# Lecture 5: Clustering

Reading: Chapter 10, Sections 3.1-2

STATS 202: Data mining and analysis

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- $K$-means clustering
- Hierarchical clustering

# $K$-means clustering

- $K$ is the number of clusters and must be fixed in advance.

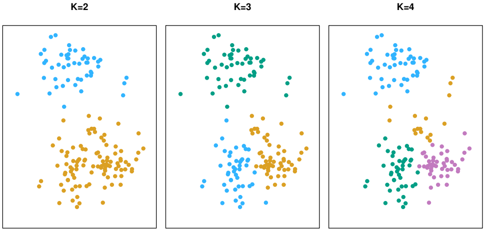
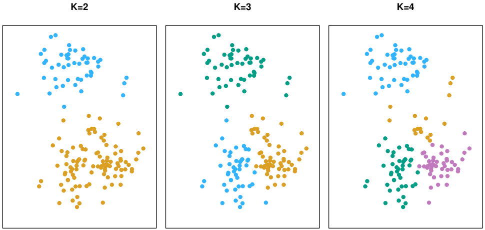



Figure 10.5

# $K$-means clustering

- $K$ is the number of clusters and must be fixed in advance.
- The goal of this method is to maximize the similarity of samples within each cluster:

$$\min_{C_1,\ldots,C_K} \sum_{\ell=1}^{K} W(C_\ell)$$

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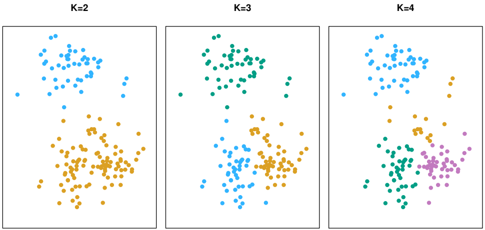


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   - Reassign each sample to the nearest centroid.

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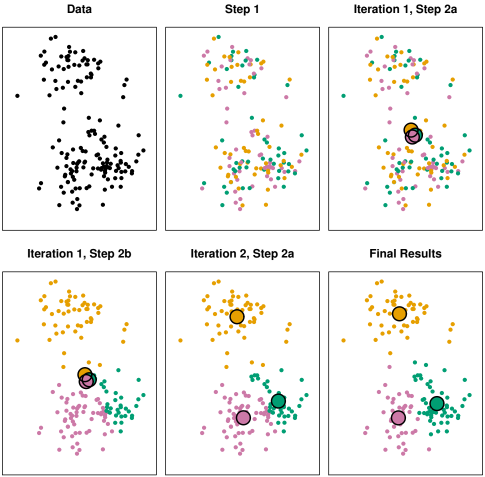


Figure 10.6

# Properties of $K$-means clustering

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- Each initialization could yield a different minimum.

# Example: $K$-means output with different initializations


Figure 10.7

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In practice, we start from many random initializations and choose the output which minimizes the objective function.

# Hierarchical clustering

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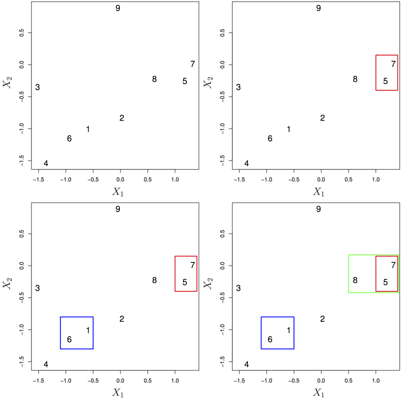
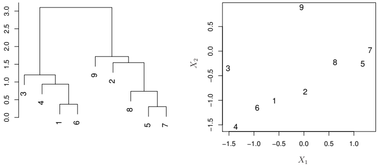
Most algorithms for hierarchical clustering are *agglomerative*.



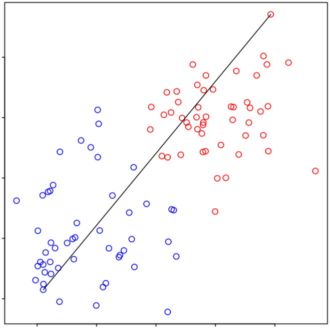




The output of the algorithm is a *dendogram*. We must be careful about how we interpret the dendogram.

