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A decision support system for vehicle relocation operations in carsharing systems

Alvina G.H. Kek [a,1], Ruey Long Cheu [b,*], Qiang Meng [c,2], Chau Ha Fung [c,3]

[a] Asia Pacific Product Scheduling, ExxonMobil Asia Pacific Pte Ltd., 18 Pioneer Road, Singapore 628498, Singapore
[b] Department of Civil Engineering, The University of Texas at El Paso, 500 West University Avenue, El Paso, TX 79968-0516, USA
[c] Department of Civil Engineering, National University of Singapore, 1 Engineering Drive 2, Singapore 117576, Singapore

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ABSTRACT

This paper presents a novel three-phase optimization-trend-simulation (OTS) decision support system for carsharing operators to determine a set of near-optimal manpower and operating parameters for the vehicle relocation problem. Tested on a set of commercially operational data from a carsharing company in Singapore, the simulation results suggest that the manpower and parameters recommended by the OTS system lead to a reduction in staff cost of 50%, a reduction in zero-vehicle-time ranging between 4.6% and 13.0%, a maintenance of the already low full-port-time and a reduction in number of relocations ranging between 37.1% and 41.1%.

1. Background and motivation

A carsharing system has a relatively small fleet of vehicles available at designation stations exclusively for use by a relatively larger group of members. Over the last decade, carsharing has emerged as an alternative to owning a vehicle. Most of this form of transportation has been taking place in the cities in Europe, North America, Japan and Singapore (Sheheen and Cohen, 2007), with more than 40 programs deployed in North America (Shaheen et al., 2006), 18 in Japan and 4 in Singapore (Barth et al., 2006). A range of market demand studies, conducted principally in Europe, has estimated a market potential of anything from 3% to 25% of the population (Millard-Ball et al., 2005), and in North America, the market potential in major metropolitan areas is estimated at 10% of individuals over the age of 21 (Shaheen et al., 2006).

In cities with high population densities, carsharing exhibits great promise in improving mobility, lowering emission level and reducing congestion problems (Britton and Associates, 1999). Simulation-based research carried out to investigate the viability of carsharing suggests that it has the potential to become economically profitable (Barth and Todd, 1999). This finding is later supported by the rapid growth of commercial carsharing companies in the US and Canada. Another mobility scenario conducted for the Sacramento region in California indicated a modest reduction in vehicle travel and emissions and a significant net economic benefit for home-based work trips (Shaheen and Rodier, 2005). To ensure that these positive effects

* Corresponding author. Tel.: +1 915 747 5717.
E-mail addresses: alvina.gh.kek@exxonmobil.com (A.G.H. Kek), rcheu@utep.edu (R.L. Cheu), cvemq@nus.edu.sg (Q. Meng), cvefch@nus.edu.sg (C.H. Fung).
[1] Tel.: +65 6660 6452.
[2] Tel.: +65 6516 5494.
[3] Tel.: +65 6516 5035.

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of carsharing remain sustainable, the number of carsharing customers must grow (Nobis, 2006). It hence becomes vital for there to be supportive regulatory legislation, financial support and easily adoptable operational tools for the continuous growth of the carsharing industry.

Conventional carsharing systems usually require users to pick up and return vehicles at the same parking lots (or stations). Stiff competition from public transportation systems and competing carsharing companies has prompted some operators which have multiple stations to provide users with flexibility in return stations. Taking it a step further, some operators provides users with flexibility in return times. A key issue that arises from such systems (having flexible return times and return stations) is the dynamically disproportionate distribution of vehicles across stations, with no pre-emptive knowledge. As a result, periodic relocation of vehicles between stations becomes necessary to ensure that there are sufficient vehicles that are spread geographically across the stations to serve user demands.

The IntelliShare research team at the University of California, Riverside proposed and experimented with two user-based relocation methods, namely trip-splitting and trip-joining, which successfully reduced the Number of Relocations (NR) required (Barth et al., 2004). These user-based relocation techniques cleverly shift the burden of relocating vehicles to the users through a price incentive mechanism. These techniques may not be viable in cities where commuters value privacy and convenience over minor cost savings in transportation. In pursuit of offering privacy, simplicity and convenience to commuters, the use of carsharing company's staff to relocate vehicles becomes necessary. This is termed operator-based relocation.

A clear need thus arises for a set of operational tools to support operator-based relocation, for carsharing systems with flexible return times and return stations. Related research in this area includes forecasting of vehicle trip demands by Cheu et al. (2006), and vehicle relocation simulation by Kek et al. (2006). The motivation behind these studies arises from a fundamental desire to enhance both operational efficiency and service level at the lowest possible cost. This paper is thus driven to focus on how to bring about an overall optimization of the operator-based vehicle relocation system. A three-phase OTS decision support system has been developed to find a near-optimal set of parameters for vehicle relocation operations. The OTS system has been tested and validated on real data provided by ICVS.

2. Multiple station carsharinig systems with fliexible return times and return stations

A carsharing system typically has a fleet of vehicles shared by a group of users. The vehicles may be picked up from and returned to designated stations. The stations are usually located at mass transit terminals, large commercial centers, and other major trip generators and attractors, making the vehicles readily accessible to the users. The operators of the carsharing systems are mostly commercial companies or non-profit cooperatives. They own the vehicles and rent the parking stalls from the property owners. Users who wish to have access to the shared vehicles must join the systems as members and pay subscriptions. Upon joining, they are then given membership identifications and passwords by the operators to enable them to have access to the vehicles with or without reservation. Once a vehicle is being picked up, the user will pay for the usage either by time, by distance or both. For the convenience of the users, the operators usually provide fuel, vehicle maintenance and insurance. The operators usually design the pricing structures which are meant for short-term use from a few hours to a few days. The carsharing operators rely on a combination of intelligent transportation systems technologies (such as a central computer server, field equipment at the stations and in-vehicle equipment with communication links) to monitor vehicle use and to implement the pricing structure. In essence, carsharing is a transportation mode that is between car rental and taxi. Carsharing is more accessible, convenient and of shorter usage duration than car rental, and yet offers more privacy compared to taxi service. More discussions on this mode of transportation can be found in Millard-Ball et al. (2005).

