Jinxi Xiac

Quiz

Linear Combinati

N.1

Dot Produc

Cross Product

Homework

Week 6

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Quiz and Exercise

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Remark

If you want to learn something, you have to spend time on it.

This question is about an m by n matrix A for which

$$Ax = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$
 has no solutions and $Ax = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ has exactly one solution.

- (a) Give all possible information about m and n and the rank r of A.
- (b) Find all solutions to Ax = 0 and explain your answer.
- (c) Write down an example of a matrix A that fits the description in part (a).



Solution

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Solution.

(a)
$$Ax = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$
 has one solution $\Longrightarrow N(A) = \{0\}$ so $r = n$. (Also, $m = 3$ since $Ax \in \mathbb{R}^3$.)

$$Ax = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$
 has no solution $\Longrightarrow C(A) \neq \mathbb{R}^3$, so $r < m$.

There are two possibilities:
$$\begin{array}{ccc} m=3 & m=3 \\ r=n=1 & r=n=2 \end{array} .$$

(b) Since
$$N(A) = \{0\}$$
 (because $Ax = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ has **1** solution), there is a unique solution to

$$Ax=0$$
, which is clearly $x=0$. (Can be either $x=\begin{bmatrix}0\\0\end{bmatrix}$ or $x=\begin{bmatrix}0\\0\end{bmatrix}$ depending on if $n=1$ or $n=2$.)

(c) A could be
$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$
 or $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ (many more possibilities).

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Linear Combination

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Define a set of vectors in \mathbb{R}^n as $\{v_1, v_2, \dots, v_k\}$. The linear combination of this set of vectors is

$$w = c_1v_1 + c_2v_2 + \cdots + c_kv_k$$

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L2 Norm

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Let $v \in \mathbb{R}^n = (v_1, codts, v_n)$, we define its norm to be

$$\sqrt{v_1^2+\cdots+v_n^2}$$

Concepts

- Normalization
- Standard Unit Vectors(bases)



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Propositions

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$$|u \cdot v| \leq ||u|| \cdot ||v||$$

$$||u+v|| \le ||u|| + ||v||$$

$$||u + v||^2 + ||u - v||^2 = ||u||^2 + ||v||^2$$

$$u \cdot v = \frac{1}{4}(||u+v||^2 + ||u-v||^2)$$

$$u \cdot v = u^T v = v^T u$$

Orthogonality and Projection

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They are much more powerful than you think. In 3D world, it is more related to planes and lines.

$$Proj_a u = \frac{u \cdot a}{||a||^2} a$$

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Definitions and Properties

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Cross Product

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Go and revise your PPT!

A few notes

- Can we change its sequence? $i \times j \times j$
- How to understand $(a \times b) \cdot c$?

More About Cross Product

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$$a \times b = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} b$$

已知:二维平面的两点X(x1, y1), Y(x2, y2), 证明X, Y两点的齐次式叉乘为过XY的直线的系数.

证明: 叉乘的定义为已知向量a = (a1,a2,a3), b=(b1,b2,b3), a叉乘b=(a2b3-a3b2, a3b1-a1b3, a1b2-a2b1) 因为XY的齐次式为 (x1,y1,1)和(x2,y2,1), 代入叉乘的定义得 (y1-y2, x2-x1, x1y2-y1x2)

定义直线的表达式为y=kx + b,将XY代入得:

y1 = kx1 + b

y2 = kx2 + b

化简后得:

k = (y2-y1)/(x2-x1)

b = y1 - ((y2-y1)/(x2-x1)) * x1

将y = kx + b 转化为 ax + by + c = 0的形式得 (a b c) = (-k, 1, -b) 化简后等于 (y1-y2, x2-x1, x1y2-y1x2)

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Need to Remember

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Need To Know

- Practice your calculation skills
- Revise the PPT
- Do not take things as granted