Lecture 19: Decision trees

Reading: Section 8.1

STATS 202: Data mining and analysis

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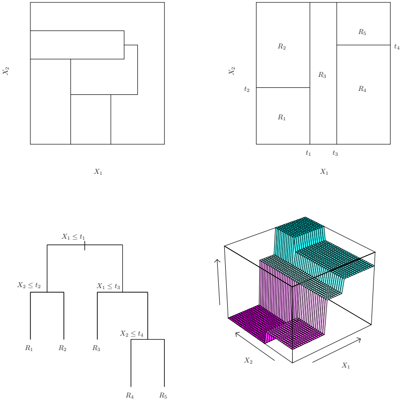
Slide credits: Sergio Bacallado

Decision trees, 10,000 foot view



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2. Predict a constant in each set of the partition.

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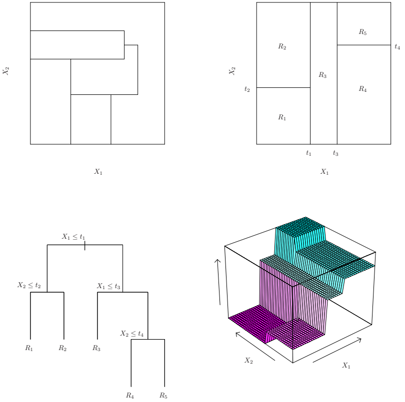

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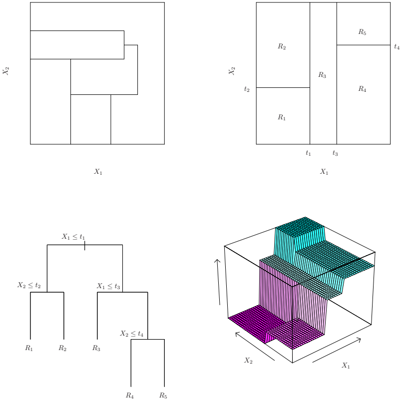

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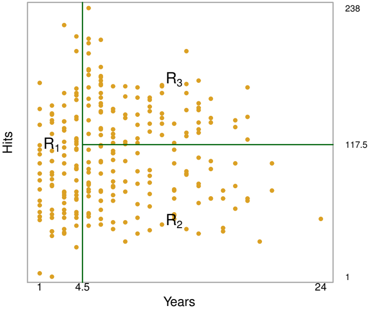
3. The partition is defined by splitting the range of one predictor at a time.

   $\rightarrow$ Not all partitions are possible.

Example: Predicting a baseball player's salary



The prediction for a point in R_i is the average of the training points in R_i.

How is a decision tree built?

- Start with a single region R_1, and iterate:

  1. Select a region R_k, a predictor X_j, and a splitting point s, such that splitting R_k with the criterion $X_j < s$ produces the largest decrease in RSS:

  $$\sum_{m=1}^{|T|} \sum_{x_i \in R_m} (y_i - \bar{y}_{R_m})^2$$

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- This grows the tree from the root towards the leaves.

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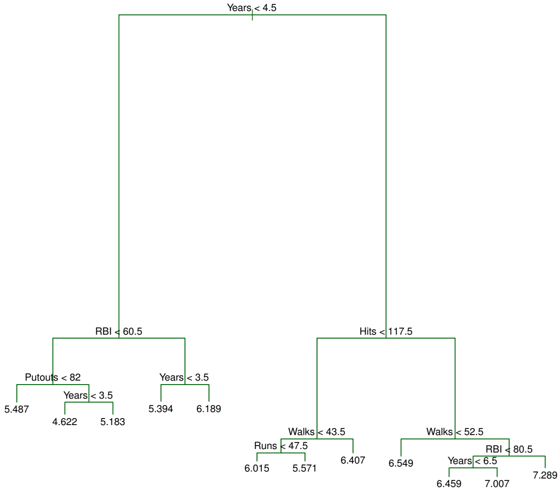

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  $\rightarrow$ In our greedy algorithm, it is possible to find good cuts after bad ones.