Carsharing operators impose various restrictions on vehicle return stations and times. In the conventional mode, a vehicle is always picked up from and returned to the same station. Users are also required to specify a return time and adhere to it, or face a penalty. This type of system is less costly for the operator to implement but inconvenient for the users. To offer more flexibility, an improved version, for an operator who has a network of stations in a city, allows users to return vehicles to another station, similar to one-way car rental. Examples of carshring systems that have adopted such a strategy include the ICVS in Singapore (Honda, 2008), Praxitele in France (Allouche et al., 1999) and IntelliShare in California (Barth and Todd, 2001a,b). These systems may face a problem of uneven distribution of vehicles at the different stations. Occasionally, stations which are popular pickup points may not have enough vehicles while stations which are popular return points may end up with too many vehicles, or not enough parking stalls. Periodically relocating vehicles between stations becomes necessary. To implement vehicle relocations, the operator sets lower and upper vehicle inventory thresholds, called relocation thresholds, at each station. When the number of vehicles at a station has exceeded its upper threshold value, the system will prompt the operator to move a vehicle to another station. On the other hand, when the number of vehicles at the station has fallen below the lower threshold value, the operator will be prompted to bring in a vehicle from another station. The user-based vehicle relocations (such as the one experimented by IntelliShare (Barth et al., 2004)) or operator-based vehicle relocations (such as the one used by ICVS) may be used. To offer even more convenience to the users, some carsharing operators (such as the ICVS) allow users even more flexibility in return times and stations. Users of this system can return vehicles whenever and wherever they like. This poses additional difficulties for the operators to maintain (1) vehicle inventories across the stations to meet user demands and (2) parking stall inventories across the stations for users to return vehicles to. Not meeting user demands means a loss in potential revenue for the operator and increased frustration for the customers, making carsharing a less attractive mode of transportation. Not having parking stalls for users to return vehicles to is effectively forcing them to rent the vehicles longer than necessary, thus increasing their usage costs.

3. Three-phase OTS decision support system

A novel three-phase OTS decision support system has been developed to assist carsharing operators with multiple stations that want to offer users flexibility in return times and return stations. The structure of this approach is presented in Fig. 1. The simulation component of the OTS system is based on the vehicle relocation simulator developed by Kek et al. (2006). In fact, the vehicle relocation simulator alone provides a means for the operators to evaluate the impact of different relocation techniques (i.e., origin-destination choices), manpower and operating parameters. However, using it to perform system optimization would mean an impractical number of simulation runs to enumerate all the possible permutations of the parameters. Therefore, an optimizer followed by a trend filter has been added as earlier phases to overcome this problem, leaving the simulator as an evaluation tool in this OTS decision support system. The concepts of the three phases are described below in this section.

Phase one of the OTS is an *Optimizer*, which receives inputs from the carsharing system on its stations' characteristics such as the number of parking stalls, inter-station vehicle relocation costs, staff costs, penalties for low level of service (LOS) for customers, typical or historical customer usage patterns and vehicle maintenance schedules. The low LOS at a station is reflected by the duration of vehicle shortage (i.e., zero-vehicle-time or ZVT) and duration of parking stall unavailability (i.e., full-port-time or FPT). The *Optimizer* then proceeds to determine the lowest-cost resource allocation, giving the optimized staff strength, staff activities, relocations and the resulting station status (number of vehicles at the stations at each time step). The availability of staff (in terms of number of drivers and their shift hours) who perform operator-based vehicle relocations and maintenance activities can be arbitrarily set in phase one and later revised through a series of heuristics in phase two. For example, one can start by assuming that all the staff is available for 24 h in a day. The shift hours and number of staff may be reduced later in phase 2 based on the relocation activities as the results of the optimization. The vehicle relocation thresholds, which are station specific, are not required at this phase as the *Optimizer* will move staff and vehicles as needed to achieve the minimum generalized cost. The relocation thresholds will be set in the next phase by examining the optimized relocation activities.

In phase two, the *Trend Filter* receives the optimized outputs from phase one and 'filters' them through a series of heuristics. A set of recommended operating parameters (staff strength and shift hours, relocation technique, relocation thresholds and whether priority should be given to maintenance jobs or relocations) is thus obtained.

Upon entering the set of operating parameters obtained in phase two into the vehicle relocation simulator developed in Kek et al. (2006), phase three of the OTS evaluates the effectiveness of this set of recommended parameters using three performance indicators, namely ZVT, FPT and NR. When ZVT occurs at a station, the station is without an available vehicle and customers requests for vehicles at that station (either by making advance reservations or walk-ins) will be rejected. Conversely, when FPT occurs, the station has no empty parking stall and users wanting to return vehicles to that station will not be able to do so. Both ZVT and FPT reduce the attractiveness of the carsharing system to users. From the operator's point of view, ZVT implies a possible loss in revenue. From the user's point of view, ZVT forces him/her to use other stations, or turn to other modes of travel, while FPT forces him/her to return the vehicle later or to another station, incurring additional usage cost. A greater value of NR means a higher cost of vehicle relocation operations. Thus, for the optimal performance of the system, the values for ZVT, FPT and NR should all be zero. The development of the *Optimizer* and *Trend Filter* are described

