Comparing Optimal Relocation Operations With Simulated Relocation Policies in One-Way Carsharing Systems

Diana Jorge, Gonçalo H. A. Correia, and Cynthia Barnhart

Abstract—One-way carsharing systems allow travelers to pick up a car at one station and return it to a different station, thereby causing vehicle imbalances across the stations. In this paper, a way to mitigate that imbalance is discussed, which is relocating vehicles between stations. For this purpose, two methods are presented, i.e., a new mathematical model to optimize the relocation operations that maximize the profitability of a carsharing service and a simulation model to study different real-time relocation policies. Both methods were applied to networks of stations in Lisbon, Portugal. Results show that relocating vehicles, using any of the methods developed, can produce significant increases in profit. For instance, in the case where the carsharing system provides maximum coverage of the city area, the imbalances in the network resulted in an operating loss of €1160/day when no relocation operations were performed. When relocation policies were applied, however, the simulation results indicate that profits of €854/day could be achieved, even with increased costs due to relocations. Using the mathematical model, the results are even better, with a reached profit of €3865.7/day. This improvement was achieved through reductions in the number of vehicles needed to satisfy the demand and the number of parking spaces needed at stations. These results demonstrate the importance of relocation operations for profitably providing a network of stations in one-way carsharing systems that covers the entire city, thus reaching a higher number of users.

Index Terms—Mathematical programming, one-way carsharing, relocation operations, simulation.

I. INTRODUCTION

IN the last decades, changes have occurred in urban transportation. Despite the greater accessibility provided by private transportation, the result has been increases in levels of congestion, pollution, and nonproductive time for travelers, particularly in peak hours [1]. There are also opportunity costs associated with using urban land for parking spaces instead of other more productive activities. In the USA, for example, automobiles spend around 90% of their time parked [2]. These issues are mitigated by public transportation, but it has other disadvantages, e.g., poor service coverage, schedule inflexibility, and lack of personalization. In addition, providing public transportation for peak-hour demand can result in idle vehicles for much of the day, resulting in inefficiencies and a high cost of service.

Strategies are needed to address these issues and to simultaneously provide people the mobility they need and desire. One strategy considered is that of carsharing. Carsharing systems involve a small to medium fleet of vehicles, which are available at several stations, to be used by a relatively large group of members [3].

The origins of carsharing can be traced back to 1948, when a cooperative known as *Sefage* initiated services in Zurich, Switzerland. In the USA, carsharing programs only appeared later in the 1980s, within the *Mobility Enterprise* program [3]. In Asian countries, such as Japan and Singapore, these systems appeared more recently.

Carsharing has been observed to have a positive impact on urban mobility, mainly by using each car more efficiently [4], [5]. The use of carsharing systems generally leads to a fall in car ownership rates and, thus, to lower car use, according to Celsor and Millard-Ball [6]. More recently, Schure *et al.* [7] has conducted a survey in 13 buildings in downtown San Francisco and concluded that the average vehicle ownership for households that use carsharing systems is 0.47 vehicles/household compared with 1.22 vehicles/household for households that do not use carsharing systems. Moreover, a study by Sioui *et al.* [8] surveyed the users of *Communauto*, which is a Montreal carsharing company, and concluded that a person who does not own a vehicle and frequently uses carsharing systems (more than 1.5 times per week) never reaches the car-use level of a person who owns a vehicle, i.e., there was a 30% difference between them. This idea is reinforced by Martin and Shaheen [9] who concluded through a survey in the USA and Canada that the average observed vehicle-kilometers traveled (VKT) of respondents before joining carsharing was 6468 km/year, whereas the average observed VKT after joining carsharing was 4729 km/year, which constitutes a decrease of 27% (1749 km/year).

Furthermore, some recent studies have concluded that carsharing systems also have positive environmental effects. For instance, Martin and Shaheen [9] noted from the VKT

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estimations presented earlier that the greenhouse gas (GHG) emissions of the major carsharing organizations in the USA and Canada can be reduced by -0.84 t GHG/year/household. Although most members increase their emissions, there are compensatingly larger reductions for other members who decrease their emissions. Moreover, Firnkorn and Müller [10], through a survey of a German carsharing company, concluded that the CO_2 emissions are decreased between 312 and 146 kg CO_2/year/average carsharing system user.

With respect to the trip configuration, carsharing systems are divided into round-trip (two-way) systems and one-way systems. Round-trip carsharing systems require users to return the cars to the same station from where they departed. This simplifies the task of the operators because they can plan vehicle inventories based on the demand for each station. It is, however, less convenient for the users because they have to pay for the time that the vehicles are parked. In one-way carsharing systems, users can pick up a car in a station and leave it at a different station [11]. In theory, therefore, one-way carsharing systems are better suited for more trip purposes than round-trip services that are typically used for leisure, shopping, and sporadic trips, i.e., short trips in which vehicles are parked for a short duration [12]. This statement is supported by various studies, including that by Costain *et al.* [13], who studied the behavior of a round-trip carsharing company in Toronto, Canada, and concluded that trips are mostly related to grocery or other household shopping purposes. A study performed in Greece by Efthymiou *et al.* [14] also concluded that the flexibility to return the vehicle to a different station from the station where it was picked up is a critical factor in the decision to join a carsharing service. However, one-way carsharing systems present an operational problem of imbalances in vehicle inventories or stocks across the network of stations due to the nonuniformity of trip demand between stations. Despite this, great effort has been made to provide these flexible systems to users in recent years.

Previous research has proposed several approaches to solve this problem, such as vehicle relocations in order to replenish vehicle stocks where they are needed [15]–[20], pricing incentive policies for the users to relocate the vehicles themselves [21], [22], operating strategies designed around accepting or refusing a trip based on its impact on the vehicle stock balance [23], [24], and station location selection to achieve a more favorable distribution of vehicles [24]. Correia and Antunes [24] proposed a mixed-integer programming (MIP) model to locate one-way carsharing stations to maximize the profit of a carsharing company, considering the revenues (the price paid by the clients) and costs (vehicle maintenance, vehicle depreciation, and the maintenance of parking spaces) and assuming that all demand between existing stations must be satisfied. In applying their model to a case study in Lisbon, Portugal, tractability issues ensued, and the model was only solvable with a time discretization of 10-min steps. The model did not allow integrating the relocation operations due to the complexity that was already reached with the location problem.

