the slope increases with the aggregate own-cost passthrough Γ, which decreases in the search cost. Second, the slope is inversely related to the degree of information asymmetry. Under full information, the slope becomes vertical, implying monetary neutrality. In the other extreme, the slope approaches $\frac{\Gamma}{1-\Gamma}$, which implies that the lower bound of the slope is governed by the aggregate own-cost passthrough, corresponding to the limit results in Theorem 3. Moreover, the aggregate own-cost passthrough is a sufficient statistic that summarizes the effects of the "deep" parameters on the slope, i.e., $\kappa, \lambda, \sigma_a$. As I have shown in Proposition 6, the own-cost passthrough in principle can vary from 0 to 1.

This Phillips curve has two important differences from the standard New-Keynesian Phillips curve. First, I rewrite the Phillips curve in (44) in the following way,

$$\hat{p} - \bar{E}\hat{p} = \frac{\Gamma}{1-\Gamma}(\hat{c} - \hat{a}) \tag{45}$$

The Phillips curve depends on the expectation of current price index instead of expectation of future inflation. In particular, the output gap is proportional to shoppers' nowcast error of price index. In a seminal paper, Coibion and Gorodnichenko (2015) test the full-information rational expectation hypothesis in various surveys, including the Michigan Survey of Consumers. They find that average household expectation under-reacts to aggregate shocks. It implies a positive gap $\hat{p} - \bar{E}\hat{p}$ for positive monetary shock and vise versa.

The distinction between static and dynamic Phillips curves highlights that firms' forward-looking behavior is not essential to explaining monetary non-neutrality. In fact, the assumption that firms are forward-looking when setting prices may conflict with evidence from anticipated VAT reforms. Buettner and Madzharova (2021) find a rapid passthrough of VAT-induced cost changes to prices within four months—two months before and two months after the reform. This finding contradicts the predictions of the standard NK model, which suggests that firms would begin raising prices at least one year prior to the reform. In contrast, my model predicts no price changes before the reform and full passthrough upon implementation, as these tax reforms are common knowledge to consumers, resulting in minimal price stickiness. In practice, additional factors influence price dynamics: intertemporal substitution of durable goods tends to elevate prices before the reform, while delays in cost passthrough within the supply chain can defer price increases until after implementation.

Second, in modern Phillips curve models, firms' expectations play a central role in driving current inflation. Even in Lucas (1972), firms' confusion about the idiosyncratic and aggregate demand shocks gives rise to monetary non-neutrality. However, the Phillips curve here presents an alternative view that household expectations can influence firms' pricing decisions as well. This has implications on how to estimate the slope of Phillips curve using the survey data. Recent efforts have been devoted to conducting new surveys of firms' expectations (Candia et al., 2023). I argue that household survey should be given enough emphasis as well.

2 Dynamic Model

I now present the full-fledged dynamic general equilibrium model. I introduce a new agent: worker. In particular, there is a representative household which consists of a single worker and a continuum of shoppers. I assume that the worker has full information about the model economy. The division between worker and shopper in a household is only used to simplify the problem. In particular, without information frictions on the worker side, we can obtain the standard Euler equation. This helps us focus on the monetary non-neutrality generated only by the frictions on the shopper side.

The timeline is as follows. Each period is divided into morning and afternoon. In the morning, the monetary authority sets the interest rate, and the worker makes decisions on the labor supply, the bond position, and the total consumption expenditure transferred to shoppers. In the afternoon, bond market closes, firms post prices, and shoppers search sequentially and they are constrained by the cash in hand. I assume that the worker and shoppers cannot communicate. Also, there is no security markets where shoppers can trade claims that are contingent on the search process and results.

Firm – Let A_{kt} denote the firm's productivity

$$\log A_{kt} = \log A_t + \sigma_a \varepsilon_{akt} \tag{46}$$

where $\varepsilon_{akt} \sim \mathcal{N}(0,1)$. A_t is the aggregate productivity. Denote $a_t = \log A_t$. It follows an AR(1) process,

$$a_t = \rho_A a_{t-1} + \sigma_A \varepsilon_{At} \tag{47}$$

where ε_{At} is the shock to aggregate productivity. It follows $\varepsilon_{At} \sim \mathcal{N}(0,1)$.

Monetary Authority – The monetary authority sets the nominal interest rate. It follows the Taylor rule,

$$i_t = \phi \hat{\pi}_t + \upsilon_{mt} \tag{48}$$

where υ_{mt} follows,

$$\upsilon_{mt} = \rho_m \upsilon_{mt-1} + \sigma_m \varepsilon_{mt} \tag{49}$$

where ε_{mt} is the monetary shock and it follows $\varepsilon_{mt} \sim \mathcal{N}(0,1)$.

Worker – The worker maximizes the expected discounted utility with discount factor $\beta \in (0,1)$ and period utility defined over the sum of values which shoppers will obtain in the afternoon. Worker can save in risk-free bonds B_t (in zero net supply) that pay an interest rate of R_t.

$$\begin{aligned} &\max_{B_t, X_t, L_t} E_0 \sum_{t=0}^{\infty} \beta^t \left(\int \left(\log \frac{X_{it}}{P_{kt}} + \frac{1}{\lambda} \epsilon_{ikt} \right) di - L_t \right) \\ \text{s.t.} \quad & X_t + B_t = W_t L_t + R_{t-1} B_{t-1} + \Pi_t \\ & X_{it} = X_t \exp\left(\sigma_x \varepsilon_{xit} - \frac{\sigma_x^2}{2} \right) \end{aligned}$$

where X_t is the total consumption expenditure transferred to all the shoppers. The division of the consumption expenditure among shoppers is random, where $\varepsilon_{xit} \sim \mathcal{N}(0,1)$ and $\int X_{it} di = X_t$. L_t is the labor effort, W_t is nominal wage, and Π_t is total nominal profits of firms. Again, the worker's problem can be simplified due to the log utility.[14]

Shopper – The shopper solves the static problem in (11) period by period given the history of signals. I assume that each shopper receives a noisy signal about $\hat{\pi}_t$ at the beginning of each period. This signal structure accommodates the learning from the price index at current period and allows us to specify a unified information structure for two type of shocks. In addition, I make the simplifying assumption that shoppers observe the past price level but do not extract information from it. Following Mondria et al. (2021) and Angeletos and Huo (2021), this assumption can be interpreted as a form of bounded rationality or inattention. The sole purpose of this assumption is to transform the Phillips curve in (45) into a relationship that directly links the output gap to the nowcast error of inflation, rather than price index. This transformation allows for a direct comparison of the impulse responses generated by our dynamic general equilibrium model with those produced by the standard NK model.

Dynamic Equilibrium – I present the proposition that describes the dynamics of consumption, inflation and interest rate in equilibrium.

Proposition 9. *The equilibrium dynamics of $\{\hat{\pi}_t, \hat{c}_t, i_t\}$ is described by the following system of three equations:*

$$\begin{aligned}
\hat{c}_t &= E_t \hat{c}_{t+1} - (i_t - E_t \hat{\pi}_{t+1}) \\
\hat{\pi}_t - \bar{E}^s \hat{\pi}_t &= \frac{\Gamma}{1-\Gamma}(\hat{c}_t - \hat{a}_t) \\
i_t &= \phi \hat{\pi}_t + v_{mt}
\end{aligned}$$

where Γ is the aggregate own-cost passthrough and $\bar{E}^s$ is the shoppers' average expectation.

Proof. See Appendix B. ■

This system is possible because (i) the division between the worker and shoppers gives us the standard Euler equation and (ii) shoppers know the last-period price index and do not extract

[14]The time-consistency is not an issue here because the uncertainty of final choice of the firm for each shopper is washed out on the household level (with that integral). For the worker, the ex-ante sum of shoppers' value is equal to the ex-post one.

information from it. These assumptions generates a system that is only different from the standard NK model in terms of the Phillips curve. This helps us focus on how our mechanism alone, which is essentially static, influences the dynamics of the system. In Appendix B, I relax both assumptions. In particular, I obtain incomplete-information Euler equation as in Angeletos and Lian (2018). In addition, I use the Phillips curve in (45) combined with a price-level targeting monetary rule. I provide a detailed discussion of the dynamics of this system.

2.1 Calibration

In this section, I present a calibration of the model. I proceed in two steps. First, I calibrate the aggregate own-cost passthrough and deep parameters that are related to the search friction. Second, I calibrate the information friction based on the empirical evidence.

Aggregate own-cost passthrough − The search cost in the model is described as a utility cost, which is hard to measure in the data. Fortunately, the aggregate own-cost passthrough is a sufficient statistic for deep parameters that are related to the search friction, i.e., search cost κ, relative importance of match utility λ and standard deviation of productivity distribution σ_a. Another property of aggregate own-cost passthrough is that it does not depend on the information friction. Therefore, we can separate the calibration of these two frictions.

Amiti et al. (2019) show that the aggregate own-cost passthrough is around 0.5 in the universe of Belgian manufacturing firms. Gopinath et al. (2011) use a retail chain database which contains information on wholesale costs and demonstrate substantial variation in own-cost passthrough estimates, with a median of around 0.5 for U.S. stores and 0.25 for Canadian stores. Own-cost passthrough estimates are often biased upward because, without controlling for all competitors' prices, the estimates may capture strategic complementarity effects. Specifically, other firms respond simultaneously to both the correlated cost shocks and change in that firm's price. Therefore, I pick $\Gamma = 0.3$ which is closed to the lower end of the their estimates.

Interestingly, Amiti et al. (2019) also show that the sum of own-cost and cross-price passthroughs cannot be rejected from being one. This seems to indicate that the passthrough from the shock to prices is close to one in the model. However, search and information frictions are more prevalent in non-tradable industries that directly engage with household, such as retail and broad service sectors. In contrast, the buyer-seller relationships in tradable industries usually involves firms on both sides and are often governed by contracts (Gopinath et al., 2011), where firms are generally better informed about prices.

