The Stowage Stack Minimization Problem with K-Rehandle Constraint

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Abstract

The container ship stowage planning problem (CSSP) has been studied all the time and many approaches have been applied to find out optimal or near-optimal solutions to this problem. In addition, many variant problems have been proposed and researched from different optimization aspects since there are many constraints in the course of actual shipping. The stowage stack minimization problem (SSMP) investigates a stowage planning problem when carriers have the obligation to ship all the given containers in different ports, with the objective to utilize the fewest number of stacks on the ship. The related research about SSMP can been seen in Avriel et al. (2000), Jensen (2010), and Wang et al. (2014), and they focused on the condition that there is no rehandle or shift, i.e., the problem they studied is the stowage stack minimization problem with zero rehandle (SSMP-ZR).

In this paper, we first propose the stowage stack minimization problem with K rehanlds (SSMP-KR) based on the stowage stack minimization problem with zero rehandle and give the detail description about it. Like most of the literature, we highlight the constraints we study while put less emphasis on other constraints.

Next, heuristic algorithms are put forward to construct solutions to the SSMP-KR. It is worth mentioning that, our algorithms are suitable for the circular route. We use Java compilation software Ecplise to execute the code for our algorithms. For the loading strategies, two different processing methods are used to cope with two different situations whether the predetermined K meets or not, respectively. When the allowed rehandles run out, the problem becomes simpler because there is no rehandling operations that we need to take into consideration. In other words, we are just dealing with the SSMP-ZR problem under this circumstance. When there are rehandling operations allowed, we have to make decisions about how to place each single container loaded in each port to make the full use of each stack and minimize the total stacks used during the shipping. On this occasion, we need to consider the following factors: what the current port is, which port is the loading container transported to, what are the current heights of all stacks, etc. Since there are many factors to be taken into consideration, we have to go on the processing of parameters tuning and we eventually get the optimal parameters combination for this difficult situation through multiple parameter adjustment procedures. As for the unloading strategies, there are still two conditions and it depends on whether the unloading container is a blocking one or not. If it is not a blocking container, we just discharge it; If it is a blocking one, we have

to unload it, reset the origin port of the unloading container and load it again. Our output for each instance using this algorithm in Ecplise includes the number of used stacks, the lower and upper bound number of used stacks, the number of used rehandles and the allowed rehandles, and the layout of ship after one loading or unloading operation.

Then, we have discussed and proved the theoretical performance guarantee of the algorithm using the math knowledge of inequation and mathematical induction. We find out the lower bound and upper bound to make sure the solutions we get are reasonable and effective, which proves the feasibility of the algorithm theoretically. In actual, we use a tip in the process of proof, we divide the stacks used into two parts: the full stacks and the partial stacks. By doing so, we can deduce the lower and upper bound faster and simpler. To evaluate the actual performance of our algorithm, we conduct experiments on a set of instances with practical size and compare the results with different values of K, and of course zero is included. We have selected a set of values of K to find out the relationship between K and the stacks we used. Through this process, we may get the approximate value of K for one particular shipping to balance the flexibility and space utilization of the ship. Whats more, instances with different number of ports, containers and heights all have been experimented to make sure our algorithms have a universality.

The results demonstrate that our heuristic approaches can generate promising solutions if we allow some rehandling operations compared with the zero rehandle solutions and random loading solutions. That is to say, the results we get use less stacks in average and it shows the problem we put forward is of practical significance and it can improve the utilization ratio of vessels with certain flexibility. It is worth mentioning that our problem with K rehandles constraint acts as a buffer and offers great flexibility for the actual problem as a result of the existence of rehandling operations. Applying this algorithm into the real shipping management can enhance the hull space utilization and reduce the unnecessary spending in the case of meeting other constraints. What's more, the novelty and practicality show the importance of this paper and it can give a reference to the future study.

Key words: Containership stowage planning, stack minimization, K-rehandle, constructive heuristic

1. Introduction

International trade plays a important role in promoting the development of world economy, promoting national income increase and expanding employment around the world. Maritime transport is the backbone of globalization and lies at the heart of cross-border transport networks that support supply chains and enable international trade. In 2015 C for the first time in UNCTAD (United Nations Conference on Trade and