In this paper, the same case study as that in [24] is considered, and station location outputs are generated using their model but now with a time discretization of 1-min. When a 10-min-based model is used, all of the travel times between stations are rounded to the next multiple of 10. Hence, users are paying for the minutes that they are not really using the vehicles. Moreover, the vehicles are also considered available only in each multiple of 10 min, whereas the reality is that they could be available earlier. Therefore, a 1-min-based model is always more realistic than a 10-min-based model or a model that considers larger time steps.

A new model is presented to optimize relocation operations on a minute-by-minute basis, given those outputs for station locations brought from the aforementioned model. Thus, the two problems, i.e., the station locations and the relocation operations, will not be considered at the same time. The objective function is the same, i.e., profit maximization, but in the relocations model, a cost for the relocation operations is also added. The vehicle relocation solutions generated with this approach, i.e., the optimal solutions, are later compared with those obtained with a simulation model that is built to evaluate different real-time vehicle relocation policies, i.e., the realistic solutions. With this comparison, the impacts of the relocation operations on the profitability of one-way carsharing systems are then analyzed, and insights into how to design and implement real-time rebalancing systems are gained.

This paper is structured as follows. In the next section, a new mathematical model is presented to optimize relocation operations, given an existing network of one-way carsharing stations. Then, a simulation model and a specification of several real-time relocation policies are presented. In the following section the case study used for testing the relocation methodologies is described, as well as the data needed and the main results reached. This paper ends with the main conclusions extracted from the study.

II. MATHEMATICAL MODEL

The objective of the mathematical programming model presented in this section is to optimize the vehicle relocation operations between a given network of stations (using a staff of drivers) in order to maximize the profit of a one-way carsharing company. In this model, all demand between existing stations is assumed to be satisfied. The notation used to formulate the model (sets, decision variables, auxiliary variables, and parameters) is shown in the following.

Sets:

$\mathbf{N} = \{1, \ldots, i, \ldots, N\}$ set of stations;

$\mathbf{T} = \{1, \ldots, t, \ldots, T\}$ set of minutes in the operation period;

$\mathbf{X} = \{1_1, \ldots, i_{t-1}, i_t,$
$\quad i_{t+1}, \ldots, N_T\}$ where i_t represents station i at minute t, i.e., the set of the nodes of a time-space network combining N stations with T minutes;

$\mathbf{A_1} = \{\ldots, (i_t, j_{t+\delta_{ij}^t}),$
$\quad, \ldots\}, i_t \in \mathbf{X}$ set of arcs over which the vehicles move between stations i and j $\forall i, j \in \mathbf{N}$, and $i \neq j$ between minute t and $t + \delta_{ij}^t$, where δ_{ij}^t is the travel time (in number of minutes) between stations i and j when the trip starts at minute t;

$\mathbf{A_2} = \{\ldots, (i_t, i_{t+1}),$
$\qquad , \ldots\} i_t \in \mathbf{X}$ set of arcs that represent the vehicles stocked in station $i \ \forall i \in \mathbf{N}$ from minute t to minute $t+1$.

Decision variables:

$R_{i_t j_{t+\delta_{ij}^t}}$ number of vehicles relocated from i to j from minute t to $t + \delta_{ij}^t \ \forall (i_t, j_{t+\delta_{ij}^t}) \in \mathbf{A_1}$;

Z_i size of station $i \ \forall i \in \mathbf{N}$, where size refers to the number of parking spaces;

a_{i_t} number of available vehicles at station i at the start of minute $t \ \forall i_t \in \mathbf{X}$.

Auxiliary variables:

$S_{i_t i_{t+1}}$ number of vehicles stocked at each station i from minute t to $t+1 \ \forall (i_t, i_{t+1}) \in \mathbf{A_2}$. This is a dependent variable that is only used for performance analysis.

Parameters:

$D_{i_t j_{t+\delta_{ij}^t}}$ number of customer trips (not including the vehicle relocation trips) from station i to station j from t to $t + \delta_{ij}^t \ \forall (i_t, i_{t+\delta_{ij}^t}) \in \mathbf{A_1}$;

P carsharing fee per minute driven;

C_{mv} cost of maintenance per vehicle per minute driven;

δ_{ij}^t travel time, which is in minutes, between stations i and j when the departure time is $t \ \forall i_t \in \mathbf{X}, j \in \mathbf{N}$;

C_{mp} cost of maintaining one parking space per day;

C_v cost of depreciation per vehicle per day;

C_r cost of relocation and maintenance per vehicle per minute driven.

Using the aforementioned notation, the mathematical model can be formulated as follows:

$$\max \pi = (P - C_{\mathrm{mv}}) \times \sum_{i_t j_{t+\delta_{ij}^t} \in \mathbf{A_1}} D_{i_t j_{t+\delta_{ij}^t}} - C_{\mathrm{mp}} \sum_{i \in \mathbf{N}} Z_i$$
$$- C_v \sum_{i \in \mathbf{N}} a_{i_1} - C_r \sum_{i_t j_{t+\delta_{ij}^t} \in \mathbf{A_1}} R_{i_t j_{t+\delta_{ij}^t}} \qquad (1)$$

subject to

$$S_{i_t i_{t+1}} + \sum_{j \in \mathbf{N}} + D_{i_t j_{t+\delta_{ij}^t}} + \sum_{j \in \mathbf{N}} R_{i_t j_{t+\delta_{ij}^t}}$$
$$- \sum_{j \in \mathbf{N}: t' = t - \delta_{ij}^t} D_{j_{t'} i_t}$$
$$- \sum_{j \in \mathbf{N}: t' = t - \delta_{ij}^t} R_{j_{t'} i_t} - S_{i_{t-1} i_t}$$
$$= 0 \ \forall i_t \in \mathbf{X} \qquad (2)$$