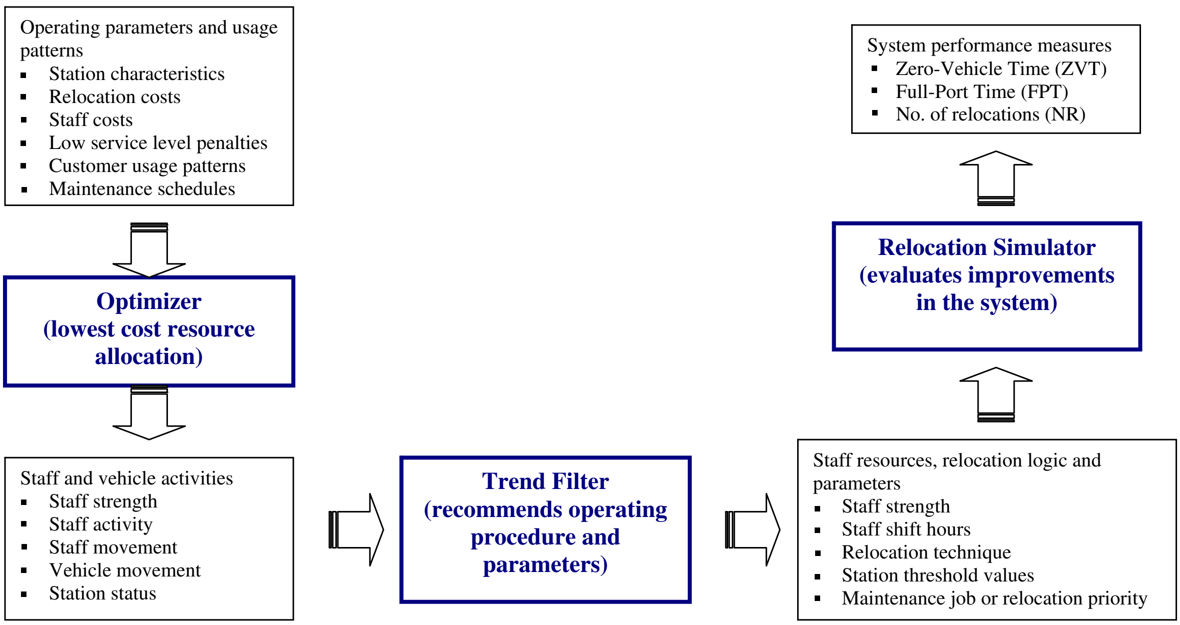

Fig. 1. Structure of the three-phase OTS decision support system.

in greater detail over the next two sections in this paper. Readers can refer to (Kek et al. (2006)) for more details of the vehicle relocation simulator which is used in phase three of the OTS. Nevertheless a brief description of the simulator is provided in a subsequent section of this paper.

4. Optimizer

4.1. Problem definition

Given a set of geographically scattered stations, with each station having a capacity (number of parking stalls) and customer pick-up and return patterns, plus the maintenance schedule for the vehicle fleet, the objective of the optimization problem is to allocate staff resource and staff activities so as to minimize the generalized cost associated with the vehicle relocation activities. The generalized cost here includes staff cost, vehicle relocation cost and penalty cost for low LOS. The staff resource refers to the number of staff and their shift hours. The staff activities refer to their job assignments including driving (relocating) vehicles between stations, bringing vehicles for maintenance and so on. This problem may be viewed as somewhat similar to the typical pickup and delivery problem, in which staff are assigned to routes that traverse between different stations while engaged in various activities, e.g., picking up and dropping off vehicles. The problem definition is described below:

(i) A staff-route can start at any one station.
(ii) At any time, each staff is engaged in exactly one of the four types of activities. The four types of activities are namely *waiting* (wait at a station for the next activity), *maintenance* (inspect or clean vehicles at a station, drive a vehicle to a gasoline station for refueling, or drive a vehicle to a workshop for maintenance), *movement* (travel between two stations without driving a vehicle) or *relocation* (drive a vehicle from one station to another).
(iii) A staff route can end at any station or in the midst of a maintenance, movement or relocation activity.
(iv) At every time step, there are three non-negative integer numbers to be monitored at each station. They are the number of available vehicles, the number of unavailable vehicles, and the number of empty parking stalls. The available vehicles refer to those ready for customers to pick up without advance reservations. The unavailable vehicles refer to those already reserved by customers or are being set aside for maintenance. The sum of available vehicles, unavailable vehicles and number of empty parking stalls must equal the station capacity.
(v) The number of available vehicles at each station varies with each time step. It is affected by vehicles relocated into and out of the station, vehicles returned to the station after maintenance, vehicles set aside or taken out for maintenance and vehicles picked up and returned by customers.
(vi) The number of unavailable vehicles at each station varies with each time step. It depends on the number of vehicles set aside or taken out for maintenance and vehicles reserved by customers.
(vii) The movement of a vehicle into and out of a station for maintenance or relocation is accompanied by a movement of staff (the driver) into and out of the same station, engaged in the maintenance or relocation activity, respectively.

4.2. Network representation

The problem is best visualized by constructing a two-dimensional time-space network as shown in Fig. 2, with the *x*-axis representing time and the *y*-axis representing space.

The time-space network has $S \times T$ nodes arranged in a rectangular grid, where S is the number of stations in the carsharing system, and T is the number of discrete time steps within the period in which the vehicle relocation operations is to be optimized. The nodes are positioned along the *x*-axis at constant time intervals (e.g., 15 minutes) and along the *y*-axis at fixed positions. Nodes in each row are for a particular station ($i = 1, 2, \ldots, S$). Nodes in each column represent all the stations at

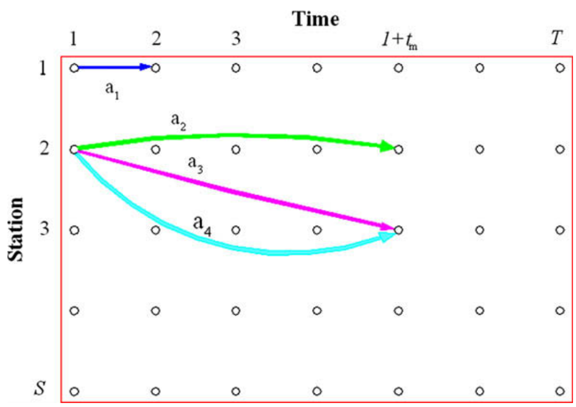

Fig. 2. Time–space network.

a time step ($t = 1, 2\ldots, T$). Note that t denotes the start of each time step. Let $\boldsymbol{N} = \{1\ldots, i\ldots, S\}$ be the set of stations. For each $i \in \boldsymbol{N}$, create T nodes representing the station i at time steps $t = 1, 2\ldots, T$, and use i_t to represent node i at time step t. The above node creation process is repeated for $i = 1, 2\ldots, S$. Denote all the $S \times T$ nodes by a row vector $\boldsymbol{V} = \{1_1, \ldots, i_{t-1}, i_t, i_{t+1}\ldots, S_T\}$.