Although the deep parameters related to the search friction are not relevant for impulse responses, it will still be valuable to check the sanity of the implied values of these parameters. To proceed, I use the estimate of elasticity of substitution from DellaVigna and Gentzkow (2019). They use NielsenIQ Retailer Scanner database and find that the average elasticity of substitution across stores and products is 0.25, implying a markup of 1.67. In the data, one retailer sell a wide range of products. In one of the extension, I generalize the model to the setup where one firm sell multiple products and there is no search friction within the store. I show that retailers charge the same markup for all products they sell. In addition, the productivity dispersion σ_a is set to be consistent with Decker et al. (2020), i.e., $\sigma_a = 0.3$. The following table shows the baseline calibration for deep parameters that do not relate to the information friction.

Table 1: Baseline calibration of the model

Parameter	Description	Value
κ	Search cost	0.22
λ	Relative importance of match utility	5.51

Notes: The table reports the calibrated values for parameters that are related to the aggregate own-cost passthrough.

Table 2: Model fit

	Moment	Model	Data	Source
M1	Average markup	1.67	1.67	DellaVigna and Gentzkow (2019)
M2	Average own-cost passthrough	0.3	0.3	Amiti et al. (2019); Gopinath et al. (2011)

Notes: The table summarizes the moments, model and data values of these moments, and the sources of the empirical values of these moments.

Table 1 summarizes the calibrated parameters, and Table 2 presents the fit of the model to data. Despite its parsimonious structure, the model is successful in matching key moments in the data. They have two implications. First, although the targeted elasticity of demand is 2.5, the implied λ+1, which represents the elasticity of demand when $\kappa \to 0$ as shown in Proposition 6, is about 6.5. This suggests that the search friction accounts for a substantial bulk of the market power. Second, Based on the calibrated values of deep parameters, we can infer the average contact per shopper ρ^{-1} equal to 1.26. This suggests a relatively large search cost and about 80% shoppers make the purchase on the first search. Therefore, Assumption 1 is plausible, as shoppers only visit a limited number of firms.

Information friction – In our Phillips curve, the output gap is proportional to the nowcast error of inflation. Therefore, it is crucial to understand how households learn about inflation. However, households may incorporate various sources of information about inflation conditional on shocks. For instance, Kumar et al. (2015) shows that households pay particular attention to salient prices, such as oil prices. Candia et al. (2023) documents that households may confuse demand shocks with supply shocks. Designing the specific information structure for each shock is out of the consideration of this paper.

To proceed, I assume that households always receive a noisy signal about inflation as a "generic" way to learn about inflation, regardless of the specific nature of underlying shocks. To isolate this "generic" learning, I will use the main inflation shock from Angeletos et al. (2020). This shock is identified by maximizing its contribution to the business-cycle variation in inflation. The key feature of this shock is that it has a very small footprint on real quantities and zero footprint on TFP. It is thus akin to the cost-push shocks in the DSGE literature. Given that this shock impacts only inflation, with no effect on other real variables, therefore, household learning relies solely on observing the inflation itself.

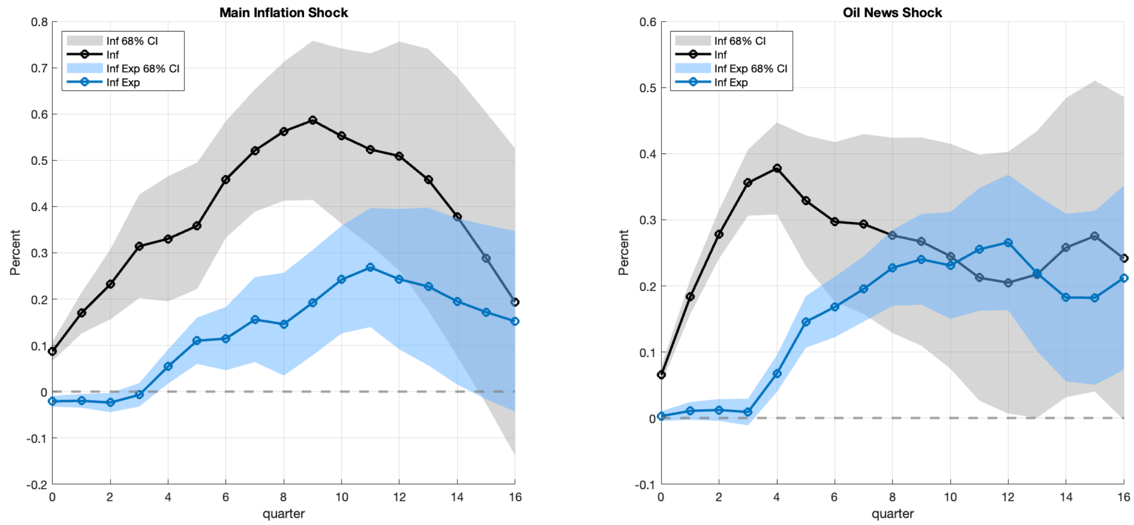
The main empirical strategy is to estimate impulse responses directly using the local projection method a là Jordà (2005). The specification is,

$$y_{t+h} = \alpha_h + \beta_h \varepsilon_t + \mathbf{\Gamma}' \mathbf{X}_t + u_{t+h} \tag{50}$$

where $\{\beta_h\}_{h=0}^{H}$ trace out the dynamic responses of the outcome. y_t is the inflation π_t and the household average inflation expectation a year ago $\bar{E}_{t-12}\pi_t$. X_t is a vector of controls. It includes 12 lags of shock itself, inflation, inflation expectation a year ago, 1-year government bond rate, growth rate of industrial production. I construct standard errors for the coefficients that are heteroskedasticity and autocorrelation robust (HAC). All reported error bands are 90% confidence intervals. In Appendix C, I show the robustness of the impulse responses across different empirical specifications.

The left panel of Figure 2 shows the impulse responses following the main inflation shock. The black line represents the impulse responses of inflation and the blue line represents the impulse

Figure 2: Impulse Responses of Inflation and Household Inflation Expectation



Notes: Dynamic responses: outcomes and forecasts. The left panel shows the responses following a main inflation shock. The right panel shows the responses following a oil news shock. The sample period is Q3 1979–Q4 2017. The black line represents the impulse responses of inflation and the blue line represents the impulse responses of inflation expectation. The shaded areas are 68% confidence intervals based on heteroskedasticity and autocorrelation robust standard errors and 12 lags. The x-axis denotes months from the shock, starting at 0. The y-axis denotes percent.

responses of inflation expectation a year ago. Since the shock is unanticipated, inflation expectations remain unchanged for the first year. They begin to adjust only when households observe and learn about the inflation. The key takeaway from this figure is that learning from inflation is slow. The inflation responses are always above the inflation expectation responses. The two error bands of the impulse responses begin to overlap at the end of the third year, and the median impulse response intersect more than four years.

In the model, I assume that the shopper receives a noisy signal about unexpected inflation $\hat{\pi}_t$ in each period. I calibrate the signal-to-noise ratio of this signal such that two impulse responses of the inflation and the inflation nowcast coincide in about 3 years after the shock. In the right panel of Figure 2, I present the impulse responses following an oil news shock identified in Känzig (2021). I find that the response of inflation expectations converges to the response of actual inflation much faster compared to the case of main inflation shocks. This is consistent with Kumar et al. (2015) who shows that households in general pay more attention to the oil prices. As a result, inflation driven by oil news shocks is more readily and quickly observed by households.

In addition, I compare our model with the standard NK model with the Phillips curve as follows,

$$\hat{\pi}_t = \kappa(\hat{y} - \hat{a}) + \beta E_t \pi_{t+1} \tag{51}$$

where $\kappa = (1 - \beta\psi)(1 - \psi)/\psi$ and ψ is the probability of a firm not being able to adjust its price, which we call Calvo parameter. Applying $E_t\pi_{t+1} = \rho\pi_t$ and ρ is the persistence of the shock,

$$\hat{\pi}_t = \frac{1}{1 - \beta\rho}\kappa(\hat{y} - \hat{a}) \tag{52}$$

To show how big the impact of the information friction and learning channel, I take the value $\kappa = \frac{\Gamma}{1-\Gamma}$. The implied value for Calvo parameter is 52.5%.

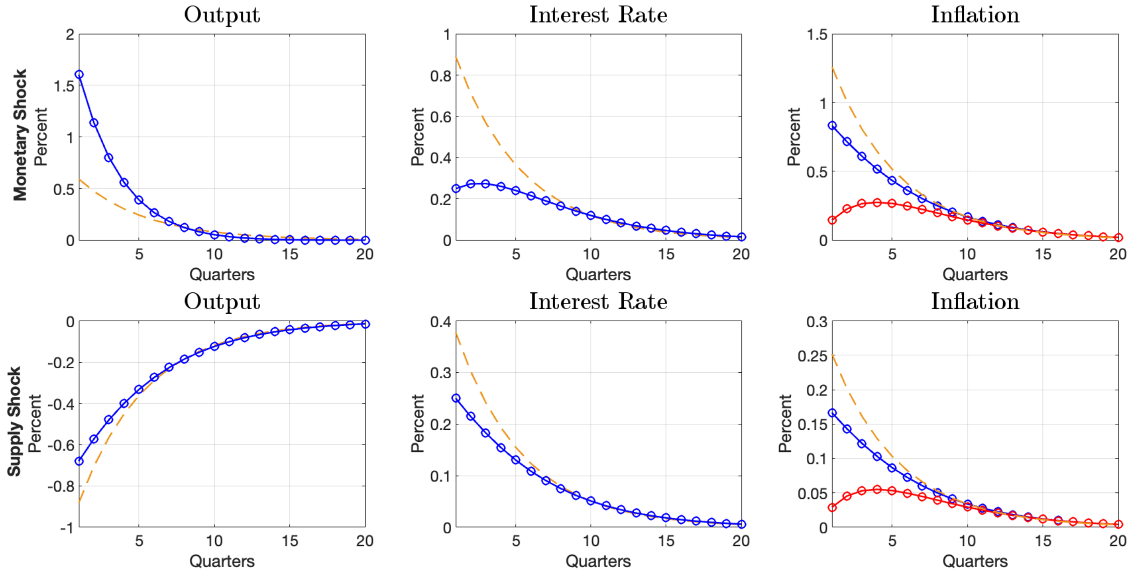
2.2 Impulse Responses of the Calibrated Model

In this section, I discuss the impulse responses of the calibrated model. The calibrated model is computed using the frequency-domain methods based on Rondina and Walker (2021), Huo and Takayama (2023), and Han et al. (2022). The Taylor rule parameter is calibrated as $\phi = 1.5$. Figure 3 presents the results. The first row shows the impulse responses of output, interest rate, inflation and inflation nowcast following a 100 basis-point interest rate cut based on the calibrated model. The second row presents the impulse responses following a 100 basis-point reduction in the aggregate productivity. In the third column, the blue line represents the inflation and the red line represents the inflation nowcast. Our theory predicts that the gap between these two lines is proportional to the output response. The dashed orange line plots the impulse responses of the standard NK model with Calvo parameter 52.5%.