$$a_{i_t} - \sum_{j_t \in \mathbf{X}} D_{i_t j_{t+\delta_{ij}^t}} - \sum_{j_t \in \mathbf{X}} R_{i_t j_{t+\delta_{ij}^t}} - S_{i_t i_{t+1}}$$
$$= 0 \ \forall i_t \in \mathbf{X} \qquad (3)$$

$$Z_i \geq a_{i_t} \quad \forall i_t \in \mathbf{X} \qquad (4)$$

$$R_{i_t j_{t+\delta_{ij}^t}} \in \mathbb{N}^0 \quad \forall \left(i_t, j_{t+\delta_{ij}^t}\right) \in \mathbf{A_1} \qquad (5)$$

$$S_{i_t i_{t+1}} \in \mathbb{N}^0 \quad \forall (i_t, i_{t+1}) \in \mathbf{A_2} \qquad (6)$$

$$a_{i_t} \in \mathbb{N}^0 \quad \forall i_t \in \mathbf{X} \qquad (7)$$

$$Z_i \in \mathbb{N}^0 \quad \forall i \in \mathbf{N}. \qquad (8)$$

The objective function [see (1)] is to maximize the total daily profit π of the one-way carsharing service, taking into consideration the revenues obtained through the trips paid by customers, relocation costs, vehicle maintenance costs, vehicle depreciation costs, and station maintenance costs. Constraints (2) ensure the conservation of vehicle flows at each node of the time-space network, and constraints (3) compute the number of vehicles at each station i at the start of time t, assuming that the vehicles destined to i at time t arrive before the vehicles originating from i at time t depart. Constraints (4) guarantee that the size of the station at location i is greater than the number of vehicles present there at each minute t. In practice, the size will not be greater than the largest value of a_{i_t} during the period of operation; otherwise, the objective function would not be optimized. Expressions (5)–(8) set that the variables must be integers and positive.

III. SIMULATION MODEL

In order to test real-time relocation policies, a discrete-event time-driven simulation model has been built using AnyLogic (AnyLogic Company, formerly XJ Technologies), which is a simulation environment based on the Java programming language. It is assumed that a trip will be performed only if there is simultaneously a station near the origin of the trip and a station near the trip's destination. The effects of congestion on the road network are captured with different link travel times throughout the day.

In each minute, trips and relocation operations are triggered, and the model updates a number of system attributes, including the number of completed minutes driven by the customers and by the vehicle relocation staff, the vehicle availability at each station, the total number of vehicles needed, and the maximum vehicle stock (i.e., the number of parked vehicles) at each station, which are used to compute the needed capacity of each station. These updated values are used to compute the objective function. It includes all revenues (the price rate paid by customers) and costs (vehicle maintenance, vehicle depreciation, parking space maintenance, and relocation operations). To satisfy all demand, a vehicle is created (the fleet size is correspondingly increased) each time a vehicle is needed in a given station for a trip and there are no vehicles available. Thus, the fleet size is an output of the simulation. The period of the simulation is between 6 A.M. and midnight, which is the same period used in [24]. At the end of the simulation run, it is possible to obtain the total profit and the total number of parking spaces needed in each station.

A. Relocation Policies

Two real-time relocation policies (1.0 and 2.0) were tested in the simulation. For each relocation policy, it is determined for each minute of the day at each station s if the status of s is that of a supplier (with an excess number of vehicles) or a demander (with a shortage of vehicles). For policy 1.0, a station s at time t is classified as a supplier if, on a previous day of operations, the number of customer trips destined for that station at instant $t + x$ exceeds or equals the number of customer trips that depart that station at the same period. Note that only the customer trips and not the repositioning trips are included in this calculation. Each station that is not designated as a supplier is classified as a demander. In this policy, x is varied between 5 and 20 min
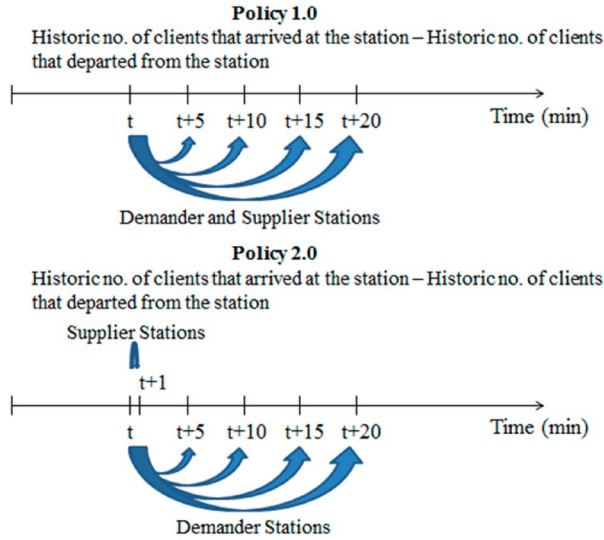
Fig. 1. Schematic representation of policies 1.0 and 2.0.

in 5-min increments to determine the supplier and the demander stations. If s is classified as a supplier, its supply is equal to the number of extra vehicles (those that are not needed for serving customer demand) at s at time t, multiplied by a relocation percentage that is a parameter. If s is classified as a demander, its demand for vehicles is set equal to the number of additional vehicles needed to serve the demand at time $t + x$. For relocation policy 2.0, the process is the same, but x is set equal to 1 min to determine the set of supplier stations, and the associated supplies are determined, as described for policy 1.0. The demander stations and their demand are determined, as in relocation policy 1.0.

A schematic representation of these policies is shown in Fig. 1.

