Introduction to Mathematical Programming
IE406

Lecture1

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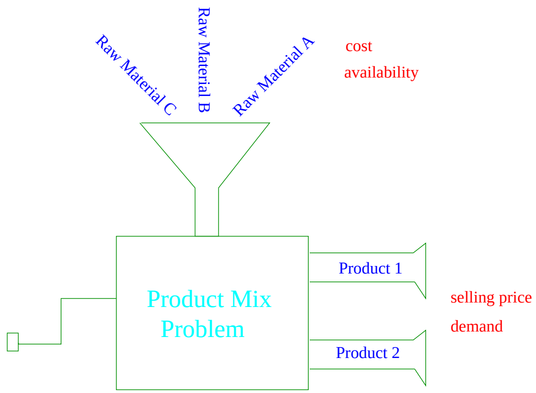
Reading for This Lecture

- Primary Reading
  - Bertsimas 1.1-1.2, 1.4-1.5
- Supplementary Reading
  - Bertsimas 1.3
  - Operations Research Methods and Models
  - Model Building in Mathematical Programming

Systems Engineering

- A *system* is a functionally related group of elements, such as

  - manufacturing systems,
  - distribution systems,
  - financial systems,
  - computer systems,
  - biological systems, and
  - political systems.



- Such systems can be *modeled* and analyzed using techniques we'll learn about in this class.
- This type of analysis is used in every industry and every sector of the economy.

What is the purpose of a model?

- The exercise of building a model can provide insight.
- It's possible to do things with models that we can't do with "the real thing."
- Analyzing models can help us decide on a course of action.

Examples of Models

- Physical Models
- Simulation Models
- Probability Models
- Economic Models
- Biological Models
- Mathematical Programming Models

Systems Modeling

- We said that a system is defined as a functionally related group of elements.
  - What is an element?
  - What is a functional relationship?
- *Systems modeling* consists of describing the relationships between elements of a given system.
- One type of model for studying such systems is called a *mathematical program*.

Mathematical Programming Models

- What does *mathematical programming* mean?
- Programming here means "planning."
- Literally, these are "mathematical models for planning."
- Also called *optimization models*.
- A *mathematical program* consists of
  - a set of *variables* that describe the state of the system,
  - a set of *constraints* that determine the states that are allowable,
  - external input *parameters* and *data*, and
  - an *objective function* that provides an assessment of how well the system is functioning.
- We control the system state by setting the values for the variables.
- The variables represent decisions that must be made in order to operate the system.
- The constraints represent specifications for system operation.
- The goal is to determine the best state consistent with operating specifications.

Forming a Mathematical Programming Model

The general form of a mathematical programming model is:

$$
\begin{array}{rcl}
min \ f(x_1, \ldots, x_n) & & \\
s.t. \ \ g_i(x_1, \ldots, x_n) & \left\{ \begin{array}{c} \leq \\ = \\ \geq \end{array} \right\} & b_i \\
(x_1, \ldots, x_n) & \in & X
\end{array}
$$

X may be a discrete set, such as $\mathbb{Z}^n$.

Types of Mathematical Programs

- The type of a mathematical program is determined primarily by
  - The form of the objective and the constraints.
  - The set X.
  - A wide range of mathematical programming model types are described at
    * the NEOS Guide, and
    * the on-line version of Operations Research Methods and Models.
- We will consider mainly linear models.
  - The objective function is *linear*.
  - The constraints are *linear*.

Solutions

- A *solution* is an assignment of values to variables.
- A solution can be thought of as a vector.
- A *feasible solution* is an assignment of values to variables such that all the constraints are satisfied.
- The *objective function value* of a solution is obtained by evaluating the objective function at the given solution.
- An *optimal solution* (assuming minimization) is one whose objective function value is less than or equal to that of all other feasible solutions.