There are three key implications about the monetary non-neutrality from the calibration analysis. First, with exogenously calibrated parameters Γ and $\{\theta_t\}$, the model exhibits substantial monetary non-neutrality. A 100 basis-point cut in the interest rate results in an initial output increase of approximately 160 basis points. This response is comparable to that in a standard NK model where the probability of not adjusting price being around 70%. This probability aligns with the value that is usually calibrated from a DSGE model (Christiano et al., 2005 and Smets and Wouters, 2007). This suggests that the degree of monetary non-neutrality generated by our mechanism is sufficient to explain and match the macro-level impulse-response evidence.

Second, the monetary non-neutrality generated by the standard NK model is three times smaller than that in our model when $\kappa = \frac{\Gamma}{1-\Gamma}$. The reason is that in the survey, household inflation expectation lags far behind the actual inflation. When we calibrate this evidence into

Figure 3: Impulse responses of price index and output in the calibrated economy



Notes: The figure plots the impulse responses of output, interest rate, inflation and inflation nowcast following a 100 basis-point interest rate cut and a 100 basis-point decrease in the aggregate productivity based on the calibrated model. In the third column, the blue line represents the inflation and the red line represents the inflation nowcast. The dashed orange line represents the impulse responses generated from a standard NK model with the Calvo parameter 52.5%. The x-axis is quarters. The y-axis is percent.

the model instead of leveraging the knowledge imposed by the rational expectations, we achieve larger monetary non-neutrality. To understand the magnitude of such difference, notice that the coefficient $1/(1-\beta\rho)$ in (52) is the counterpart of $1/(1-\theta)$ in our model. In practice, the discount factor β is close to one and the persistence of the shock is usually large to match the persistent effect of the shock in the literature, e.g., $\rho = 0.8$. Then, $1/(1-\beta\rho) = 5$. In contrast, the implied information friction in the initial period from our calibration is $1/(1-\theta) = 1.2$, making the slope of the Phillips curve about four times smaller in our model.

Third, since the output response depends on the gap between actual inflation and the inflation nowcast, rapid learning can close this gap before the shock fully dissipates. Consequently, the persistence of the output response is shorter than the persistence of the monetary shock. This is consistent with the empirical evidence. As shown in Figure 3 of Bauer and Swanson (2023), following a monetary shock, industrial production reaches its trough in a year, while the CPI continues to decline and approximately reaches its trough after 40 months. This suggests that the persistence of the output response is shorter than that of the inflation response. Our mechanism provides an endogenous explanation for the less persistent output response.

Now, I turn to supply shock. Following a 100 basis-point reduction in the aggregate productivity is rather not surprising, the output declines and the inflation increases by less than 100 basis points. This occurs because prices remain below the full-information benchmark, which sustains aggregate demand above the natural output level and the higher labor supply in equilibrium. The persistence of the output response inherits the shock's persistence, when the gap between inflation and inflation nowcast narrows to zero. This differs from the case of a monetary shock, as the aggregate productivity shock directly impacts output.

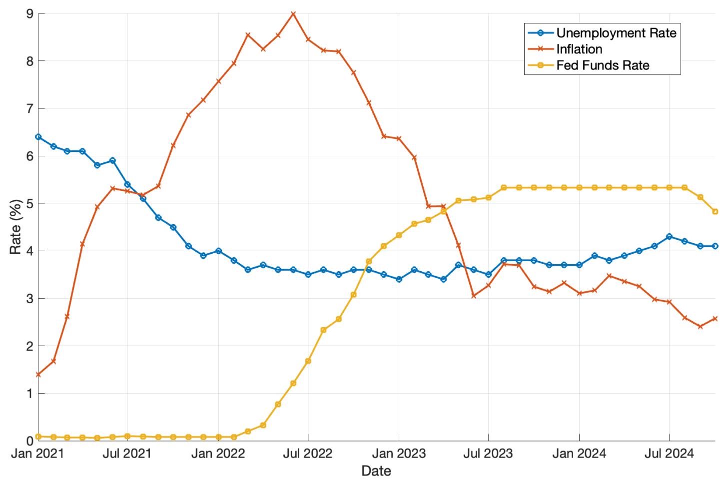
Overall, our mechanism can generate substantial monetary non-neutrality, comparable in magnitude to the calibrated Calvo parameter commonly used in DSGE literature. The model sheds light on how the survey data can be directly linked to the output gap. The persistence of the output response is endogenously determined, with faster learning leading to lower persistence.

Applications – I consider the application of the model to explaining the post-pandemic high inflation episode.

Figure 4 shows the unemployment rate, inflation and Fed funds rate after pandemic. Blanchard and Bernanke (2023) shows that supply shocks have a significant impact at the outset of the recent inflationary surge, while labor-market tightness exerts more persistent effects. Benigno and Eggertsson (2023) have the similar conclusion with some difference in the empirical strategy. Then, two puzzles emerges. First, the supply-induced inflation shoots high right after the pandemic, contrary to the expectations of many economists and policymakers who anticipated muted inflation due to the flat Phillips curve. Second, despite the Fed's aggressive interest rate hikes beginning in August 2022, demand-driven inflation declined rapidly without a substantial rise in unemployment.

Most of research on the first puzzle attributes the inflationary surge to the Phillips curve being near vertical. Benigno and Eggertsson (2023) proposes a non-linear New Keynesian Phillips curve where labor becomes more costly when the vacancy-to-unemployed ratio exceeds a certain value. Harding et al. (2023) shows that the non-linearity can arise due to a quasi-kinked demand schedule for goods produced by firms. However, these theories fail to address the second puzzle, as supply shortages have subsided. Instead, I propose a unified explanation for both puzzles through the information channel in our model.

Figure 4: Post-Pandemic Unemployment Rate, Inflation and Fed Funds Rate

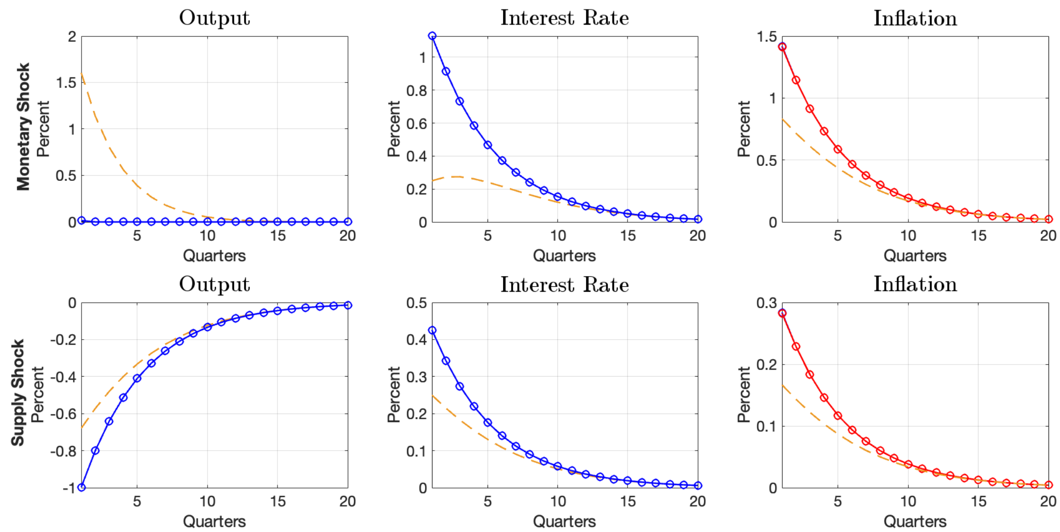


Notes: The figure shows the time series of unemployment rate, inflation and interest rate from Jan 2021 to Oct 2024. The x-axis is date. The y-axis is rate.

When the inflation first picked up, there was extensive public discussion and media coverage surrounding the onset of supply chain disruptions and labor shortages due to concerns about infections. Simultaneously, many restaurants posted notices informing consumers that they needed to increase prices because labor and material costs had risen. These public signals likely reduced information asymmetry, enabling households to learn more quickly about the rising costs faced by firms. Translating this scenario into the model setup implies that the signal regarding unexpected inflation becomes highly precise. As a result, the Phillips curve slope steepens, and cost changes are rapidly passed through to prices. The second row of Figure 5 illustrates the impulse responses following a supply shock when information is very precise. The orange dashed line represents the impulse responses based on our benchmark calibrated model. The increase in inflation is doubled relative to the benchmark calibration. The output and unemployment rate did not show a significant decline in reality compared to the model, probably due to the concurrent fiscal stimulus.

In the second phase, as the Fed raised interest rates, inflation began to decline. When information about inflation becomes precise, the slope of the Phillips curve becomes nearly vertical as shown in (8), resulting in minimal changes to real economic activity. The first row of Figure 5 illustrates the impulse responses to a demand shock under precise information. It is clear that the output response is close to zero.

Figure 5: Impulse Responses of Inflation and Household Inflation Expectation



Notes: The figure shows the impulse responses of output, interest rate, inflation and inflation nowcast following a 100 basis-point decrease in the aggregate productivity in a calibrated model with a large signal-to-noise ratio. The orange dashed line represents our benchmark calibrated model. In the third column, the blue line represents the inflation and the red line represents the inflation nowcast. The x-axis is quarters. The y-axis is percent.

Policy Implication – The main policy lesson from this application is that the slope of the Phillips curve is endogenous to the level of information available on the consumer side. Conventional Phillips curves, derived from reduced-form assumptions such as infrequent price adjustments and menu costs, are vulnerable to the Lucas Critique. Specifically, economic shocks are not isolated from the broader society; while individuals may not immediately recognize the shock itself, they often discuss its consequences. Certain shocks, in particular, may trigger a sequence of public events, policy communications, and announcements—such as FOMC meetings and direct fiscal transfers to households—which significantly increase households' awareness of the shock, resulting in a steeper slope of Phillips curve. This has important implications for the conduct of monetary policy. The monetary authority should adopt a more aggressive stance when information asymmetry is low, as prices are more responsive to the shock in such environments.