For each time t, given these calculated values of the vehicle supply or demand at each station, the relocation of vehicles between stations is determined by solving a classic transportation problem. The objective is to find the minimum cost distribution of vehicles from m origin nodes (representing the supplier stations) to n destination nodes (representing the demander stations), with costs that are equal to the total travel time. An artificial supply node and an artificial demand node are added to the network, with all the supply and demand concentrated at the respective artificial nodes. The artificial supply node is connected to the supply nodes, which are linked to the demand nodes, and finally, the demand nodes are linked to the artificial demand node (as shown Fig. 2). For each arc, the following three parameters are defined, i.e., the cost of the arc (the travel time), the lower bound on the arc flow (the minimum number of vehicles), and the upper bound on the arc flow (the maximum number of vehicles). On each arc from the artificial supply node to a supply node i, the lower and upper bounds on the flow equal the supply at i, and the travel time on the arc is 0. For each arc between a supply node at station i and a demand node j, the lower bound on the flow is zero, and the upper bound is the minimum of the supply of vehicles at i and the number of vehicles demanded at j. On each arc between a demand node j and the artificial demand node, the lower and upper flow
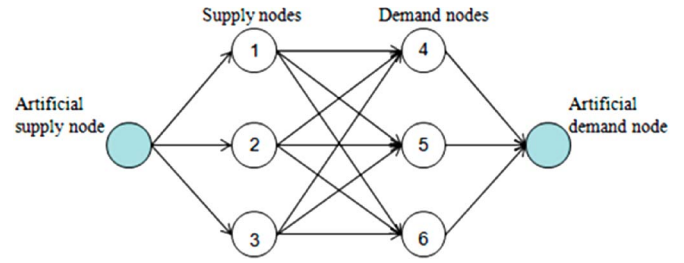

Fig. 2. Minimum cost flow algorithm scheme.

bounds equal the demand at j, and the travel time on the arc is 0. When there is imbalance between the total supply and the total demand, either one extra supply node or one extra demand node is created.

In the simulation, an optimal relocation is determined using a minimum cost network flow algorithm that is available in the simulation programming language Java [25].

As aforementioned, for each simulation run, two tuning parameters, i.e., the relocation percentage and x, are defined. The relocation percentage multiplied by the supply (of vehicles) at a supplier station represents the value of the supply input to the transportation algorithm. x represents the time period used for the minute-by-minute calculation for each station to determine its status as either a supplier or demander of vehicles.

Using relocation policies 1.0 and 2.0 as a starting point, three variants of these two policies were developed for each of them. The first is that each supplier station is required to keep at least one vehicle at that station at all time steps, i.e., its supply is equal to the number of extra vehicles minus 1 at time t, multiplied by the relocation percentage (policies 1.A and 2.A). The second is that the distribution of vehicles at each station at the start of the day is set to that generated by the mathematical model defined in the previous section (policies 1.B and 2.B). In addition, the third is the same as the second with priority given to the stations with the greatest demand for vehicles (policies 1.C and 2.C). In practice, this is done through artificially reducing the travel time to those stations that need a higher number of vehicles, thus making them more attractive as a destination for the vehicles, according to the assignment method explained earlier. The travel times to a demander station are reduced as a function of the relative magnitude of the demand at that station. For example, if the demand at station s equals or exceeds 10% of the total demand for vehicles at all demander stations, the travel times between the supplier stations and station s are decreased by 10% (which is done by multiplying the travel times by 0.9).

A schematic representation of the methodology that is used in this paper is presented in Fig. 3.

IV. LISBON CASE STUDY

The case study used in this paper is the same as in [24]. It is the municipality of Lisbon, which is the capital city of Portugal. Lisbon has been facing several mobility problems, such as traffic congestion and parking shortages due to the increase in car ownership and the proliferation of urban expansion areas in the periphery that is not served by public transportation.
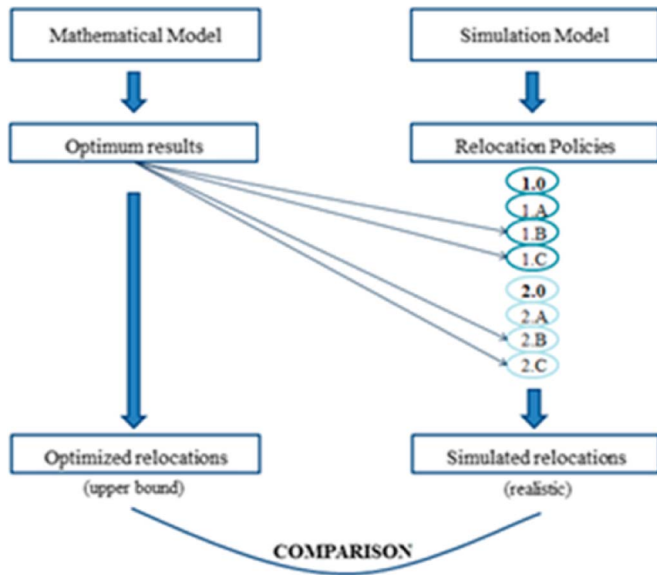
Fig. 3.   Schematic representation of the methodology used.

Moreover, public transportation, even with the improvements that have been achieved, was not able to restrain the growth in the use of private transportation for commuter trips. For these reasons, the municipality of Lisbon is a good example where different alternative transportation modes, such as carsharing, may be implemented.