Next, define arcs between the nodes in the time-space network to represent staff activities (waiting, maintenance, movement and relocation). There are four sets of arcs. Each set of arcs represents one of the four types of activities. For each node $i_t \in \boldsymbol{V}$, create an arc that represents a waiting activity at the same station i from time step t to time step $t + 1$. Denote this set of waiting arcs as $\boldsymbol{A}_1 = ,\ldots, a_1(i_t, i_{t+1})\ldots,\}$. For each node $i_t \in \boldsymbol{V}$, create an arc that represents a maintenance activity at station i from time step t to time step $t + t_m$, where t_m is the fixed number of time steps required for all regular maintenance activities. Denote this set of *maintenance* arcs as $\boldsymbol{A}_2 = \{\ldots, a_2(i_t, i_{t+t_m}), \ldots\}$. During maintenance, a vehicle may be taken out of station i by the staff and returned to the same station t_m time steps later. For each node $i_t \in \boldsymbol{V}$, create a set of $S - 1$ arcs that represent movements of staff from stations i to j, $\forall j \in \boldsymbol{N} j \neq$ i, from time step t to time step $t + t_{ij}$, where t_{ij} is the travel time (in number of time steps) from stations i to j. Denote this set of movement arcs as $\boldsymbol{A}_3 = \{\ldots, a_3(i_t, i_{t+t_{ij}}), \ldots\}$. For each node $i_t \in \boldsymbol{V}$, create a set of $S - 1$ arcs that represent vehicle relocation activities from stations i to j, $\forall j \in \boldsymbol{N}$, $i \neq j$, from time steps t to $t + t_{ij}$, where t_{ij} is the driving time from stations i to j, $\forall i, j \in \boldsymbol{N}$, $i \neq j$. Denote this set of relocation arcs as $\boldsymbol{A}_4 = \{\ldots, a_4(i_t, i_{t+t_{ij}}), \ldots\}$. Note that, the driving time to relocate a vehicle and the movement time (without driving a vehicle) between stations i and j are assumed to take the same number of time steps equal to t_{ij}.

A sample of each set of arcs can be seen in Fig. 2. Arc a_1 represents a waiting activity at station 1 from time steps 1 to 2; arc a_2 represents a maintenance activity at station 2 from time steps 1 to $1 + t_m$; arc a_3 represents a movement activity from stations 2 to 3 from time steps 1 to $1 + t_{23}$ (t_{23} is the travel time from stations 2 to 3); arc a_4 represents a relocation activity from stations 2 to 3 from time steps 1 to $1 + t_{23}$.

Finally, define a set of staff available to carry out these activities, $\boldsymbol{L} = \{1, \ldots, k, \ldots, W\}$. Here, W is the maximum number of staff available.

4.3. Mixed integer programming formulation

The mixed integer programming formulation for this problem involves seven types of decision variables:

x^k: Binary variable for staff usage, taking the value 1 if staff k is ever used from $t = 1, \ldots, T$, and 0 otherwise, $\forall k \in \boldsymbol{L}$.
$y^k_{i_t i_{t+1}}$: Binary variable associated with $\boldsymbol{A}_1$, taking the value 1 if staff k waits at station i from time steps t to $t + 1$, and 0 otherwise, $\forall (i_t, i_{t+1}) \in \boldsymbol{A}_1, k \in \boldsymbol{L}$.
$z^k_{i_t i_{t+t_m}}$: Binary variable associated with $\boldsymbol{A}_2$, taking the value 1 if staff k maintains a vehicle at station i from time steps t to $t + t_m$, or drive a vehicle out of station i at time step t for maintenance and return the vehicle to the same station at time step $t + t_m$, and 0 otherwise, $\forall (i_t, i_{t+t_m}) \in \boldsymbol{A}_2, k \in \boldsymbol{L}$.
$u^k_{i_t j_{t+t_{ij}}}$: Binary variable associated with $\boldsymbol{A}_3$, taking the value 1 if staff k moves from station i at time step t to station j at time step $t + t_{ij}$, and 0 otherwise, $\forall (i_t, j_{t+t_{ij}}) \in \boldsymbol{A}_3, k \in \boldsymbol{L}$.
$v^k_{i_t j_{t+t_{ij}}}$: Binary variable associated with $\boldsymbol{A}_4$, taking the value 1 if staff k relocates a vehicle from station i at time step t to station j at time step $t + t_{ij}$, and 0 otherwise, $\forall (i_t, j_{t+t_{ij}}) \in \boldsymbol{A}_4, k \in \boldsymbol{L}$.
$d^r_{i_t}$: Integer variable representing the number of rejected customer demand for vehicles at station i from time steps $t - 1$ to t, $\forall i_t \in \boldsymbol{V}$.
$s^r_{i_t}$: Integer variable, representing rejected customer return of vehicles at station i from time steps $t - 1$ to t, $\forall i_t \in \boldsymbol{V}$.

The four binary variables, $y^k_{i_t i_{t+1}}, z^k_{i_t i_{t+t_m}}, u^k_{i_t j_{t+t_{ij}}}$ and $v^k_{i_t j_{t+t_{ij}}}$ are associated with the sets of arcs, $\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{A}_3$ and $\boldsymbol{A}_4$ respectively. In addition, there is a k variable (that denotes staff k) associated with each arc. This means that there can be a maximum of W staff activities being carried out on any arc.