2.3 Extensions

I discuss two extensions. In the first extension, I allow multiple goods to be produced by one firm. The firm produces goods with different productivity for each good.

$$\log A_{kj} = \log A + a_k + a_{kj}$$

where a_{kj} is the productivity of producing good j by firm k. It is i.i.d following $a_{kj} \sim \mathcal{N}(0, \tilde{\sigma}^2_{ap})$. I assume that there is no search frictions when shopping within a firm. Shoppers decide which firm to purchase form and then buy the CES aggregation of all the goods in the firm. Let P_k denote the CES price index of multiple goods in firm k. We have the following result.

Proposition 10. *Each firm charges same markup over all the products it sells.*

$$P_{kj} = \frac{e_k}{e_k - 1} \frac{W}{A_{kj}} \tag{53}$$

where e_k is the elasticity of demand uniform for all j. It is determined by

$$P_k = \frac{e_k}{e_k - 1} \frac{W}{A_k}; e_k = \lambda \frac{\int X\big(g\big(\lambda(v^*(x) + p_k)\big)\big) d\Phi_x(x)}{\int X\big(1 - G\big(\lambda(v^*(x) + p_k)\big)\big) d\Phi_x(x))} + 1 \tag{54}$$

where P_k is the CES price index of P_{kj}. The passthrough of product-level productivity shocks a_{kj} increases toward one when the number of products increases.

This extension speaks to the empirical literature on passthrough of exchange rate shock to retail prices. Goldberg and Hellerstein (2013) and Nakamura and Zerom (2010) find complete passthrough of wholesale prices to retail prices for beer and coffee sales in retail stores.[15] The proposition shows that when there are many goods in one store, the passthrough of product-specific idiosyncratic shocks is closed to one. However, this does not mean that the passthrough of aggregate shocks and firm-specific shocks is complete.

In the second extension, I generalize the static general equilibrium model to accommodate finite labor supply elasticity. The details of setup and equilibrium are delegated to Appendix B. I present the following result.

Proposition 11. *The Phillips curve, when the elasticity of labor supply is η, is given by,*

$$\hat{p}_t = \frac{\Phi}{1 - \Phi - \frac{\eta(1-\tilde{\Phi})}{1+\eta}} (\hat{c}_t - \hat{a}_t) \tag{55}$$

where $\tilde{\Phi} \leq 1$ is defined in the Appendix B. $\tilde{\Phi} = 1$ if $\theta_t = 1$. ε_{mt} is the monetary shock.

[15] See Gopinath et al. (2011) for a summary of this literature.

This proposition presents a Phillips curve with an additional term capturing the effect of incomplete information on labor supply. Due to the log utility assumption, changes in the nominal wage should not influence labor supply. However, under incomplete information, shifts in search behavior following shocks lead to a different customer allocation across firms. For instance, in the case of an expansionary monetary shock, increased search activity by consumers drives higher demand for high-productivity firms, resulting in a more efficient allocation of demand. Consequently, the aggregate labor demand increases. In equilibrium, both labor supply and nominal wage increase, exerting upward pressure on the price index. As a result, the slope of the Phillips curve becomes steeper.

3 Empirics

In this section, I provide empirical support for the key mechanism of the model: unanticipated inflation increases search activities due to information frictions about marginal costs on the consumer side. To support this logic, we must establish two ingredients in the data: (i) the presence of information frictions and (ii) the reaction of search behavior under incomplete information about unanticipated inflation. The former is addressed in Section 2.1. Here, we focus on the second component.

In particular, I utilize a detailed consumer panel dataset that includes information on households' shopping trips, spending, store choices, and demographic characteristics to examine whether higher inflation is associated with increased measures of search behavior. My contribution to the literature lies in developing a novel measure of search activities that more accurately captures search efforts and aligns more closely with the predictions of standard sequential search models. Finally, I present evidence on the secular decline of search activities and explore how this trend may be connected to the decline in the slope of the Phillips curve.

3.1 Search Activities and Unanticipated Inflation

I begin by presenting evidence on how consumers adjust their search behavior in response to unanticipated inflation.

Data − The data source is the NielsenIQ Consumer Panel Data set.[16] The sample period is 2006 Q1 - 2019 Q4. NielsenIQ tracks the shopping behavior of average 55,000 households every year. Each household uses in-home scanners to record purchases. Households also record any deals used that may affect the price. These households represent a demographically balanced sample of households in 49 states and about 3,000 counties in the United States. Each household stays in the panel for 30 quarters on average. The dataset has over 1,000 NielsenIQ-defined product modules that are organized into over 100 product groups and covers around 30% of all expenditure on goods in the CPI.[17] The dataset contains information about each shopping trip the household takes, such as the retailer, the spending on each product defined as a barcode, the product module and group that the product belongs to, and the date of transaction. Moreover, the data includes households' demographic information such as age, education, employment, marital status, which are updated annually. I aggregate the dataset to the quarterly level. I only consider consumers live in the Metropolitan Statistical Area.[18]

Measurement − The search protocol described and characterized in the model simplifies the actual shopping process. In practice, shoppers tend to make the majority of their purchases at a primary retail store, while visiting other stores for specific needs. For instance, during the sample period, shoppers allocate an average of 65% of their total spending on a given product group to the store they visit the most frequently. Additionally, shoppers often purchase multiple items in a single trip, and most stores offer a wide range of product groups. As a result, search behavior may manifest as a reallocation of spending across stores for the same product group. For example, a shopper may initially buy milk from store A and cheese from store B but, after a shock, begin purchasing both milk and cheese from store A while reducing cheese purchases from store B. This

[16] Researcher's own analyses calculated (or derived) based in part on data from Nielsen Consumer LLC and marketing databases provided through the NielsenIQ Datasets at the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business. The conclusions drawn from the NielsenIQ data are those of the researcher(s) and do not reflect the views of NielsenIQ. NielsenIQ is not responsible for, had no role in, and was not involved in analyzing and preparing the results reported herein.

[17] For further discussion of the NielsenIQ data, see Broda and Weinstein (2010).

[18] People living in the countryside may exhibit different search behaviors compared to those in urban areas. They are more likely to engage in "one-stop" shopping due to limited store availability. Their lower search activity may not indicate a lack of willingness to search but rather reflect physical constraints imposed by the environment.

suggests that simply counting the number of trips and the number of distinct stores shopper visits may not fully capture the search effort, even though it is the definition of a search in the model.

To account for the empirical patterns of shopping, I construct the measure of search effort as follows. First, for each consumer and each product group, I find the store she visit most frequently in the entire sample period. I call that store the *routine store*. Second, if she does not buy from the routine store or she buys from multiple stores for this product group, I record the total spending that she spends on the stores other than the routine store. Repeat this process for all the product groups. Finally, I define the non-routine share of spending as the ratio of all the non-routine spending over all groups and the total spending in the given quarter. The non-routine share captures the *extra* search effort to look for the products that shoppers usually buy from the routine store.

I also construct other two measures of search activities. The first is the number of trips each household takes in a quarter. Second, I measure the number of distinct retail stores visited by consumers in a quarter to capture the distribution of trips across retailers. Both measures captures some aspects of the primary measure. Table 1 presents the summary statistics for these search measures. On average, consumers' non-routine share of spending is 35.6%. They make about 3.5 shopping trips per week, visit approximately 12 different retailers per quarter. The variance of these measures is substantial. This is consistent with our model in which search is primarily driven by idiosyncratic match utility shocks and productivity shocks which may have substantial variance.

Table 3: Descriptive Statistics of Household Search Behavior

	Mean	S.D.	10th Percentile	90th Percentile	Observations
Non-routine Share(%)	35.6	25.2	3.23	71.1	2,846,354
Number of Trips	40.5	29.3	11	80	2,846,354
Number of Distinct Retailers	11.7	7.27	4	22	2,846,354

Notes: The table reports summary statistics for key household search behavior variables in my data. *Non-routine Share* is the share of non-routine spending in a quarter. *Number of Trips* is the total number of shopping trips per quarter per household. *Number of Retailers* is the distinct retailers visited by a household per quarter.

Unanticipated Inflation – Following the literature, I assume that consumers use historical data to forecast the current inflation. In particular, consumers estimate a simple OLS regression of inflation on four lags of the inflation for food and drinks and the unemployment rate. The residual from this regression is the unanticipated inflation, which has a mean of -7.8 bp and a standard deviation of 51 bp in the sample period. I focus on inflation for food and drinks because the majority of NielsenIQ products fall into this category. Additionally, I consider the measure based on the inflation for overall goods and services. The anticipated inflation in this case has a mean of -7.5 bp and a standard deviation of 76 bp. I normalize the two series of unanticipated inflation, so their units are standard deviation in our sample. Appendix C provides further details of regression and discusses several robustness checks.

Impact of Unanticipated Inflation on Consumers' Search Behavior − To assess the changes in search behavior after an unanticipated inflation shock, our baseline specification is:

$$y_{it+1} = \lambda_i + \beta\tilde{\pi}_t + X_{it} + e_{it} \tag{56}$$

where t is time; i represents consumer. λ_i is the consumer fixed effect. $\tilde{\pi}_t$ is the unanticipated inflation. β is the coefficient of interest. It measures the magnitude of the correlation between the unanticipated inflation and the consumers' search behavior. y_{it+1} is the non-routine share of spending in the next quarter. I use next-period value for two reasons. First, it avoids reverse causality because inflation and consumer search behavior are jointly determined in theory. Second, it may take time for consumers to change their shopping habits. X_{it} is the time-varying consumer controls. These controls include consumer age, employment, education, marital status, having children or not, and consumer i's total spending in time t. As pointed out by Aguiar and Hurst (2007), these variables have large affect on the pattern of shopping behavior.