A. Data

The data needed are the following: a carsharing trip matrix, a set of candidate sites for locating stations, the driving travel times, and the costs of operating the system. The trip matrix was obtained through a geocoded survey that was conducted in mid-1990s and updated in 2004 in the Lisbon Metropolitan Area (LMA). The survey data contains very detailed information on the mobility patterns of the LMA, including origins and destinations, the time of the day, and the transportation mode used for each trip. This survey was filtered through some criteria, such as the age of the travelers, the trip time, the trip distance, the time of the day in which the trip is performed, and the transportation mode used, in order to only consider the trips that can be served by this system, resulting in 1777 trips. The candidate station locations were defined by considering a grid of squared cells (with 1000-m long sides) over Lisbon and by associating one location with the center of each cell. The result was a total of 75 possible station locations. This is obviously a simplification. To implement a carsharing system in a city, a detailed study of appropriate locations would be necessary. Travel times were computed using the transportation modeling software VISUM (PTV Group), considering the Lisbon network and the aforementioned mobility survey, and they were expressed in minutes. The carsharing system is available 18 h/day, between 6:00 A.M. and 12:00 A.M. The morning and afternoon peaks correspond to the periods between 8:00 A.M. and 10 A.M., and 6:00 P.M. and 8 P.M., respectively. To compute the costs related to the vehicles, it is considered an "average" car, whose initial cost is €20 000, and this car is mainly driven in a city. The following
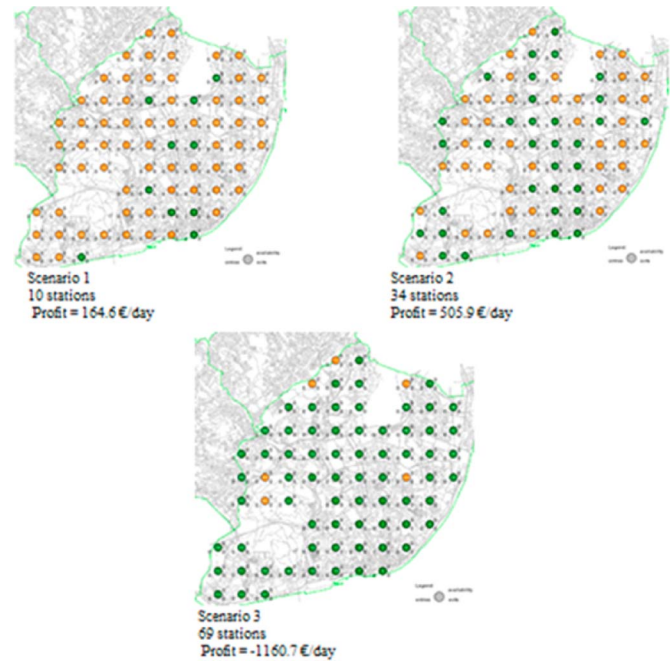

Fig. 4.   Location model solutions.

costs of running the system were calculated as realistically as possible.

C_{m1} cost of maintaining a vehicle, i.e., €0.007/min. This cost was calculated using a tool that is available on the Internet that was developed by a German company, i.e., INTERFILE [26], and it includes the insurance, fees and taxes, fuel, and maintenance and wear of the vehicle;

C_v cost of depreciation per vehicle, i.e., €17/day, which is calculated using the same aforementioned tool [26] and considering that the vehicles are used for three years. It was also considered that the company fully needed financing for the purchase of the vehicles, with an interest rate of 12%, and the vehicles' residual value is equal to €5000;

C_r cost of relocating a vehicle, i.e., €0.2/min since the average hourly wage in Portugal is €12/h;

C_{m2} cost of maintaining a parking space, i.e., €2/day; this cost is smaller than the parking fee in a low-price area in Lisbon, considering that the city authorities would be able to give support to these types of initiatives.

Carsharing price per minute P was considered €0.3/min, which is based on the rates of *car2go* [27].

The station location model [24] was run for three scenarios, with a minute-by-minute discretization of time (note that this model does not include vehicle relocations). The three networks used in this paper are the three networks found in [24], as well as the trip matrix used. In the first scenario, the number of stations was constrained to be just 10 (considered a small network). In the second scenario, the stations were freely located to maximize profit (any number, any location). In the third scenario, the stations were located to satisfy all demand in the city (where there is demand, there is a station). The results, including the station locations, the number of stations, and the associated profits, are presented in Fig. 4.

TABLE I
RESULTS FOR THE DIFFERENT RELOCATION POLICIES

Station network (scenarios)	Indicators	Optimization of the station locations	Best results for each policy							
			1.0	2.0	1.A	1.B	1.C	2.A	2.B	2.C
69 (full demand attended)	x (min)	--	5	10	15	5	10	10	10	10
	Best relocation %	--	50	90	100	60	80	100	40	90
	Vehicles	390	264	273	262	264	257	267	318	222
	Parking spaces	739	533	490	550	412	409	480	415	334
	Time driven (min)	23711	23711	23711	23711	23711	23711	23711	23711	23711
	Time of relocations (min)	0	4008	2921	4800	4346	5169	2967	2661	9051
	Demand proportion/Travel time decreasing	--	--	--	--	--	0.7/0.3 - 0.9/0.1	--	--	0.1/0.9
	Profit (€/day)	-1160.7	591.7	742.1	433.3	766.1	726.5	854.9	179.1	695.1
34 (free optimum)	x (min)	--	5	5	5	5	15	5	5	5
	Best relocation %	--	0	0	10	0	10	0	0	0
	Vehicles	121	121	121	121	126	125	121	126	126
	Parking spaces	241	241	241	240	195	195	241	195	195
	Time driven (min)	10392	10392	10392	10392	10392	10392	10392	10392	10392
	Time of relocations (min)	0	0	0	4	0	54	0	0	0
	Demand proportion/Travel time decreasing	--	--	--	--	--	0.1/0.9 - 0.3/0.7	--	--	all equal
	Profit (€/day)	505.9	505.9	505.9	507.1	512.9(*)	519.1(**)	505.9	512.9(*)	512.9(*)
10 (small network)	x (min)	--	5	5	5	5	5	5	5	5
	Best relocation %	--	0	0	0	0	0	0	0	0
	Vehicles	22	22	22	22	22	22	22	22	22
	Parking spaces	42	42	42	42	29	29	42	29	29
	Time driven (min)	2125	2125	2125	2125	2125	2125	2125	2125	2125
	Time of relocations (min)	0	0	0	0	0	0	0	0	0
	Demand proportion/Travel time decreasing	--	--	--	--	--	all equal	--	--	all equal
	Objective (€/day)	164.6	164.6	164.6	164.6	190.6(*)	190.6(*)	164.6	190.6(*)	190.6(*)

(*) no relocations occur, profit achieved only by bringing the initial availability from optimization

(**) this profit is achieved using relocations and bringing the initial availability from optimization

B. Results

The optimum relocation operations were determined using model (1)–(8), and all relocation policies were simulated with all possible parameter combinations for each of the three station location solutions (scenarios) generated with the approach of the work in [24]. The value of x was varied between 5 and 20 min in 5-min increments, as it is referred to in Section III. This range was selected because most travel times are between these two values. The relocation percentage was varied between 0% (no relocations) and 100% (all available vehicles in the supplier stations can be relocated) in 10% increments. For policies 1.C and 2.C, simulation results were generated for the following combinations of parameters: 0.1/0.9 (more than 10% of the demand in a station, with a 90% decrease in travel time), 0.3/0.7, 0.5/0.5, 0.7/0.3, and 0.9/0.1. In the end, the number of simulation runs was 1920.