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Development) records C world seaborne trade volumes1 were estimated to have exceeded 10 billion tons. Shipments expanded by 2.1 per cent, a pace notably slower than the historical. Containerization is an important driver of the global economy, and the container has become a mainstay of worldwide trade. Other than a short period of decline in 2009, the number of containers shipped by the world's shipping lines on container vessels has been steadily increasing for the past several decades. As we all know, logistics is the main link in international trade. In view of the complexity of logistics in international trade, multimodal transport can bring a great convenience. Furthermore, the perfect fit between container transport and multimodal transport makes the international trade easier and more convenient. Container-shipping is a kind of modern transportation tool which has a lot of advantages in speed, security, and quality comparing with the break-bulk cargo ship. Containerization has revolutionized cargo shipping. containerization is one of the catalysts of global trade. It has resulted in a tremendous growth of the maritime freight industry. Nowadays, over 80% of world trade is carried by the maritime freight industry, which operates the container transportation business Zhang & Lee (2016). In addition, according to World Shipping Council, around 52 percent of the value of world international seaborne trade today is being moved in containers. This mode is deeply accepted by shipper and carrier, and is becoming the main trend of freightage in the world. It shows powerful vitality and promising prospects.

The container stowage plan is a pivotal link to containers transportation,and the objective of which is to draw a plan of loading and discharging sequence of contains for containership in Zhang et al. (2008). Actual examples show that containership stowage plans not only influence the income of shipping company from transportation but also have direct relation to the safety of ships and freights. These problems have troubled and aroused the interests of both scholars and commercial shipping organization in many countries since 1970s. With the development of container-shipping, more and more containers are transported by sail. Over the past two decades there has been a continuous increase in demand for cost efficient containerized transportation. Up to now, no satisfying solution has development of container transportation trade. Thus, it makes higher demand of the problem.

A container ship transports cargo in containers, and the ship layout varies from vessel to vessel. The most frequently used addressing notation for storage locations in a container ship is the bay-row-tier system. The cargo space of a container ship is split into several 20-ft-long areas. These areas are termed 20-ft bays. In addition, two contiguous 20-ft bays form a 40-ft bay which can be used to accommodate 40-ft containers. Each bay consists of container stacks placed along the length of ship (see Figure 1). All bays are divided into an over deck and an deck area, separated by structures called hatches. In order to gain access to the containers stored under deck, we must remove all the containers on the deck, including the hatch over. The container position in the ship is uniquely indexed by (bay, row, tier). Such three-tuple is called a slot. On the under

deck, the parts of a bay are divided into stacks/slots that are one container wide, and are composed of two Twenty-foot Equivalent Unit (TEU) stacks and a single Forty-foot Equivalent Unit (FEU) stack (see Figure 2). In our paper, we assume that all the containers are of the same size, (e.g. 20 TEUs) and each slot can be fully occupied by one container.

Figure 1: The arrangement of cargo space in a container vessel.

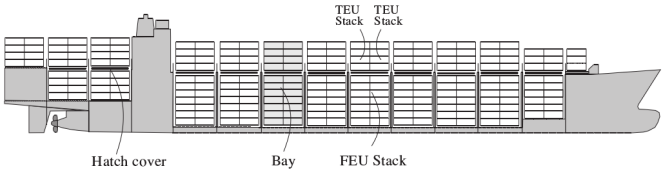
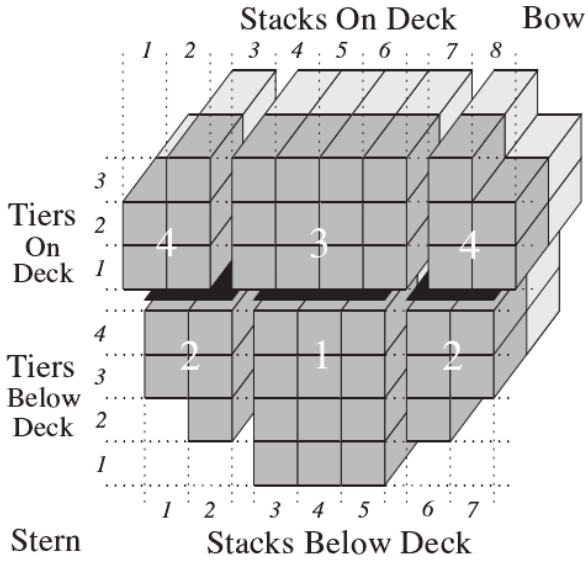


Figure 2: a front view of a vessel bay.