The results are presented in Table 4. The first column indicates that one standard deviation (51 bp) increase in unanticipated food and drink inflation leads to a 26.5 bp increase in the non-routine share of spending. This suggests that consumers respond to higher prices by engaging in more active search, shifting purchases to stores outside their routine stores. The magnitude of the response is modest, which is about a half (26.5 bp/51 bp). The second column uses the unanticipated overall inflation. It shows a much smaller increase in the non-routine share and not statistically significant. The effect is smaller because the NielsenIQ data primarily covers food and beverages, and search behavior is more sensitive to inflation in these sectors. The third and fourth columns introduce controls for the number of shopping trips and the number of distinct stores visited. The coefficients decrease only slightly, suggesting these variables capture a very limited portion of search effort. This suggests that consumers allocate most of their search efforts to substituting products within

Table 4: Non-routine Share of Spending and Unanticipated Inflation

Dep. var.: Non-routine Share	(1)	(2)	(3)	(4)
Unanticipated inflation (F&D)	0.265***		0.253***	0.234***
	(0.023)		(0.023)	(0.023)
Unanticipated inflation (overall)		0.022		
		(0.018)		
Number of trips			0.033***	-0.028***
			(0.002)	(0.003)
Number of distinct stores				0.327***
				(0.011)
Observations	2,660,735	2,660,735	2,660,735	2,660,735
Consumer fixed effect	√	√	√	√
Consumer varying effect	√	√	√	√

Notes: The table reports the estimates in specification (56). Each observation is at the consumer×quarter level covering from 2006 Q1 to 2019 Q4. The coefficient represents the corresponding change in different measures of search behavior after a standard deviation increase in unanticipated inflation. Consumer fixed and time-varying effects are controlled. Standard errors are clustered at the consumer level. *Significant at the 10% level; **Significant at the 5% level; ***Significant at the 1% level.

the same categories across the stores they have already visited, rather than increasing trips or visiting new stores.

The estimated coefficient may be biased downward for several reasons. First, consumers only record purchases from stores included in the NielsenIQ dataset, which predominantly covers large retail stores. As a result, our measure may not fully capture the non-routine share of spending if consumers switch to stores not included in the dataset or to online purchases. Second, substantial substitution within a product group could contribute to the bias. For example, consumers may trade down to lower-quality goods within the same store (Jaimovich et al., 2019) after an increase in inflation. However, we do not observe the time spent in each shopping trip. Finally, the inflation shock may not be entirely passed on to retail prices, as suppliers may absorb part of the shock in their wholesale costs. This incomplete pass-through could further dampen the observed relationship between inflation and non-routine spending.

Overall, this evidence supports a key aspect of the main mechanism. As prices rise after an aggregate shock, consumers are incentivized to search for alternatives. The evidence indicates that: (i) there is information frictions about the unanticipated inflation and (ii) The search response is economically large.

3.2 Secular Decline in Non-routine Share and Shopping Time

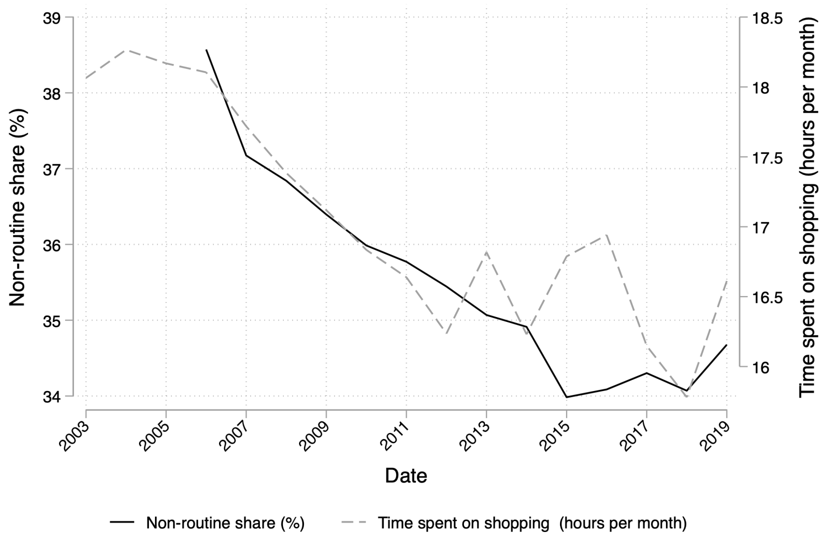
In this final section, I show the secular decline in both non-routine share of spending and shopping time. I discuss the potential explanations and how they are related to the secular decline in the slope of Phillips curve (Del Negro et al., 2020).

I use data from the 2003–2019 waves of the American Time Use Survey (ATUS) conducted by BLS. The ATUS sample is drawn from individuals exiting the Current Population Survey (CPS). Each wave is based on 24-hour time diaries, where respondents report their activities from the previous day in detailed time intervals. The ATUS includes over 400 detailed time-use categories, providing rich insights into daily activity patterns. Specifically, I use the time each respondent reports in the category "obtaining goods" as a proxy for shopping time.

To map the empirical objects to the model concepts, we interpret the non-routine share of spending as a measure of the number of searches in the model, while the total shopping time represents the total search cost for an average shopper. Specifically, the total search cost is defined as the average number of searches multiplied by the search cost per search, i.e., $\rho^{-1}\kappa$. Figure C.1 illustrates that, in our calibrated model, the total search cost increases as the search cost per search rises, while there is a reduction in the number of searches. In the context of empirics, higher search frictions decrease the non-routine share of spending while increasing the total shopping time.

Figure 6 shows that the non-routine share of spending declines by 5 percentage points over the sample period, while the average time spent on shopping declines 2 hours per month. Based on our model, this empirical pattern contradicts the view that search frictions have decreased due to improvements in information and communication technology (ICT) (Ellison and Ellison, 2018). One possible alternative explanation is the rise of market power driven by a few dominant retail chains, which has crowded out local retail stores (Hausman and Leibtag, 2007, Federal Trade Commission, 2023). In addition, DellaVigna and Gentzkow (2019) shows that the stores within a chain tend to set uniform prices for identical products. Both factors contribute to a decline in the dispersion of the price distribution, reducing the average value of additional searches and the threshold in the

Figure 6: Time Trend of Non-Routine Share of Spending and Shopping Time



Notes: The figure plots the average non-routine share of spending and shopping time over time. I use the American Time Use Survey (2003-2019) to compute the average shopping time.

model. It results in the lower aggregate total passthrough as shown in Proposition 6. Figure C.2 illustrates how the aggregate total passthrough increases with the variance of productivity shocks.

In sum, both rising market concentration and price uniformity reduces the price dispersion, potentially explaining the secular decline in the slope of the Phillips curve.

4 Conclusion

This paper develops a novel framework for understanding monetary non-neutrality, driven entirely by consumer-side frictions. At the core of the model is the information asymmetry about marginal costs between consumers and firms. Specifically, the framework integrates a heterogeneous firm block and incomplete consumer information into a standard sequential search model. When consumers observe a price increase, they attribute it to adverse productivity shocks rather than increases in nominal wages, prompting them to search for outside options. Firms, in turn, internalize this consumer search behavior, limiting the passthrough of cost changes to prices. The framework also accommodates aggregate supply shocks, providing a toolbox for analyzing a wide range of shocks as in the standard NK model.

This paper further presents a Phillips curve that relates the output gap to the nowcast error of inflation. It emphasizes the role of household expectations in determining the slope of Phillips curve. After calibrating the model to moments drawn from the literature and empirical evidence, the dynamic framework generates substantial monetary non-neutrality, highlighting the importance of consumer-side mechanisms in macroeconomic dynamics.

To empirically examine how search activities respond to unanticipated inflation, I propose a new measure of search activity, leveraging the rich data on consumer behavior from the NielsenIQ Consumer Panel. The results show that an unanticipated inflation spike significantly increases search activity, providing empirical support for the model's key mechanism. Lastly, I document a secular decline in search activity and connect this trend to rising market concentration and price uniformity. These structural changes in the retail landscape offer insights into the observed secular decline in the slope of the Phillips curve.

Several further topics of inquiry are left for future research. First, although this paper focuses on final goods markets, the framework can be extended to any market with many sellers and buyers, such as upstream and downstream firms in supply chains. An interesting extension would be embedding this model into production network models. Second, applying the model to the labor market could yield valuable insights. Given the parallels in the literature between search behavior in goods and labor markets, this extension might be straightforward. Workers' incomplete information regarding the average posted wage could influence their job-search decisions, prompting firms to adjust wage-setting and potentially generating wage stickiness. Finally, to better calibrate the model and assess the quantitative importance of the mechanism, a micro-foundation for search costs is necessary. A spatial and industrial organization model would be a promising candidate.

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Appendix

A Proofs and Derivations in Static Model

Proof of Proposition 1:

Proof. We consider the case where the first draw does not need the search cost. Shoppers are randomly assigned a firm for free in the first round of search. This guarantees that they always participate the market.

I prove that for given x, if $\int U(v|x)\psi(y|x)dy = -\infty$, then $v^*(x) = -\infty$; if $\int U(v|x)\psi(y|x)dy > -\infty$, there exists a unique threshold defined as follows.

$$v^*(x) = -\frac{\kappa}{1-\Psi(v^*(x)|x)} + \frac{\int_{v^*(x)}^{\infty} y\psi(y)dy}{1-\Psi(v^*(x)|x)} \tag{57}$$

First, given x, I consider two limits. When $v \to -\infty$,

$$-\kappa + U(v|x)\int_{-\infty}^{v}\psi(y|x)dy + \int_{v}^{\infty}U(y|x)\psi(y|x)dy = -\kappa + \int U(y|x)\psi(y|x)dy$$

Then, if $\int U(v|x)\psi(y|x)dy > -\infty$, $U(v|x) > v$. Intuitively, when the search cost is lower than the value of search when the agent has the worst value whatsoever, the agent should choose to continue searching. In the other limit where $v \to \infty$,

$$-\kappa + U(v|x)\int_{-\infty}^{v}\psi(y|x)dy + \int_{v}^{\infty}U(y|x)\psi(y|x)dy = -\kappa + U(v|x)$$

Then, $U(v|x) = v$. Intuitively, when the agent has the best value whatsoever, the agent should stop search because she cannot get any other good that offers better value.