For all the scenarios, the mathematical model was run in an i7 processor at 3.40 GHz, with a 16-GB random access memory computer under a Windows 7 64-bit operation system using Xpress, which is an optimization tool that uses branch-and-cut algorithms for solving MIP problems [28]. The solutions found were always optimal. Xpress took about 206 min to reach the optimal solution for scenario 1, 5 min for scenario 2, and 8.3 s for scenario 3. The time that the model took to run is reasonable even for the bigger scenario with 69 stations located. The factor that influences how quickly the solutions are achieved is the number of stations doubtlessly.

With respect to the simulation model, there was the need to run it many more times than the optimization routine, but each time, the model only took a few seconds to run.

In Table I, the best simulation results for each relocation policy are shown.

Analyzing Table I and comparing with the solution with no relocations, policy 1.0 achieves better results only for the 69-station scenario, increasing from €1160.7/day (losses) to €591.7/day (profit). This profit is achieved by setting the x parameter equal to 5 min and the relocation percentage equal to 50%. Similar results to those of policy 1.0 are evident for policy 2.0, but policy 2.0 achieves a greater profit (€742.1/day), with the relocation percentage set to 90%, and x is equal to 10 min.

Policy 1.A achieves better results (a profit of €433.3/day) when compared with the solution with no relocations only for the 69-station scenario, using a relocation percentage that is equal to 100% and x that is equal to 15 min. This profit, however, is lower than the profits reached by using policies 1.0 and 2.0.

For policy 1.B, it is possible to improve the profits for all scenarios compared with the model with no relocations; however,
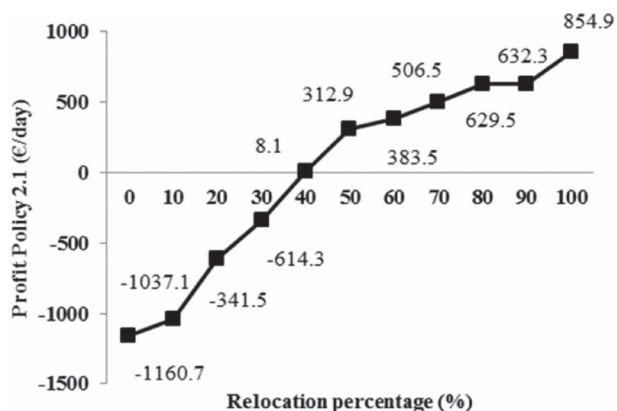
Fig. 5. Evolution of profit for the best relocation policy found with 69 stations located, and parameter x is equal to 10 min.

for the 34-station and 10-station scenarios, profit increases are achieved by using the initial availability of vehicles at each station brought from model (1)–(8). The profit is €766.1/day for the 69-station scenario, using a relocation percentage that is equal to 60% and x that is equal to 5 min. For the scenarios with 34 stations and 10 stations, however, the increase in profit is very low.

With respect to policy 1.C, the results are better than the solution with no relocations for the 69-station scenario. The best result, i.e., €726.5/day, is achieved for two fraction-of-demand and fraction-of-travel-time scenarios, i.e., (0.7/0.3) and (0.9/0.1), using a relocation percentage that is equal to 80% and x that is equal to 10 min. For the 34-station scenario, the profit is €519.1/day, which is similar to that obtained with no relocations (€512.9/day).

For policy 2.A, the results are similar to those for policy 1.A, but with greater profit (€854.9/day), using a relocation percentage that is equal to 100% and x that is equal to 10 min. The results for policies 2.B and 2.C are similar to those obtained for 1.B and 1.C.

Policy 2.0 is better than policy 1.0 for the 69-station scenario; policy 1.A is worse than policy 2.A; and policies 1.B and 1.C are better than policies 2.B and 2.C. For the network with the optimum number of stations located (34 stations), policy 1.C is better than policy 2.C, whereas policies 1.A and 1.B are similar in effectiveness to policies 2.A and 2.B. Finally, for the 10-station scenario, the best profit is reached when no relocations occur and the initial availability of vehicles at each station is brought from model (1)–(8). The small network that was tailored to the demand data makes it difficult to improve the profit with relocations.

Although only the best results are presented in Table I, it is important to note that, with variations in the relocation percentage and x parameters, the objective function values fluctuate greatly. This is shown in Fig. 5 for the 69-station scenario and policy 2.A. With x equal to 10 min, variations in the relocation percentage result in variations in the objective function value from €1037.1/day to €854.9/day. These parameters must be appropriately calibrated for each city and travel pattern scenario to produce the best results.

As a general conclusion, with relocations, improvements in profit are achieved through a combination of reductions in the number of vehicles and/or in the number of parking spaces. These reductions offset the corresponding increases in staff costs and vehicle maintenance costs resulting from the relocations. For the 69-station scenario, the greatest profit is reached with policy 2.A, which allows a reduction of 31.5% in the number of vehicles and a reduction of 35.0% in the number of parking spaces relative to the scenario with no relocations. The time spent with vehicle relocations in this case is 2967 min/day (about 50 h/day). However, policy 2.C allows the greatest reduction in the number of vehicles (43.1%) and in the number of parking spaces (54.8%) but requires a threefold increase in the relocation time (9051 min/day). This illustrates that minimizing vehicles and parking spaces does not necessarily maximize profit.