# Notion of distance between clusters

At each step, we link the 2 clusters that are "closest" to each other.

Hierarchical clustering algorithms are classified according to the notion of distance between clusters.



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The distance between 2 clusters is the *maximum* distance between any pair of samples, one in each cluster.

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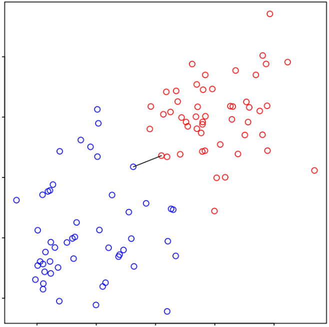

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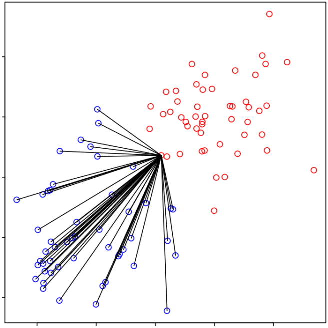

**Average linkage:**
The distance between 2 clusters is the average of all pairwise distances.

# Example

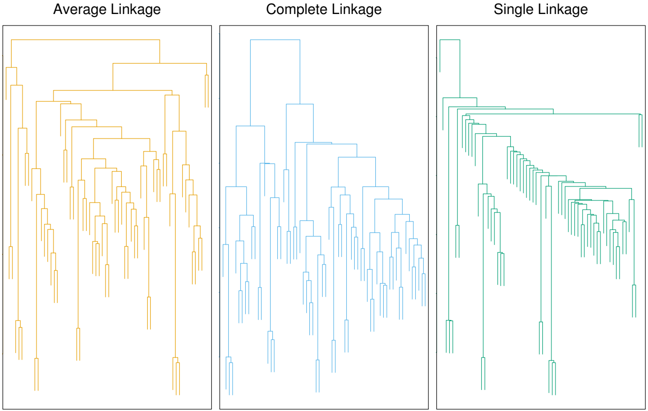


Figure 10.12

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  - Most important: temper your conclusions.

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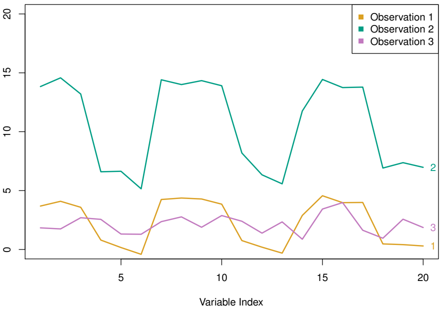
- Does Euclidean distance capture dissimilarity between samples?

# Correlation distance

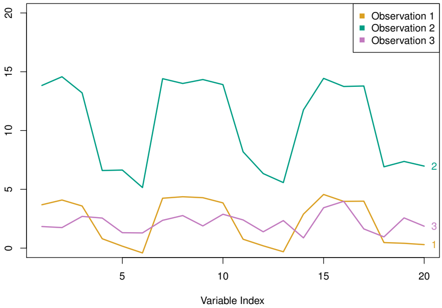
**Example:** Suppose that we want to cluster customers at a store for market segmentation.

- Samples are customers
- Each variable corresponds to a specific product and measures the number of items bought by the customer during a year.



# Correlation distance

- Euclidean distance would cluster all customers who purchase few things (orange and purple).



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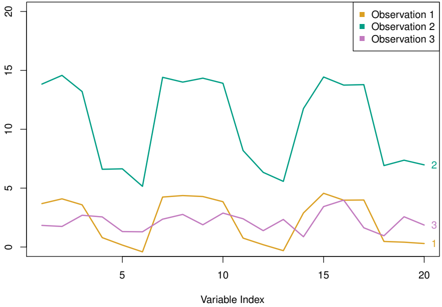

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- Perhaps we want to cluster customers who purchase *similar* things (orange and teal).
- Then, the **correlation distance** may be a more appropriate measure of dissimilarity between samples.