Second, I claim that if $U(v|x) > v$, then $U'(v|x) = 0$. To prove the claim, suppose there exist values of v on its support (at least at $-\infty$) such that $U(v) > v$. Then, $U(v)$ satisfies,

$$U(v|x) = -\kappa + U(v|x)\int_{-\infty}^{v}\psi(y|x)dy + \int_{v}^{\infty}U(y|x)\psi(y|x)dy \tag{58}$$

Equivalently,

$$U(v|x) = -\frac{\kappa}{1-\Psi(v|x)} + \frac{\int_{v}^{\infty}U(y|x)\psi(y|x)dy}{1-\Psi(v|x)} \tag{59}$$

It is easy to show that

$$U'(v|x) = \frac{\psi(v|x)}{(1-\Psi(v|x))^2}\left\{\int_{v}^{\infty}U(y|x)\psi(y|x)dy - U(v|x)(1-\Psi(v|x)) - \kappa\right\} = 0 \tag{60}$$

The second equation holds due to (59). Then, it is easy to see that $U(v|x)$ is a constant in $(-\infty, v^*]$, which is equal to $v^*(x)$, and then $U(v) = v$ in (v^*, ∞). v^* is unique. In addition, for $v < v^*$,

$$-\kappa + U(v|x)\int_{-\infty}^{v}\psi(y|x)dy + \int_{v}^{\infty}U(y|x)\psi(y|x)dy = -\kappa + v^*(x)\Psi(v^*(x)) + \int_{v^*(x)}^{\infty}y\psi(y|x)dy = v^*(x) \tag{61}$$

The search problem in (11) is therefore simplified to

$$U(v|x) = \max\{v, v^*(x)\} \tag{62}$$

This implies that the value of an additional search does not depend on the state v. No matter what state the shopper has, she always compare the state with $v^*(x)$.

Now, we consider the case in which $\int U(v|x)\psi(y|x)dy = -\infty$ for any x. This happens when firms charge arbitrarily high prices, which implies $\psi(y|x)$ is a Dirac function at $y = -\infty$. Then, it is optimal to accept any firm in the first round and the resulting $v^*(x) = -\infty, \forall x$. ■

Proof of Proposition 2:

Proof. We take first-order condition of firm's profit in (17) with respect to P_k. It is easy to show that we can always express the pricing strategy as in (18) and define the elasticity of demand as:

$$e_k = -\frac{\partial \log D(P_k)}{\partial \log P_k} = -\frac{\partial\Big(\int X\big(1 - G\big(\lambda(v^*(x) + p_k)\big)\big)d\Phi_x(x)\Big)}{\partial p_k} + 1 \tag{63}$$

■

Proof of Lemma 1:

Proof. We are interested in the case in which consumers search actively. We first define the markup elasticity,

$$\Lambda_k = -\frac{d\log \mu_k}{dp_k} = \frac{1}{e_k(e_k - 1)}\frac{\partial e_k}{\partial p_k}$$

Since G is log-concave, $\frac{\partial e_k}{\partial p_k} > 0$. Therefore, the markup elasticity is positive. We can further write out the definition,

$$\Lambda_k = \frac{\frac{g'}{g}\frac{1-G}{g} + 1}{\frac{1}{\lambda}\frac{1-G}{g} + 1}$$

Here, I omit the argument of functions for simplicity. The argument is $\lambda(v^*(w) + p_k)$. Note that equilibrium price is a function of $v^*(w)$ and a_{kt}. I now rewrite the LHS of (20) in terms of the integral over productivity distribution, which is exogenous,

$$\int\int_{\lambda(v^*(w)+p^*(v^*(w),a))}^{\infty}\big(\frac{1}{\lambda}\epsilon - p^*(v^*(w),a) - v^*(w)\big)g(\epsilon)d\epsilon\phi_a(a)da = \kappa \tag{64}$$

where $\phi_a(a)$ is the pdf of productivity distribution. Fix $p^*(v^*(w),a)$, the LHS is decreasing in $v^*(w)$. However, higher $v^*(w)$ also decreases optimal prices. To know the net effect of these two forces, we need to derive $\frac{\partial p(v^*(w),a_k)}{\partial v^*(w)}$. From now on, we use $x_k(v^*(w))$ to denote $x(v^*(w),a_k)$ for any variable x.

$$\frac{\partial p_k(v^*(w))}{\partial v^*(w)} = \frac{\partial mu_k(v^*(w))}{\partial v^*(w)} = -\frac{1}{e_k(v^*(w))(e_k(v^*(w))-1)}\frac{\partial e_k(v^*(w))}{\partial v^*(w)} \tag{65}$$

It is easy to show that

$$\frac{\partial e_k(v^*(w))}{\partial v^*(w)} = \lambda^2\frac{g'(1-G)+g^2}{(1-G)^2}\left(1+\frac{\partial p_k(v^*(w))}{\partial v^*(w)}\right) \tag{66}$$

Combine the above equation with (65), we have:

$$\frac{\partial p_k(v^*(w))}{\partial v^*(w)} = -\frac{\Lambda_k}{1+\Lambda_k} \tag{67}$$

Since $\Lambda_k > 0$, $\frac{\partial p_k(v^*(w))}{\partial v^*(w)} \in (-1,0)$. This implies that $\frac{\partial(v^*(w)+p_k(v^*(w)))}{\partial v^*(w)} \in (0,1)$. Therefore, the LHS of (20) decreases in $v^*(w)$. ■

Proof of Theorem 1:

Proof. First, notice that since G is log-concave, μ_k decreases with P_k. From $P_k = \mu_k\frac{W}{A_k}$, we know that there is a unique solution for each optimal price P_k given the full-information threshold $v^*(w)$. Then, from Lemma 1, we know that there exists a unique solution $v^*(w)$ to 20 given that the prices are computed through the first-order conditions as in (24). The equilibrium in which shoppers search actively exists and is unique because $v^*(w)$ exists and is unique.

As Diamond (1971) points out famously, there are always a continuum of equilibria where shoppers do not search and firms charge very high prices, i.e., $v^*(w) = -\infty$ and $P_k = \infty, \forall k$. ■

Remarks on Computation of the Full-information Equilibrium:

Remark 1 (Remark on Computing Steady-State Equilibrium)**.** *The proof of Theorem 1 provides insights on the computational method for the full-information equilibrium.*

1. *First guess a* $v^*(w)$

2. *Calculate the optimal price distribution given $v^*(w)$*

3. *Plug guessed $v^*(w)$ and derived price distribution into 20 and check if LHS is equal to the given search cost κ.*

4. *Increases guessed $v^*(w)$ if LHS is larger than search cost, according to Lemma 1. Vice versa.*

5. *Loop the procedure 1-4 until the difference between the LHS and the RHS of 20 is smaller than the given tolerance*

Proof of Theorem 2:

Proof. Suppose the nominal wage increases from w to w'. Notice $\Delta w = w' - w$ does not need to be small. We guess that all the optimal prices increase proportionally, i.e., $p'_k = p_k + \Delta w$. Then the price distribution $f(p|w)$ shifts to the right and becomes $f(p - \Delta w|w')$. The threshold is determined by,

$$\int \int_{\lambda(v^*(w')+p)}^{\infty} \big(\frac{1}{\lambda}\epsilon - p - v^*(w')\big) g(\epsilon) d\epsilon f(p - \Delta w|w') dp = \kappa \tag{68}$$

Let $z = p - \Delta w$. Then we can rewrite the LHS of the above equation,

$$\int \int_{\lambda(v^*(w')+\Delta w+z)}^{\infty} \big(\frac{1}{\lambda}\epsilon - z - (\Delta w + v^*(w'))\big) g(\epsilon) d\epsilon f(z|w') dz = \kappa \tag{69}$$

This implies that $v^*(w') = v^*(w) - \Delta w$. According to (24), the elasticity of demand is given by,

$$\begin{aligned} e'_k &= \lambda \frac{g\big(\lambda(v^*(w') + p'_k)\big)}{1 - G\big(\lambda(v^*(w') + p'_k)\big)} + 1 \\ &= \lambda \frac{g\big(\lambda(v^*(w) - \Delta w + p_k + \Delta w)\big)}{1 - G\big(\lambda(v^*(w) - \Delta w + p_k + \Delta w)\big)} + 1 \\ &= e_k \end{aligned}$$

Since the elasticity of demand does not change, the optimal prices is given by,

$$p'_k = \log\big(\frac{e_k}{e_k - 1}\big) + w' - a_k = p_k + w' - w = p_k + \Delta w \tag{70}$$

Therefore, we verify the guess that the optimal prices increase proportionally with the nominal wage. Based on Theorem 1, the equilibrium is unique. Therefore, the above constructed equilibrium is the only equilibrium when the nominal wage is w'. ■

Proof of Proposition 3: Here, I provide separate proofs for two parts of Proposition 3. I start with the proof of the first part.