In Table II, for each of the three network scenarios, results are compared for the solutions to the station location model without relocations [24], the solutions applying the relocation optimization model (1)–(8), and the best performing simulated relocation policy.

The results for the simulated relocation policies are far from the optimal relocation solutions, showing that it is difficult to design effective real-time strategies based on fixed rules. A case in point is the 34-station scenario in which the optimized relocations contribute to an improvement in profit of about €1262/day, whereas the real-time relocation policies only improve the profit to about €13/day.

Nevertheless, it is important to observe that the policies evaluated in this paper were able to make profitable the 69-station scenario that serves all demand in the city. Relocation policies, then, can help carsharing companies to provide sustainable services to greater numbers of people in expanded geographic areas.

V. CONCLUSION

The most convenient carsharing systems for users are one-way systems; however, as detailed in literature, these systems require vehicle repositioning to ensure that the vehicles are located where they are needed [17], [21], [22]. Several approaches have been proposed to try to solve this problem, such as an operator-based approach [15], [16] and a station-location approach [24]. With the operator-based approach, the stock of vehicles at stations is adjusted by relocating the vehicles to locations where they are needed.

In this paper, we have presented two independent tools that can be combined, i.e., a mathematical model for optimal vehicle relocation and a discrete-event time-driven simulation model with several real-time relocation policies integrated. Kek *et al.* [15], [16] also developed an optimization model and a simulation model, but in their work, only the optimization model allows determining the relocation operations. The simulation model is only used to evaluate the performance of the systems when the relocation operations that were determined by the optimization model are performed. Nair and Miller-Hooks [17] developed a stochastic MIP model to optimize vehicle relocations, which has the advantage of considering demand uncertainty. However, they did not develop a simulation model and a way of determining relocation operations in real time.

TABLE II
RESULTS FOR THE DIFFERENT PROBLEMS

Models	69 stations		34 stations		10 stations	
	Profit (€/day)	Improvements (€/day)	Profit (€/day)	Improvements (€/day)	Profit (€/day)	Improvements (€/day)
Optimization of station locations	-1160.7	--	505.9	--	164.6	--
Optimization of relocation operations	3865.7	5026.4	1768.1	1262.2	322.0	157.4
Simulation with the best relocation policy	854.9	2015.6	519.1	13.2	190.6	26

Barth *et al.* [18] presented a queuing-based discrete-event simulation model and three ways of deciding when relocations should be performed, one of which, which is called "historical predictive relocation, " is similar to what is proposed in the relocation policies presented here. However, there has been a higher number of policies and combination of parameters tested in this paper than in [18]. Moreover, Barth *et al.* [18] did not develop an optimization model and ways of combining both optimization and simulation. With respect to Barth *et al.* [19], an aggregated approach was developed. They only studied a measure to determine if the whole system needs relocations or not, whereas in this paper, each station has been treated individually.

The developed optimization model was applied to the case study first introduced by Correia and Antunes [24]. Using the alternative networks of stations that were obtained for the city of Lisbon, the relocation approaches that have been developed in this paper were evaluated and compared.

The optimized relocation decisions for these networks indicated significant potential improvements in the system profit. For instance, the solution covering all demand for the entire city (containing 69 stations) has an estimated daily loss of €1160, but when operations are expanded to include optimal relocation decisions, this estimated daily loss is transformed into an estimated daily profit of about €3800. There are also significant economic improvements in the other networks (containing 34 and 10 stations).

The optimal solutions to the relocation model provide upper bounds on the economic gains that are achievable with relocations because the inputs to the optimization model require *a priori* knowledge of the full pattern of daily trip demands. To evaluate the impacts of real-time relocation operations in this research, relocation policies were devised and executed in a simulation model. For the largest network of stations that these simulated, real-time relocation strategies are estimated to improve the profitability significantly, reaching a profit of about €855/day with the best relocation policy. This is far from the optimum; however, it is implemented in real time, making it more likely to be achieved in a real operation when vehicles are not reserved one day in advance. For the smaller networks, the correspondingly smaller improvement is explained by the fact that the station locations in these networks were specifically chosen to reduce the need for repositioning by using the model in [24]. By integrating the results of the relocation optimization model with the relocation policies (e.g., using in the simulation the optimization's initial vehicle availability at each station), improved results are achieved for the relocation policies.

The main conclusion that has been drawn from this paper is that relocation operations should be considered when setting up station-based one-way carsharing systems. Important effort must be made into more deeply studying what has been defined in this paper as the real-time relocation policies to be implemented in the day-to-day operation of these systems, thus allowing the sustainability of full network coverage of this service in a city. The fact that by introducing relocation policies it was possible to transform the worst profitable network (69 stations) into the most profitable network encourages research into expanding the methods to estimate when and how many vehicles should be relocated between stations [29].

With respect to the transferability of both models (the mathematical model and the simulation model) to another city, it is important to remember that mathematical models always have a computation time that is dependent on the problem dimension. Thus, as the city size increases, i.e., the number of carsharing stations, the computation time should also increases due to the increasing number of decision variables. Regarding the simulation model, this problem is nonexistent. Therefore, it can be applied to any city independently of its dimension.

Moreover, the results that have been presented in this paper are very sensitive to changes in the travel demand. Hence, the simulation model that has been built in this paper should be improved in future projects to increase the realism of the day-to-day operation of such transportation system, including the stochastic trip variability and the travel time.