The known constants are:

c_{ij}: Fixed cost of a movement or relocation trip from stations i to j, $\forall i, j \in \boldsymbol{N}$, $i \neq j$.
c_x: Fixed cost of utilizing one staff.
c_d: Fixed cost of rejecting the demand of one customer-vehicle trip.
c_s: Fixed cost of rejecting the return of one vehicle by a customer.
r_{i_0}: Number of available vehicles at station i at time step $t = 0$, $\forall i \in \boldsymbol{N}$.
$\bar{r}_{i_0}$: Number of unavailable vehicles at station i at time step $t = 0$, $\forall i \in \boldsymbol{N}$.
d_{i_t}: Demand for vehicles at station i from time steps $t - 1$ to t, $\forall i_t \in \boldsymbol{V}$, from historical data.
s_{i_t}: Number of vehicles returned by customers at station i from time steps $t - 1$ to t, $\forall i_t \in \boldsymbol{V}$, from historical data.
m_{i_t}: Number of returned vehicles in need of maintenance at station i from time steps $t - 1$ to t, where $m_{i_t} \leqslant s_{i_t}$, $\forall i_t \in \boldsymbol{V}$.
p_i: Number of parking stalls at station i, $\forall i \in \boldsymbol{N}$.

Two additional dependent variables are:

r_{i_t}: Number of available vehicles at station i at time step t, $\forall i_t \in \boldsymbol{V}$.
$\bar{r}_{i_t}$: Number of unavailable vehicles at station i at time step t, $\forall i_t \in \boldsymbol{V}$.

The mixed integer linear programming formulation for the problem is:

$$\text{Min} \quad Z = c_{ij}\left(\sum_{(i_t,j_{t+t_{ij}})\in A_3}\sum_{k\in L} u^k_{i_t j_{t+t_{ij}}} + \sum_{(i_t,j_{t+t_{ij}})\in A_4}\sum_{k\in L} v^k_{i_t j_{t+t_{ij}}}\right) + c_x\sum_{k\in L} x^k + c_d\sum_{i_t\in V} d^r_{i_t} + c_s\sum_{i_t\in V} s^r_{i_t} \tag{1}$$

subject to

$$\sum_{i\in N} y^k_{i_1 i_2} + \sum_{i\in N} z^k_{i_1 i_{1+t_m}} + \sum_{\substack{i,j\in N\\ i\neq j}} u^k_{i_1 j_{1+t_{ij}}} + \sum_{\substack{i,j\in N\\ i\neq j}} v^k_{i_1 j_{1+t_{ij}}} = x^k \quad \forall k\in L \tag{2}$$

$$y^k_{i_{t-1} i_t} + z^k_{i_{t-t_m} i_t\,(i_{t-t_m},i_t)\in A_2} + \sum_{(j_{t-t_{ji}},i_t)\in A_3} u^k_{j_{t-t_{ji}} i_t} + \sum_{(j_{t-t_{ji}},i_t)\in A_4} v^k_{j_{t-t_{ji}} i_t} \tag{3}$$

$$- y^k_{i_t i_{t+1}} - z^k_{i_t i_{t+t_m}} - \sum_{(i_t,j_{t+t_{ij}})\in A_3} u^k_{i_t j_{t+t_{ij}}} - \sum_{(i_t,j_{t+t_{ij}})\in A_4} v^k_{i_t j_{t+t_{ij}}} = 0 \quad \forall i_t\in V, k\in L, t>1 \tag{4}$$

$$r_{i_t} = r_{i_{t-1}} + \sum_{(j_{t-t_{ji}},i_t)\in A_4}\sum_{k\in L} v^k_{j_{t-t_{ji}} i_t} - \sum_{(i_t,j_{t+t_{ij}})\in A_4}\sum_{k\in L} v^k_{i_t j_{t+t_{ij}}} \tag{5}$$

$$+ \sum_{\substack{(i_{t-t_m},i_t)\in A_2\\ k\in L}} z^k_{i_{t-t_m} i_t} + (s_{i_t} - s^r_{i_t}) - (d_{i_t} - d^r_{i_t}) - m_{i_t} \quad \forall i_t\in V \tag{6}$$

$$\bar{r}_{i_t} = \bar{r}_{i_{t-1}} - \sum_{k\in L} z^k_{i_t i_{t+t_m}} + m_{i_t} \forall i_t\in V \tag{7}$$

$$r_{i_t} + \bar{r}_{i_t} \leqslant p_i \quad \forall i_t\in V \tag{8}$$

$$d^r_{i_t} \leqslant d_{i_t} \quad \forall i_t\in V \tag{9}$$

$$s^r_{i_t} \leqslant s_{i_t} \quad \forall i_t\in V \tag{10}$$

$$x^k = (0,1) \quad \forall k\in L \tag{11}$$

$$y^k_{i_t i_{t+1}} = (0,1) \quad \forall (i_t, i_{t+1})\in A_1, k\in L \tag{12}$$

$$z^k_{i_t i_{t+t_w}} = (0,1) \quad \forall (i_t, i_{t+t_m})\in A_2, k\in L \tag{13}$$

$$u^k_{i_t i_{t+t_{ij}}} = (0,1) \quad \forall (i_t, i_{t+t_{ij}})\in A_3, k\in L \tag{14}$$

$$v^k_{i_t i_{t+t_{ij}}} = (0,1) \quad \forall (i_t, i_{t+t_{ij}})\in A_4, k\in L \tag{15}$$