Proof of Part 1. First, we can rewrite 14 in terms of integrating over the exogenous productivity distribution,

$$\int\int\int_{\lambda(v^*(x)+p^*(a,w))}^{\infty}\big(\frac{1}{\lambda}\epsilon - p^*(a,w) - v^*(x)\big)g(\epsilon)d\epsilon\phi_a(a)dah(w|x)dw = \kappa \tag{71}$$

where $v^*(x)$ is implicitly determined by the above equation. On the first order, the posterior belief of the nominal wage collapses to a Dirac function at $E(w|x)$. Also, combining the shopper's expected price conditional on x as shown in (27), on the first order, the above equation becomes,

$$\int\int_{\lambda(v^*(x)+\bar{p}_k+\varphi(a)E(\hat{w}|x))}^{\infty}\big(\frac{1}{\lambda}\epsilon - \bar{p}_k - \varphi(a)E(\hat{w}|x) - v^*(x)\big)g(\epsilon)d\epsilon\phi_a(a)da = \kappa \tag{72}$$

Take the total derivative on $E(\hat{w}|x)$ on both sides,

$$\int\Big(\varphi(a) + \frac{dv^*(x)}{dE(\hat{w}|x)}\Big)\big(1 - G(\lambda(v^*(x) + \bar{p}_k + \varphi(a)E(\hat{w}|x)))\big)\phi_a(a)da = 0$$

On the first order, it can be written as follows,

$$\frac{dv^*(x)}{dE(\hat{w}|x))} = -\int\varphi(a)\bar{\omega}(a)\phi_a(a)da = -\int\varphi_k\bar{\omega}_k dk \tag{73}$$

where $\bar{\omega}(a)$ is the expenditure share of firms with productivity a in the full-information equilibrium where $w = \bar{w}$. Then, the threshold, on the first order, is given by,

$$v^*(x) = v^*(\bar{w}) - \frac{\partial v^*(x)}{\partial v^*(x)}v^*(x) = v^*(\bar{w}) - \Phi E(\hat{w}|x)$$

Recall $\hat{p} = \Phi\hat{w}$. We have the result. ■

Proof of Part 2 . First, plug the result in Part 1 into the elasticity of demand in (19), the elasticity becomes,

$$e_k = \lambda\frac{\int Xg\big(\lambda(v^*(\bar{w}) - \Phi E(\hat{w}|x) + p_k)\big)d\Phi_x(x)}{\int X\big(1 - G\big(\lambda(v^*(\bar{w}) - \Phi E(\hat{w}|x) + p_k)\big)\big)d\Phi_x(x)} + 1 \tag{74}$$

with some abuse of notation, Φ is the aggregate total passthrough and $\Phi_x(x)$ is the cdf of information sets. Let $y = \lambda\Phi\big(E(\hat{w}|x) - \bar{E}(\hat{w})\big)$ and its pdf is $\phi_y(y)$, which is a Gaussian distribution with mean zero and standard deviation $\sigma = \lambda\Phi\theta\sigma_s$. Further, we denote $z = \lambda(v^*(w) - \Phi\bar{E}(\hat{m}) + p_k)$. The elasticity is rewritten as follows,

$$e_k = \lambda\frac{\int Xg(z+y)\phi_y(y)dy}{\int X\big(1 - G(z+y)\big)\phi_y(y)dy} + 1 \tag{75}$$

It is easy to show that the first-order approximation to the ratio is equivalent to separately approximating numerator and denominator and then combining them. Following this result, we first expand the numerator.

$$g(z+y) = g(z) + g'(z)y + \frac{g''(z)}{2}y^2 + \mathcal{O}(y^3)$$

Substitute this expansion into the integral and also notice that $X \propto \exp(y + \bar{w})$,

$$\begin{aligned}\int \exp(y)g(z+y)\phi_y(y)dy &= \int (1+y)\Big(g(z) + g'(z)y + \frac{g''(z)}{2}y^2 + \mathcal{O}(y^3)\Big)\phi_y(y)dy \\ &= g(z) + \int \Big(\big(g'(z) + \frac{g''(z)}{2}\big)y^2 + \mathcal{O}(y^3)\Big)\phi_y(y)dy \\ &= g(z) + \big(g'(z) + \frac{g''(z)}{2}\big)\sigma^2 + \mathcal{O}(\sigma^3)\end{aligned}$$

Therefore, $\int g(z+y)\phi_y(y)dy \to g(z)$ on the order of σ_s^2, which is second-order term. Similarly, $\int \big(1 - G(z+y)\big)\phi_y(y)dy \to 1 - G(z)$ on the order of σ_s^2. On the other hand, the average expectation of nominal wage shock $\bar{E}(\hat{w})$ approaches zero on the order of σ_s. Therefore, the elasticity, on the first order, is given by,

$$e_k = \lambda\frac{g(z)}{1-G(z)} + 1 = \lambda\frac{g(\lambda(v^*(w) - \Phi\bar{E}(\hat{w}) + p_k))}{1 - G(\lambda(v^*(w) - \Phi\bar{E}(\hat{w}) + p_k))} + 1 \tag{76}$$

■

Proof of Lemma 2:

Proof. First, log price is given by,

$$p_k = \log\mu_k + w - a_k$$

Total differentiate on both sides,

$$\hat{p}_k = \frac{\partial \log\mu_k}{\partial p_k}\hat{p}_k + \frac{\partial \log\mu_k}{\partial p}\hat{p} + \hat{w} \tag{77}$$

We further define $\gamma_{kt} = (1 - \frac{d\log\mu_k}{dp_k}\Big|_{\hat{w}=0})^{-1}$ and $\xi_k = \frac{d\log\mu_k}{dp}\Big|_{\hat{w}=0}\gamma_k$. Then, we have,

$$\hat{p}_k = \gamma_k\hat{w} + \xi_k\hat{p} \tag{78}$$

Since $\hat{p} = \Phi\hat{w}$, the total passthrough for individual firm satisfies,

$$\varphi_k = \gamma_k + \Phi\xi_k$$

Integrate on both sides,

$$\Phi = \frac{\Gamma}{1 - \Xi}$$

where $\Gamma = \int \gamma_k\bar{\omega}_k dk$, $\Xi = \int \xi_k\bar{\omega}_k dk$. ■

Proof of Theorem 3: I first prove the "Incompleteness" part of the Theorem. Then I prove an important lemma and finally I prove the comparative statics results.

Proof of Incompleteness . First, notice that for any function $f(w)$, the first-order approximation to its derivative is,

$$f'(w) = f'(\bar{w}) + f''(\bar{w})\hat{w} \tag{79}$$

Then, $f'(w)|_{\hat{w}=0} = f'(\bar{w})$ on the first order. I claim that the approximation gives the same result if we reverse the order of the operations. Let's first take the first-order approximation to $f(w)$,

$$f(w) = f(\bar{w}) + f'(\bar{w})\hat{w} \tag{80}$$

Then if we take derivative w.r.t. $\hat{w}$ and then make $\hat{w} = 0$, we get the same result.

Now, following this result, $\frac{\partial e_k}{\partial p_k}\Big|_{\hat{w}=0}$ and $\frac{\partial e_k}{\partial p}\Big|_{\hat{w}=0}$ can be computed by first first-order approximating e_k as we did in Proposition 3 and then take derivative to $\hat{p}_k$ and $\hat{p}$. For simplicity, I omit the argument of the functions, in particular, $z = z(v^*(\bar{w}) + \bar{p}_k)$ for any function z. It is straightforward to show that, on the first order,

$$\frac{\partial e_k}{\partial p_k}\Big|_{\hat{w}=0} = \lambda^2 \frac{g'(1-G) + g^2}{(1-G)^2} \tag{81}$$

We can derive that γ_k does not depend on θ. In addition, we have

$$\frac{\partial e_k}{\partial p}\Big|_{\hat{w}=0} = -\theta\lambda^2 \frac{g'(1-G) + g^2}{(1-G)^2} \tag{82}$$

Then, we have $\frac{\partial \log \mu_k}{\partial p} = -\theta \frac{\partial \log \mu_k}{\partial p_k}$. According to Lemma 2, we have

$$\xi_k = \theta(1 - \gamma_k) \tag{83}$$

I claim that $\varphi_k < 1, \forall k$ and notice that to prove it, it is sufficient to prove $\gamma_k + \xi_k < 1, \forall k$. We can write out $\gamma_k + \xi_k$,

$$1 - (\gamma_k + \xi_k) = (1 - \gamma_k)(1 - \theta) \tag{84}$$

Since $-\frac{d \log \mu_k}{dp_k} > 0$, $\gamma_k < 1$. ■

Before turning to the result on "Composition", I first prove the following lemma,

Lemma A-1. *Let* $\Lambda_k = -\frac{d\log\mu_k}{dp_k}\Big|_{\hat{w}=0}$ *denote the steady-state markup elasticity. When* G *is Gumbel distribution, it has the following property,*

$$\frac{d\Lambda(y)}{dy} < 0$$

In addition, $\Lambda(y) \to 0$ *as* $y \to \infty$ *and* $\Lambda(y) \to \infty$ *as* $y \to -\infty$.

Proof. To simplify notation, we denote $g(y_k) = g(\lambda(v^*(\bar{w}) + \bar{p}_k))$ and $G(y_k) = G(\lambda(v^*(\bar{w}) + \bar{p}_k))$. We assume that G follows standard type-I extreme-value distribution, i.e., $G(y) = \exp(\exp(-y))$, $g(y) = \exp(-y - \exp(-y))$. It also follows that $\frac{g'}{g} = e^{-y} - 1$.

First, since G is log-concave, g is also log-concave. This implies the following property,

$$(\frac{g'}{g})' < 0; \quad (\frac{1-G}{g})' < 0$$

Additionally, using G is Gumbel, we have $\frac{1-G}{g} \to 1$ as $y \to \infty$ and therefore $\frac{1-G}{g} > 1$.

Second, we write out markup elasticity,

$$\Lambda = \frac{\frac{g'}{g}\frac{1-G}{g} + 1}{\frac{1}{\lambda}\frac{1-G}{g} + 1} \tag{85}$$

Then the derivative of markup elasticity w.r.t y is given by,

$$\begin{aligned}
\frac{d\Lambda(y)}{dy} &= \left(\frac{g'}{g}\right)'\left(\frac{1-G}{g}\right)^2\frac{1}{\lambda} - \left(\frac{1-G}{g}\right)'\frac{1}{\lambda} + \left(\frac{g'}{g}\frac{1-G}{g}\right)' \\
&= -e^{-y}\left(\frac{1-G}{g}\right)^2\frac{1}{\lambda} + \frac{1}{\lambda}(1 + \frac{g'}{g}\frac{1-G}{g}) + \left(\frac{g'}{g}\frac{1-G}{g}\right)' \\
&< -e^{-y}\left(\frac{1-G}{g}\right)\frac{1}{\lambda} + \frac{1}{\lambda}(1 + (e^{-y} - 1)\frac{1-G}{g}) + \left(\frac{g'}{g}\frac{1-G}{g}\right)' \\
&= \frac{1}{\lambda}\left(1 - \frac{1-G}{g}\right) + \left(\frac{g'}{g}\frac{1-G}{g}\right)'
\end{aligned}$$

The inequality is due to $\frac{1-G}{g} > 1$. The first term of the last equation is negative. We need to deal with the second term. Notice that

$$\left(\frac{g'}{g}\frac{1-G}{g}\right)' = \left(\frac{g'}{g}\right)'\left(\frac{1-G}{g}\right) + \left(\frac{g'}{g}\right)\left(\frac{1-G}{g}\right)'$$

It is straightforward that when $y < 0$, $\frac{g'}{g} = e^{-y} - 1 > 0$. Then, $\left(\frac{g'}{g}\frac{1-G}{g}\right)' < 0$. We now focus on the case where $y > 0$.

$$\begin{aligned}\left(\frac{g'}{g}\frac{1-G}{g}\right)' &= -e^{-y}\frac{1-G}{g} + (1-e^{-y}) - (1-e^{-y})^2\frac{1-G}{g} \\ &< -e^{-y}\frac{1-G}{g} + (1-e^{-y}) - (1-e^{-y})\frac{1-G}{g} \\ &= (1-e^{-y}) - \frac{1-G}{g} \\ &< 0\end{aligned}$$

The first inequality is due to $1 - e^{-y} \in (0, 1)$ when $y > 0$. The second inequality is due to $\frac{1-G}{g} > 1$. Combine all together, we have

$$\frac{d\Lambda(y)}{dy} < 0$$

For limit results, first recall $\frac{1-G}{g} \to 1$ as $y \to \infty$. Plug into (85), we get $\Lambda \to 0$ when $y \to \infty$. Second, since $\frac{1-G}{g} \to \infty$ when $y \to -\infty$, $\Gamma \approx \lambda g'/g \to \infty$ since $g'/g \to \infty$. ■

Next, we turn to the result on composition and comparative statics.

