Linear Algebra: An Introduction

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About the Course *Linear Algebra I*

- Yes, this is a math course which is not that easy.
- However, it is extremely important, as a fundamental course. It is fascinating when it comes to scientific research.
- There are a number of ways to learn it outside of the classroom.
- Feel free to raise suggestions and seek help.

About the Schedule

- Homework accounts for 7% in total grade. We will give points by the degree of completion.
- The seminar is scheduled on every Wednesday at SPST 1-105, starting at 20:00. I think it is from week 3 to week 15(?)
- After each lecture, there will be around 5 questions.
- And the DDL for homework is 12 noon every Monday of the following week.
- Please submit your homework on BlackBoard in time. If you need extra time, please contact me in advance.

What Will Be Discussed During the Seminar ?

- Homework
- Summary of the last week's lectures
- Maybe a few more exercises...

Concepts

- A tactic to solve linear equations
- Three elementary row operations
- Row echelon form and reduced row echelon form
- Pivot position, leading variables ...

Keys

Remarks

- All row echelon forms of a matrix have the same number of zero rows, and the same pivot positions;
- Every matrix has a unique reduced row echelon form.

Maybe later in the course

$$A = LU$$

$A\mathbf{x} = \mathbf{b}$

$$\begin{cases} a_{11}x_1 + \cdots + a_{1n}x_n = b_n \\ \vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n = b_m \end{cases} \Longrightarrow \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ & \vdots & \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

Homogeneous Linear System

Basic form: $A\mathbf{x} = \mathbf{0}$

Corollary

There are exactly two possibilities for a homogeneous linear systems:

- It has only the trivial solution.
- The system has infinitely many solutions (including the trivial solution)

Some Explanations

Why?

If A is the augmented matrix of a linear system, then the number of effective equations in the system is exactly r.

Corollary

Suppose that a linear system has coefficient matrix A and augmented matrix B, where $A \in M_{m\times n}$ and $B \in M_{m\times(n+1)}$

- It is inconsistent if and only if rank(A) $<$ rank(B).
- It is consistent if and only if rank(A) $=$ rank(B).
- It has exactly one solution if and only if rank(A) $=$ rank(B) $=$ n.
- It has infinitely many solutions if and only if rank(A) $=$ rank(B) $<$ n.

Operations

My point of view

- The most fascinating operation in *this course* is **multiplication**
- To a certain point, if sizes are matched, the calculation is accurate
- Please learn carefully in next lecture, for *linear combination* will be introduced!

Resource

After Class

- **Visual Explanation**
- Chinese Course
- MIT Course Recordings
- Search in *Zhihu*, and many other blog platforms