$$d^r_{i_t} \geqslant 0 \quad \forall i_t\in V \tag{16}$$

$$s^r_{i_t} \geqslant 0 \quad \forall i_t\in V \tag{17}$$

$$r_{i_t} \geqslant 0 \quad \forall i_t\in V \tag{18}$$

$$\bar{r}_{i_t} \geqslant 0 \quad \forall i_t\in V \tag{19}$$

The objective function (1) minimizes the total generalized cost, taking into consideration movement and relocation costs, staff cost and penalty costs of rejecting the demand for or return of vehicles from customers. Note that it is possible to expand the objective function if there is a difference in the costs for the day and night operations. In the objective function, the staff movement cost and vehicle relocation cost are the same (denoted by c_{ij}). It is possible to have different sets of costs for staff movement and vehicle relocation trips. Constraint (2) serves the dual purpose of assigning a non-zero value to x^k when staff k is used from time step $t = 1$ and restricting staff k to only performing one type of activity at $t = 1$. Constraint (4) ensures the conservation of a staff's activities at each node at i_t. It restricts each staff to begin exactly one new activity after the previous one is completed. Constraints (6) and (7) update the number of available and unavailable vehicles respectively. The number of available vehicles at a station is adjusted by the vehicles relocated into and out of the station, vehicles returned to the station after maintenance, vehicles completed maintenance at the station, vehicles returned and picked up by customers. The number of unavailable vehicles at a station is adjusted by vehicles taken out for maintenance from the station and vehicles requiring maintenance at the station. Constraint (8) ensures that the sum of available and unavailable vehicles does not exceed the station's capacity at any time step. Constraint (9) ensures that the rejected demands for vehicles does not exceed the requested demands for vehicles at a station at time step t. Constraint (10) ensures that the number of rejected returns of vehicles does not exceed the number of requested returns of vehicles. Constraints (11)–(15) impose binary conditions on the variables $x^k, y^k_{i_t i_{t+1}}, z^k_{i_t i_{t+t_m}}, u^k_{i_t j_{t+t_{ij}}}$ and $v^k_{i_t j_{t+t_{ij}}}$ respectively. Constraints (16)–(19) impose non-negativity conditions on the variables $d^r_{i_t}, s^r_{i_t}, r_{i_t}$ and $\bar{r}_{i_t}$ respectively.

This optimization problem is thus run from time steps $t = 1,\ldots, T$. As mentioned earlier, the staff strength W and length of staff shift hours can be arbitrarily set in this phase and later revised by the *Trend Filter* in phase two. A good estimate of the upper bound of W is to scan the station status in the raw data to see, at the maximum, how many stations have vehicles that need to be simultaneously relocated. No relocation threshold is assumed at this stage. These thresholds will be deduced in phase two based on the optimal solutions obtained in phase one. The *Optimizer* developed here is a mixed integer linear programming model and is commonly solved using the branch-and-bound technique (Winston, 2004).

5. Trend filter

This *Trend Filter* 'filters' the optimized results obtained from phase one through a series of heuristics to produce a recommended set of operating parameters. The key information extracted from the filter includes staff strength and shift hours, relocation techniques, station threshold values and whether priority should be given to maintenance jobs or relocation trips.

5.1. Selection of staff strength and shift hours

A set of recommend staff strength and shift hours may be derived by observing the optimized $x^k, z^k_{i_t i_{t+t_m}}, u^k_{i_t j_{t+t_{ij}}}$ and $v^k_{i_t j_{t+t_{ij}}}$ values over time steps. A summation of all the x^k values provides an initial estimate of the recommended staff strength. This value is then adjusted depending on the $z^k_{i_t i_{t+t_m}}, u^k_{i_t j_{t+t_{ij}}}$ and $v^k_{i_t j_{t+t_{ij}}}$ values, which depict staff work load.

A sample activity graph for one staff on a typical day is shown below in Fig. 3. A non-zero variable value in any of the *z*, *u* or *v* variables (with a fixed *k* value) at time step *t* indicates that the staff is engaged in carrying out a maintenance, movement or relocation activity respectively. In this figure the shift hours was arbitrarily set at eight hours per shift in phase one, i.e., three runs per day on the *Optimizer* (0000 h to 0800 h, 0800 hrs to 1600 h and 1600 h to 0000 h). The initial estimate of the staff strength, based on the optimized x^k values was no staff from 0000 h to 0800 h and one staff from 0800 h to 0000 h. It can be observed from Fig. 3 that the staff is essentially only active from 1200 h to 2200 h. A more efficient choice of shift hours is to have no staff from 0000 h to 1200 h, one staff from 1200 h to 2200 h and no staff from 2200 h to 0000 h.

5.2. Selection of relocation technique

Two techniques of relocating vehicles were proposed in (Kek et al., 2006), namely *shortest tim*e and *inventory balancing*. Relocating vehicles by the *shortest time* technique means moving vehicles to or from a neighboring station in the shortest possible time (including staff movement time, if necessary). Relocating vehicles by the *inventory balancing* technique means filling a station which has a shortage of vehicles with a vehicle from another station which has an oversupply of vehicles. The recommended relocation techniques for use during different time periods of a day may be derived by observing the optimized $v^k_{i_t j_{t+t_{ij}}}$ values from phase one. A non-zero value in $v^k_{i_t j_{t+t_{ij}}}$ implies a relocation from stations *i* to *j*. This trip may be classified as a *shortest time* relocation or an *inventory balancing* relocation as follows. When a vehicle is relocated from stations *i* to *j*, it is either because station *i* is experiencing an oversupply, station *j* is experiencing a shortage or both. It is thus defined that if a relocation from station *i* to *j* is followed by a reduction in available vehicles in station *j* (in the next time step, due to customer pickup or sending for maintenance) and at the same time an increase in available vehicles in station *i* (in the next time step, due to customer return or completion of maintenance), the relocation is considered *inventory balancing*. Otherwise, the relocation is considered *shortest time* since the *Optimizer* is cost-minimizing, and time is reflected in the various cost components in the objective function. Through an observation of the classified relocation techniques across the time steps, consistent relocation techniques for use during different time periods of the day are thus recommended.

5.3. Selection of relocation thresholds

There are a total of four relocation thresholds for each station: two critical thresholds and two buffer thresholds (Kek et al., 2006). When the number of available vehicles in the station goes above the high critical threshold or below the low critical threshold, a relocation request is generated from and to the station respectively. When a relocation is required at one station, the supporting station will only allow a vehicle to be taken out or brought in if the number of vehicles in the

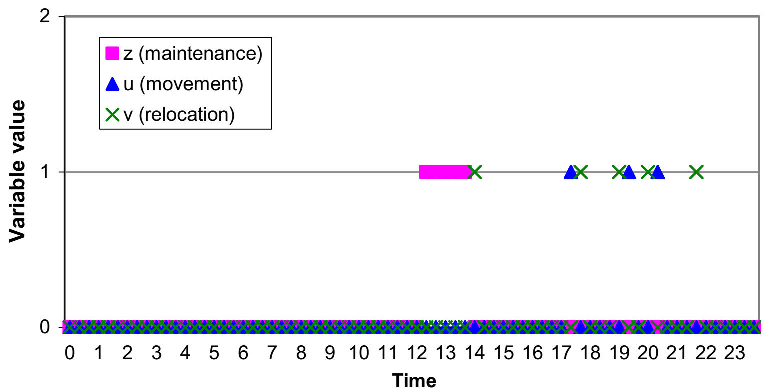

Fig. 3. Activity graph for one staff on a typical day.