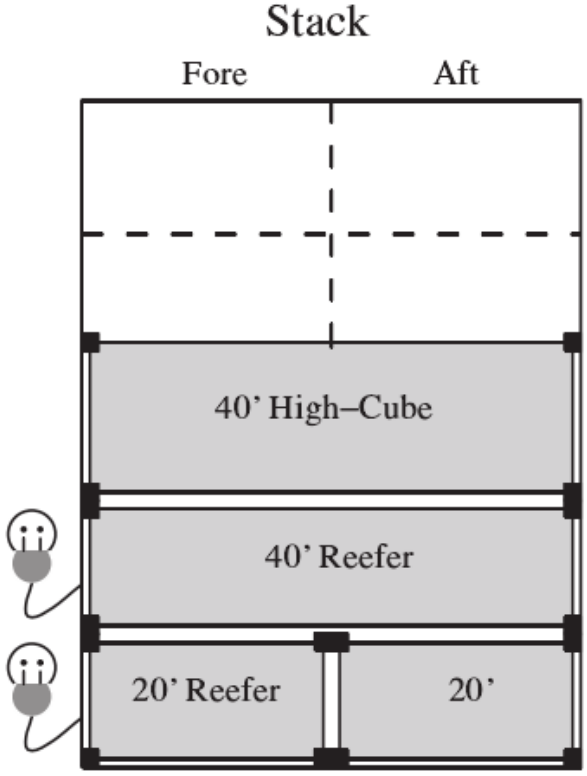


Some slots that have power plugs are known as reefer slots. A standard ISO container is usually 20 or 40 ft long, 8 ft wide and 8.6 ft high. 40 foot containers can be 8.6 ft high, or 9.6 ft high for high-cube containers (see Figure 3).

Each container has a weight, height, length and port where it has to be unloaded (discharge port). Unloading and loading movements of same containers in the same port are called rehandles. Rehandles arise either when we want to unload containers destined for current port which however are beneath those destined for subsequent ports, or when we want to reorder the sequence of containers to prevent more shifts in the future. Usually it is called necessary shifts in the former case and voluntary shifts in the latter.

The previous research ignores the importance of the capacity of ship and give much emphasis on the balance of ship, the number of rehandle and so on. Sometimes we need think about the importance of capacity as well. For example, if we can use each stack sufficiently, the required number of ships will be reduced and thus the cost decreases. The problem mentioned in this paper give a priority on how to reduce the use of

Figure 3: a side view of a partially loaded stack.



stack with K rehandle as much as possible. Through the research on this problem, another aspect of container stowage planning is exhibited and a lot of shipping cost can be saved by using what we propose in this paper. On the other hand, it represents a novel thinking perspective to make a research about this kind of problem and it will promote the development of academic shipping optimal problem.

We put forward the stowage stack minimization problem with K rehandle constraint problem (SSMP-KR) in this paper. Literally speaking, it is a container stowage planning problem whose objection is to find out minimization stack with K rehandle constraint on a given condition. The heuristic algorithm in this paper uses the greedy rule to finish the loading and unloading operation for every container to be loaded and unloaded in each port along the voyage. Therefore, the solution we get may be the approximately optimal rather than the global optimal.

The contribution of our paper lies in three aspects. First, the previous research usually focused on how to reduce the rehandles or shifts , our paper initially considers how to get the optimal container stowage plan when giving the allowed number of rehandles. Second, the problem can also be regarded as a extended problem of the stowage stack minimization problem with zero rehandle, which is a combinatorial optimization problem. Furthermore, we can get the different results when the value of K is different, and zero rehandle can be a benchmark for others to compare with. By comparing different results, we eventually choose a appropriate value of K to balance the complexity of quay operations and the space utilization of the cabin.

Third, we put forward a heuristic algorithm to solve the problem proposed in this paper and the result shows the good performance of our algorithm. The result can help us know the stowage plan visibly and make a analysis about what the appropriate K is. Therefore, the problem and the algorithm presented in our paper are both novel and worthy of further study.

The remainder of this paper is organized as follows: we review extant research works in Section 2 and the formal definition of our problem is provided in Section 3. In Section 4 we present our heuristic algorithm for the problem, as well as explain the ideas behind and specific details about the algorithm. Experiments and associated analysis are illustrated in Section 5. Concluding remarks close the paper in Section 6.

2. Literature Review

The container stowage problem can be divided into two classes: one is management and programming at the container terminal, the other is stowage plan for containership. This paper mainly focuses on the latter. And sorted by research method, this problem will be divided into six categories: simulation based on probability, heuristic procedures, mathematical programming approach, decision support system, expert system and evolutionary computation. The following are some description about these categories of containership stowage plan.

Since 1970s, the stowage plan problem has been studied by shipping organization for commercial demand. They tried to find a good solution which would satisfy fewer shifting and higher efficiency when loading and discharging containers.