Proof of Composition and MCS. First, notice that to show φ_k decreases in a_k, it is sufficient to show that γ_k decreases in a_k according to (84).

From Lemma 2, $\gamma_k = (1 + \Lambda_k)^{-1}$. Since Λ_k decreases in P_k, it implies γ_k increases in P_k. Since P_k decreases in a_k, it implies γ_k decreases in a_k.

Next, we prove the comparative statics results. We first consider the comparative statics on κ. From Lemma 1, an increase in the search cost κ implies lower threshold $v^*(\bar{w})$. According to Lemma A-1, the steady-state markup elasticity decreases in $v^*(\bar{w})$. This implies a uniform decrease in γ_k. As a result, φ_k decreases in κ for any k according to (84). When $\kappa \to 0$, $v^*(\bar{w}) \to \infty$ and therefore $\Lambda_k \to 0$, resulting in $\gamma_k \to 1$ for any k. This implies $\varphi_k = 1$ for any k. When $\kappa \to \infty$, $v^*(\bar{w}) \to -\infty$, and therefore $\Lambda_k \to \infty$, resulting in $\gamma_k \to 0$ for any k. This implies $\varphi_k = 0$ for any k.

Then, we consider the comparative statics on θ. An increase in the information friction, which means a decrease in θ, induces lower φ_k, according to (84). When $\theta \to 0$, $\Phi = \Gamma$ according to Lemma 2. When $\theta \to 1$, $\varphi_k = 1$ according to (84). ■

Proof of Proposition 4:

Proof. From (83), and Lemma 2, we have,

$$\hat{p}_k = \gamma_k \hat{w} + (1-\gamma_k)\theta\hat{w} \tag{86}$$

$$= \gamma_k \hat{w} + (1-\gamma_k)\bar{E}\hat{w} \tag{87}$$

Aggregate, we have

$$\hat{p} = \Gamma\hat{w} + (1-\Gamma)\bar{E}\hat{w} \tag{88}$$

The aggregation requires every firm is individually rational, so they know (87). Suppose every shopper believe that firms and other shoppers are rational. Then every shopper believes the above condition holds. Shoppers' average expectation of $\hat{p}$ satisfies,

$$\bar{E}\hat{p} = \Gamma\bar{E}\hat{w} + (1-\Gamma)\bar{E}^2\hat{p} \tag{89}$$

where $\bar{E}^2[\cdot] = \bar{E}[\bar{E}[\cdot]]$ denotes the second-order belief. Iterating ad infinitum, the change in the actual price index $\hat{p}$ can be expressed in terms of the higher-order beliefs of the monetary shock $\hat{w}$:

$$\hat{p} = \Gamma\sum_{h=0}^{\infty}(1-\Gamma)^h\bar{E}^h\hat{w} \tag{90}$$

"Iterating ad infinitum" amounts to imposing common knowledge of rationality. The first iteration requires that shoppers know that firms and other shoppers are rational, the second iteration requires that shoppers know that others know they are rational and firms are rational, and so on. ■

Proof of Proposition 5:

Proof. Since the markup μ_k decreases in p_k, it increases in a_k. It is easy to show that $\lim_{a\to-\infty} e_k = 1-\lambda\frac{g'}{g} = 1+\lambda$ and $\lim_{a\to\infty} e_k = 1$. Then, $\lim_{a\to-\infty}\mu_k = \frac{\lambda+1}{\lambda}$ and $\lim_{a\to\infty}\mu_k = \infty$.

From Lemma 2, $\gamma_k = (1+\Lambda_k)^{-1}$. Also, we know $\xi_k = \theta(1-\gamma_k)$. Since γ_k decreases in a_k as shown in the proof of Theorem 3, ξ_k increases in a_k. Also, in the limit when $a\to\infty$, $p\to-\infty$. Then $\Lambda\to\infty$ and $\gamma\to 0$, $\xi\to\theta$. In the opposite limit, $p\to\infty$, and $\Lambda\to 0$, implying $\gamma\to 1$ and $\xi\to 0$. ■

Proof of Proposition 6:

Proof. See Proof of Theorem 3. ■

Proof of Proposition 7:

Proof. See Proof of Theorem 3. ■

Proof of Proposition 8:

Proof. In the case of monetary shock, recall the labor supply condition,

$$\hat{p} + \hat{c} = \hat{w} \tag{91}$$

Combine with the definition of the aggregate passthrough,

$$\hat{p} = \frac{\Phi}{1-\Phi}\hat{c} \tag{92}$$

Plug in the condition in Proposition 4, we have

$$\hat{p} = \frac{\Gamma}{1-\Gamma}\frac{1}{1-\theta}\hat{c} \tag{93}$$

In the case of aggregate supply shock, two equations are as follows,

$$\hat{p} + \hat{c} = \hat{w}$$

$$\hat{p} = \Phi(\hat{w} - \hat{a})$$

Substitute $\hat{w}$, we achieve $\hat{p} = \frac{\Phi}{1-\Phi}(\hat{c} - \hat{a})$. Again, leveraging Proposition 4, we have the result.

■

B Appendix for Proofs and Calibration in Dynamic Model

Proof of Proposition 9:

Proof. We can rewrite worker's problem as:

$$\max_{B_t, X_t, L_t} E_0 \sum_{t=0}^{\infty} \beta^t \left(\log X_t - L_t\right)$$

$$\text{s.t.} \quad X_t + B_t = W_t L_t + R_{t-1}B_{t-1} + \Pi_t$$

It is easy to show that the standard Euler equation holds. Recall we have,

$$\hat{p}_t - \bar{E}\hat{p}_t = \frac{\Gamma}{1-\Gamma}(\hat{c}_t - \hat{a}_t) \tag{94}$$

Since we assume that households know $\hat{p}_{t-1}$, we have:

$$\begin{aligned}(\hat{p}_t - \hat{p}_{t-1}) - (\bar{E}\hat{p}_t - \hat{p}_{t-1}) &= \frac{\Gamma}{1-\Gamma}(\hat{c}_t - \hat{a}_t)\\ \hat{\pi}_t - \bar{E}\hat{\pi}_t &= \frac{\Gamma}{1-\Gamma}(\hat{c}_t - \hat{a}_t)\end{aligned}$$

■

Dynamic Model without Simplifying Assumptions Now, we get rid of the simplifying assumptions. I only briefly describes the model. Most of the setup is similar to the model laid out in the main text. There is a continuum of shoppers indexed by $i \in I$. In the morning, they make consumption-saving and labor supply decisions. In the afternoon, they go shopping with given consumption expenditure that they have allocated to consumption in the morning.

In the morning, the shopper maximizes the following discounted utility:

$$\max_{B_{it}, X_{it}, L_{it}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\log \frac{X_{it}}{P_{kt}} + \frac{1}{\lambda} \epsilon_{ikt} - L_t \right)$$
$$\text{s.t.} \quad X_{it} + B_{it} = W_{it} L_{it} + R_{it-1} B_{it-1} + \Pi_{it}$$

where X_{it} is the consumption expenditure left for the shopping in the afternoon. Notice that W_{it} and R_{it} are shopper-specific and I assume $w_{it} = w_t + \zeta_{it}$, $i_{it} = i_t + u_{it}$, $\pi_{it} = \pi_t + v_{it}$, where i_t is the log-deviation of R_t following the convention. These shocks $\zeta_{it}, u_{it}, v_{it}$ are IID across i and t, independent of one another, and independent of any other random variable in the economy. Shocks follow normal distribution with their respective variances. They "noise up" the information that each shopper can extract from the available market signals. I assume that shoppers only treat the current-period price index $\hat{p}_t$ as their signal to align with the information structure in the main text. In the afternoon, shoppers have the same search problem. The following proposition characterize the three-equation system,

Proposition 12. *The equilibrium dynamics of $\{\hat{p}_t, c_t, i_t\}$ is described by the following system of equations:*

$$\hat{c}_t = -\sigma \left\{ \sum_{k=1}^{\infty} \beta^{k-1} \bar{E}_t^s \left[i_{t+k} - \hat{\pi}_{t+k+1} \right] \right\} + \frac{1-\beta}{\beta} \left\{ \sum_{k=1}^{\infty} \beta^k \bar{E}_t^s \left[\hat{c}_{t+k} \right] \right\}$$
$$\hat{p}_t - \bar{E}^s \hat{p}_t = \frac{\Gamma}{1-\Gamma} (\hat{c}_t - \hat{a}_t)$$
$$i_t = \phi_\pi \hat{p}_t + \varepsilon_{mt}$$

where Γ is the aggregate passthrough.

The first equation represents incomplete-information Euler equation. Under incomplete information, households need to form expectations of future aggregate consumption and inflation, which in turn depends on how other households respond to the shock. This forms a dynamic beauty contest game as described in Angeletos and Lian (2018). The second equation presents the

Phillips Curve as in the main text. The third equation represents the price-level targeting rule. It guarantees that this system will converges to its steady state as the shock dissipates. Otherwise, if we use the inflation-targeting rule, the price level will never return to its original value.