supporting station is at and above the low buffer threshold or at and below the high buffer threshold, respectively. The buffer threshold values are thus naturally bounded by the critical threshold values, i.e., the high critical threshold value is greater than the high buffer threshold value while the low critical threshold value is smaller than the low buffer threshold value. Choosing more extreme critical threshold values (which gives a larger range between the upper and lower limits) and less extreme (a smaller range, and hence more conservative) buffer threshold values would trigger fewer relocation requests and would allow fewer relocations to be carried out, thus reducing relocation cost but possibly compromising the LOS. Conversely, more conservative critical thresholds (a smaller range) and extreme buffer thresholds (a larger range) would trigger more relocation requests and would allow more relocations to be carried out, thus maintaining a higher LOS but at the expense of increased relocation cost.

A recommended set of relocation thresholds may be derived by observing the optimized r_{i_t} values (number of available vehicles at station i at time step t) together with the relocation technique identified. Where relocations are identified as *shortest time*, the low buffer threshold of the supporting station is taken to be the minimum r_{i_t} value of the supporting station from which the vehicle is removed. Similarly, the high buffer threshold of the supporting station is taken to be the maximum r_{i_t} value of the supporting station to which the vehicle is inserted. For critical thresholds however, there is a need to look at all the relocation trips that belong to the *inventory balancing* and *shortest time* relocation techniques. The high critical threshold of the requesting station is taken to be the minimum r_{i_t} value of the requesting station from which the vehicle is removed. Similarly, the low critical threshold of the requesting station is taken to be the maximum r_{i_t} value of the requesting station to which the vehicle is inserted. Given the advantage of perfect knowledge in the *Optimizer*, a conservative allowance of one and two vehicles may be recommended for the buffer and critical thresholds, respectively. A set of four threshold values for each station is thus derived.

5.4. Selection of job priority

This refers to the decision to give priority to either maintenance jobs or relocation trips when both are required. Depending on the cost structure of the carsharing operation, it may be more cost-effective to give priority to either maintenance jobs or relocation trips at different time periods of a day. Once again, a recommended priority may be deduced by observing the optimized $v^k_{i_t j_{t+t_{ij}}}$ and r_{i_t} values. A non-zero value of $v^k_{i_t j_{t+t_{ij}}}$ coupled with a positive value of r_{i_t} implies that priority is being given to relocation trips over maintenance jobs. That is, relocations are still being carried out although there are available vehicles at a station. Otherwise, the default priority is given to maintenance jobs. Through observations of this priority across time steps, a recommended set of priorities for different time periods of the day is thus derived.

6. Relocation simulator

A vehicle relocation simulation model has been developed to mimic the logic used by a typical multiple station carsharing system with fliexible return times and stations. The simulation model's logic and operations have been reported in Kek et al. (2006). The developed model is based on time-stepping simulation, whose inputs are the operational set-up of the stations (station capacities, critical and buffer thresholds), travel time between stations, historical activities (time stamps and stations) of user pick-up, user return, and maintenance of vehicles. At simulation time step of very minute, the simulation logic updates the inventory level at each station based on the vehicle activities and checks the inventory level against the station's buffer and critical thresholds. Based on available staff strength, staff shift hours and relocation techniques used, the simulation logic then triggers off and simulates the relocations. Key outputs (ZVT, FPT, NR) which describe the system performance are tabulated and analyzed for the simulated duration. The simulation model had been validated with three months of data collected from ICVS. Using the same input data, the simulation model produced the performance indicators that closely matched the ones recorded by the actual system Kel et al. (2006).

7. Computational runs

The new three-phase OTS decision support system was tested using one plus three months of commercially operational data provided by ICVS to evaluate its effectiveness. The purpose of this test was to see how much the OTS decision support system can help to improve the ICVS's relocation operations form the actual operations by reducing the vehicle relocation cost and/or improving the LOS. One month of data (prior to the three months) with 1236 customer-trips was first passed through the *Optimizer*. The data was presented to the optimizer in eight-hour blocks. That is, the data was divided into a series of eight-hour segments, and independent optimization runs were made for the different segments. The mixed integer programming problem with one segment of data ($T = 32$ 15-minute intervals per eight hours) had 8210 variables. In each segment, the staff strength W was assumed to be the same number employed by ICVS, and each staff's shift hour was artificially set to eight hours. As mentioned, the staff strength and shift hours were later reduced/modified as recommended by the *Trend Filter*. The Trend Filter may recommend a shift hour for a staff which may span across two eight-hour segments. The branch and bound algorithm was applied with a node selection strategy to branch on the best bound, i.e., branching was always done on the pending node giving the smallest value to the objective function. The model was coded into ILOG OPL Studio, Version 3.7.1 and solved using the ILOG CPLEX 9.1 Mixed Integer Programming module (ILOG, 2005).

The cost coefficients used were estimated from publicly available data, with c_x being valued at S$47 per shift (eight hours multiplied by the minimum average wage rate), c_s and c_d equally valued (based on potential commercial losses, calculated from the published fare structure) at S$271.58 on weekdays (Mondays to Fridays, 0800 h to 1900 h), S$397.70 on weekends (Saturdays, Sundays and public holidays, 1900 h to 0800 h the following day) and S$833.30 for overnight (1900 h to 0800 h) and an estimated c_{ij} that ranged between S$1.50 to $9.00.

The optimized results were then passed through the *Trend Filter* to extract operating parameters (staff strength, staff shift hours, relocation techniques, station threshold values and whether priority should be give to maintenance jobs or relocation trips). The optimized results for weekdays and weekends were examined separately, so that different operating parameters were recommended for weekdays and weekends. The filtered results suggested a 50% reduction in staff strength with minor adjustments to shift hours during weekdays and weekends, use of inventory balancing vehicle relocation technique throughout, less conservative station threshold values and priority to be given to maintenance jobs over relocation trips.