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  - Select the optimal tree T_i by cross validation.

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  - Choose the optimal α (the optimal T_i) by cross validation.

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4. For each tree T_i, average the 10 test errors, and select the value of α that minimizes the error.

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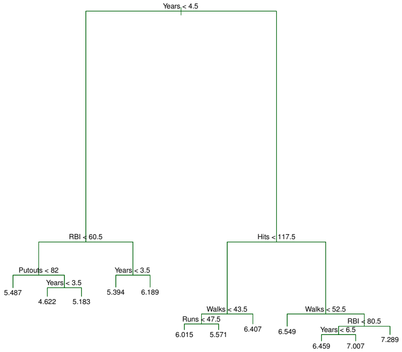
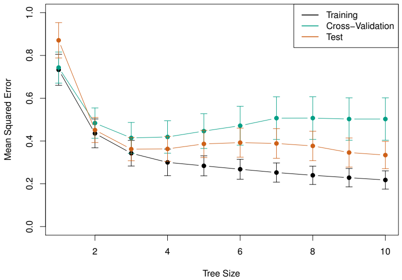
Note: We are doing all fitting, **including the construction of the trees**, using only the training data.

Example. Predicting baseball salaries





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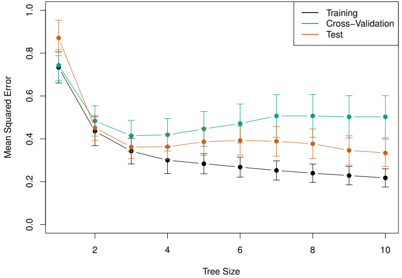

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- We predict the response by **majority vote**, i.e. pick the most common class in every region.
- Instead of trying to minimize the RSS:

$$\sum_{m=1}^{|T|} \sum_{x_i \in R_m} (y_i - \bar{y}_{R_m})^2$$

we minimize a classification loss function.

Classification losses

- The 0-1 loss or misclassification rate:

$$\sum_{m=1}^{|T|} \sum_{x_i \in R_m} \mathbf{1}(y_i \neq \hat{y}_{R_m})$$

- The Gini index:

$$\sum_{m=1}^{|T|} q_m \sum_{k=1}^{K} \hat{p}_{mk}(1 - \hat{p}_{mk}),$$

  where $\hat{p}_{m,k}$ is the proportion of class k within R_m, and q_m is the proportion of samples in R_m.

- The cross-entropy:

$$-\sum_{m=1}^{|T|} q_m \sum_{k=1}^{K} \hat{p}_{mk} \log(\hat{p}_{mk}).$$

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- **Motivation for the Gini index:**

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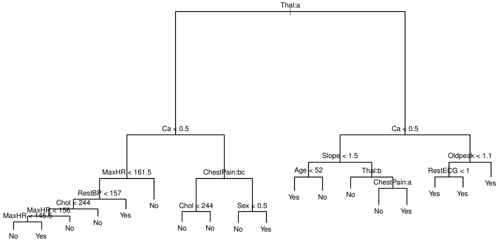
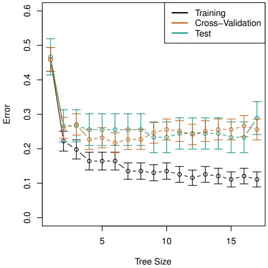
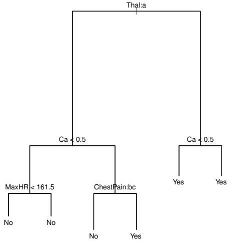
- It is typical to use the Gini index or cross-entropy for growing the tree, while using the misclassification rate when pruning the tree.

Example. Heart dataset.







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- They easily handle qualitative predictors and missing data.
- **Downside: they don't necessarily fit as well!**