REFERENCES

[1] D. Schrank, T. Lomax, and S. Turner, "TTI's 2010 urban mobility report," Texas Transp. Inst., College Station, TX, USA, 2010.
[2] *National Household Travel Survey: Summary of Travel Trends*, U.S. Department of Transportation, Washington, DC, USA, 2001.
[3] S. Shaheen, D. Sperling, and C. Wagner, "A short history of carsharing in the 90's," *J. World Transp. Policy Pract.*, vol. 5, no. 3, pp. 16–37, Sep. 1999.
[4] T. Litman, "Evaluating carsharing benefits," *Transp. Res. Rec., J. Transp. Res. Board*, no. 1702, pp. 31–35, 2000.
[5] T. Schuster, J. Byrne, J. Corbett, and Y. Schreuder, "Assessing the potential extent of carsharing—A new method and its applications," *Transp. Res. Rec., J. Transp. Res. Board*, no. 1927, pp. 174–181, 2005.
[6] C. Celsor and A. Millard-Ball, "Where does carsharing work?: Using geographic information systems to assess market potential," *Transp. Res. Rec., J. Transp. Res. Board*, no. 1992, pp. 61–69, 2007.
[7] J. Schure, F. Napolitan, and R. Hutchinson, "Cumulative impacts of carsharing and unbundled parking on vehicle ownership & mode choice," presented at the 91st Annual Meeting of the Transportation Research Board, Washington, DC, USA, 2012, Paper 12-1490.
[8] L. Sioui, C. Morency, and M. Trépanier, "How carsharing affects the travel behavior of households: A case study of Montréal, Canada," *Int. J. Sustainable Transp.*, vol. 7, no. 1, pp. 52–69, Jan. 2013.
[9] E. Martin and S. Shaheen, "Greenhouse gas emission impacts of carsharing in North America," *IEEE Trans. Intell. Transp. Syst.*, vol. 12, no. 4, pp. 1074–1086, Dec. 2011.
[10] J. Firnkorn and M. Müller, "What will be the environmental effects of new free-floating carsharing systems? The case of car2go in Ulm," *Ecol. Econ.*, vol. 70, no. 8, pp. 1519–1528, Jun. 2011.
[11] S. Shaheen, A. Cohen, and J. Roberts, "Carsharing in North America: Market growth, current developments, and future potential," *Transp. Res. Rec., J. Transp. Res. Board*, no. 1986, pp. 116–124, 2006.

[12] M. Barth and S. Shaheen, "Shared-use vehicle systems: Framework for classifying carsharing, station cars, and combined approaches," *Transp. Res. Rec., J. Transp. Res. Board*, no. 1791, pp. 105–112, 2002.

[13] C. Costain, C. Ardron, and K. Habib, "Synopsis of users' behaviour of a carsharing program: A case study in Toronto," in *Proc. 91st Annu. Meet. Transp. Res. Board*, Washington, DC, USA, 2012, pp. 421–434.

[14] D. Efthymiou, C. Antoniou, and P. Waddell, "Which factors affect the willingness to join vehicle sharing systems? Evidence from young Greek drivers," in *Proc. 91st Annu. Meet. Transp. Res. Board*, Washington, DC, USA, 2012, pp. 1–19.

[15] A. Kek, R. Cheu, and M. Chor, "Relocation simulation model for multiple-station shared-use vehicle systems," *Transp. Res. Rec., J. Transp. Res. Board*, no. 1986, pp. 81–88, 2006.

[16] A. Kek, R. Cheu, Q. Meng, and C. Fung, "A decision support system for vehicle relocation operations in carsharing systems," *Transp. Res. E, Logist. Transp. Rev.*, vol. 45, no. 1, pp. 149–158, Jan. 2009.

[17] R. Nair and E. Miller-Hooks, "Fleet management for vehicle sharing operations," *Transp. Sci.*, vol. 45, no. 4, pp. 105–112, Nov. 2011.

[18] M. Barth and M. Todd, "Simulation model performance analysis of a multiple station shared vehicle system," *Transp. Res. C, Emerging Technol.*, vol. 7, no. 4, pp. 237–259, Aug. 1999.

[19] M. Barth, J. Han, and M. Todd, "Performance evaluation of a multi-station shared vehicle system," in *Proc. IEEE Intell. Transp. Syst. Conf.*, Oakland, CA, USA, 2001, pp. 1218–1223.

[20] D. Jorge, G. Correia, and C. Barnhart, "Testing the validity of the MIP approach for locating carsharing stations in one-way systems," in *Proc. 15th Edition Euro Working Group Transp.*, Paris, France, 2012, pp. 138–148.

[21] W. Mitchell, C. Borroni-Bird, and L. Burns, "New mobility markets," in *Reinventing the Automobile: Personal Urban Mobility for the 21st Century*. Cambridge, MA, USA: MIT Press, 2010, pp. 130–155.

[22] A. Febbraro, N. Sacco, and M. Saeednia, "One-way carsharing: Solving the relocation problem," in *Proc. 91st Annu. Meet. Transp. Res. Board*, Washington, DC, USA, 2012, pp. 113–120.

[23] W. Fan, R. Machemehl, and N. Lownes, "Carsharing: Dynamic decision-making problem for vehicle allocation," *Transp. Res. Rec., J. Transp. Res. Board*, no. 2063, pp. 97–104, 2008.

[24] G. Correia and A. Antunes, "Optimization approach to depot location and trip selection in one-way carsharing systems," *Transp. Res. E, Logist. Transp. Rev.*, vol. 48, no. 1, pp. 233–247, Jan. 2012.

[25] H. Lau, "Network Flow," in *A Java Library of Graph Algorithms and Optimization*. Boca Raton, FL, USA: CRC Press, 2007, pp. 254–272.

[26] iNTERFILE, accessed Jun. 10, 2012. [Online]. Available: http://www.excel.interfile.de/Autokostenrechner/autokostenrechner.html

[27] car2go, accessed Jul. 2, 2012. [Online]. Available: http://excel.interfile.de/Autokostenrechner/autokostenrechner.html

[28] "Getting started with Xpress," Leamington Spa, U.K., 2008, Release 7.

[29] D. Jorge and G. Correia, "Carsharing systems demand estimation and defined operations: A literature review," *Eur. J. Transp. Infrastruct. Res.*, vol. 13, no. 3, pp. 201–220, Sep. 2013.

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