This set of parameters was then entered into the simulator and evaluated with three months of test data. This data set was selected for its maximum range of vehicle to trip-station ratio, thus enabling a better assessment of the potential benefits from across a wider range of system setup, when new vehicles and/or stations are added to the system. When there are x vehicles, y customer trips a day and z stations, this ratio is calculated as $x/(yz)$. A low ratio implies a high intensity of vehicle usage and/or a wide spread of vehicles across the stations. It is important to note that the value of y simply indicates the trip frequency (which reflects the intensity of vehicle pick-ups and returns) and not vehicle utilization (vehicle-hour used or vehicle-km traveled). The three performance indicators, namely ZVT, FPT and NR were used to gauge the system performance. Due to the confidentiality of the operational data from ICVS, all ZVT and FPT values presented in this paper have been scaled by the factor m to dimensionless values. Similarly, NR has also been similarly scaled by a factor p.

Besides being influenced by the station set-ups and operating parameters, the system performance indicators are also primarily influenced by the vehicle to trip-station ratio. The results generated from the simulation are plotted against their vehicle to trip-station ratios. A comparison of the performance indicators when the system was operating with the OTS-generated parameters and with the current parameters used by ICVS (referred to as the base model) is shown in Fig. 4. The percentage improvements in the performance indicators are presented in Table 1.

It can be observed from Fig. 4 that all the three performance indicators are either maintained or reduced. Because all the FPT, ZVT values have been scaled by a factor m, and all the NR values by a factor p, the improvements from the base model to the one generated by the OTS decision support system are best expressed in percentages. As mentioned earlier, lower ZVT and FPT values correspond to a better LOS for customers, while a lower NR value means a reduction in relocation cost. Fig. 4 shows that the ZVT levels were reduced for the various vehicles to trip-station ratios. From Table 1, it can be seen that ZVT levels show consistent improvements ranging between 4.6% and 13.0% from the base model. It is also obvious from Fig. 4 that NR levels have been significantly reduced by up to 41.1%. The magnitude of the improvements in the FPT is relatively smaller compared to that of the ZVT, as suggested in Fig. 4. This is because the FPT in the base model already have small values, due to the conservative setting of the upper critical and buffer threshold values in the base model to favor zero or low FPT. Any slight reduction in the FPT is therefore a significant percentage improvement. The computed percentage improvements in FPT listed in Table 1 range between 0% to 71.3%.

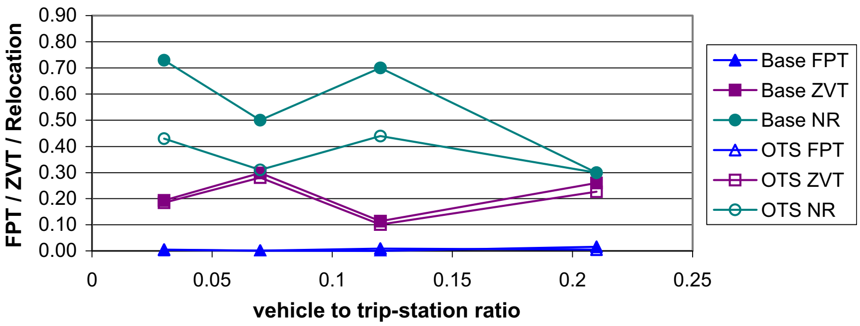

Fig. 4. Comparison of OTS against the base model.

Table 1
Percentage improvement in performance indicators of OTS from base model

Vehicle to trip-station ratio	Improvement (%)		
	ZVT	FPT	NR
0.03	4.6	0.0	41.1
0.07	6.0	8.1	38.0
0.12	11.2	0.0	37.1
0.21	13.0	71.3	0.0

The above performance surpasses the results of the previous simulations conducted by Kek et al. (2006) which uses iterative methods to select the parameters for the vehicle relocation operations. Although the iterative approach is an improvement from the base case, it is not easy to find the optimal or near-optimal combination of the parameters. The previous simulations generated a cost savings of up to 12.8%, resulting in a trade-off relationship between the indicators from the base case (see Fig. 4 in (Kek et al., 2006), where a reduction in ZVT is compromised by an increase in NR, and vice versa, using a different data set from the ICVS). This new three-phase OTS decision support system enables all the three performance indicators to be consistently maintained or improved, coupled with a 50% reduction in staff cost.

8. Summary

This paper studies for the first time the interesting and practical problem of finding a set of near-optimal parameters for vehicle relocation operations in a multiple-station carsharing system with flexible return times and return stations. A three-phase decision support system is proposed and developed to solve this problem. The contributions of this research are (1) the formulation of the vehicle relocation problem as a mixed integer programming problem in phase one; and (2) the introduction of heuristics that convert the optimized outputs to more practical and hence near-optimal operating parameters in phase two. Simulation tests, based on a set of commercially operational data, have produced statistics that indicate a better system performance than the existing system. The three-phase decision support system recommends a set of parameters for vehicle relocation operations, enabling a reduction in staff cost of 50%, an improvement of ZVT between 4.6% and 13.0%, a maintenance of the low FPT level and a reduction in NR between 37.1% and 41.1%. Other intangible benefits such as increased operational efficiency and potential increase in profit are also present. This decision support system provides a means for carsharing operators (with fliexible return times and return stations) to enhance both operational efficiency and customer service levels at a lower cost.

It is important to note the versatility of this decision support system and its adaptability to a wide variety of multiple-station carsharing systems with fliexible return timess and return stations. The operators can easily apply this model to their unique systems to identify a recommended set of operating parameters, effectively removing excess staff resources in their system and bringing about enhanced service levels and increased operational efficiency. Another potentially useful application of this model is when the operator has opened a new station. After operating the new station for a month, data from all the stations may be entered and processed through the decision support system for a recommended set of operating parameters.

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