Shields (1984) introduced a computer preplanning system (CAPS) which had been used by American President Lines. The CAPS produced stowage plan by using Monte Carlo theory. Expert system is a most active research and practical field of Artificial Intelligent (AI). Dillingham & Perakis (1986) introduced their researches on expert system of stowage based on rules. Each of the suggested container movements was displayed graphically as the decisions were made. Database in the system could not only accomplish the stowage plan, but also track the location of the container. It was highly interactive, the user could interrupt the system at any time when the planning proceeded and overrode or revoked suggestions that the system had made. Wilson et al. (2001) outlined a computer system that generates good sub-optimal solutions to the stowage pre-planning problem. This is achieved through an intelligent analysis of the domain allowing the problem to be divided into sub-problems: a generalized placement strategy and a specialised placement procedure. This methodology progressively refines the arrangement of containers within the cargo-space of a container ship until each container is specifically allocated to a stowage location.

Scott & Chen (1978) constructed a model, in which containers were sorted into ten classes based on some characteristics and then three heuristic rules were used to assign gradually each type of containers to slots of

the ship by four steps, while the shifting problem had not been considered in their model. Avriel et al. (1998) presented a 0-1 binary linear programming formulation based on minimizing the number shifting in a single bay of a ship calling at a given number of ports. Further, they showed that finding the minimum number of columns for which there was a zero shifts stowage plan is equivalent to finding the coloring number of circle graphs. Botter & Brinati (1991) created a model called complete mathematical model for the stowage plan problem, which used binary decision variables to determine the containers unloading and loading sequence for each port. The objective function of this model for the stowage plan involved two important factors: the number of re-stowing along the route and the longitudinal crane movement along the quay during the container loading and unloading operation. The advantages were that the model considered every port the ship calling at as a different stage of the whole system, which made the model much more correspond to the actual cases. But too many factors were considered in the model, the dimension of formulization was too large to solve because of large number of binary variables and constraints arising in the model. Literatures have shown that binary linear programming formulation for such problems is impracticable for real life problems due to the large number of binary variables and constraints.

Avriel et al. (1998) and Avriel et al. (2000) focused on stowage planning in order to minimize the number of unproductive shifts (temporary unloading and reloading of containers at a port before their destination ports in order to access containers below them for unloading). Aspects of ship's stability and other real-life constraints are not considered. A binary linear programming (LP) model is formulated. Due to the proven NP-hardness of the problem, a so-called "Suspensory Heuristic" — based on earlier work by Avriel & Penn (1993) — is developed in order to solve even large problem instances. The heuristic assigns slots in a bay to containers dynamically with respect to the sequence of ports in a vessel's route. Haghani & Kaisar (2001) proposed a MIP model for developing loading plans in order to minimize the time that a vessel spends at port, and the container handling cost which shifts caused by is highly influenced by the number of unproductive but necessary an unsatisfactory arrangement of containers. In Ambrosino et al. (2004) the problem of stowing containers into a containership has been faced by evaluating an exact 0-1 Linear Programming model, that is not practically useful for large cases. This has been modified by an approach proposed by the authors, consisting of heuristic preprocessing and prestowing procedures that also allow the relaxation of some constraints of the exact model. The proposed approach exhibits very good performance in terms of both solution precision and computational time. Moreover, in the performance evaluation with real size cases, it also guarantees another very important maritime performance index, in term of handling operations/hour. In Delgado et al. (2012), they focus on the slot planning phase of this approach and present a Constraint Programming and Integer Programming model for stowing a set of containers in a single bay section. This so-called slot planning problem is NP-hard and often involves stowing several

hundred containers. Using state-of-the-art constraint solvers and modeling techniques, however, we were able to solve 90% of 236 real instances from our industrial collaborator to optimality within 1 second. Thus, somewhat to our surprise, it is possible to solve most of these problems optimally within the time required for practical application. Ambrosino et al. (2015) proposed a Mixed Integer Programming (MIP) heuristic aimed at determining stowage plans in circular routes for container ships so as to give support for the ship coordinator and the terminal planner.

Webster & Van Dyke (1970) first tried to solve the problem by using computer. They put forward a heuristic algorithm that could create a random stowage plan. What's more, Ding & Chou (2015) developed a heuristic algorithm which can generate stowage plans with a reasonable number of shifts for such problems. The algorithm, verified by extensive computational experimentations, performs better than the Suspensory Heuristic Procedure (SH algorithm) in Avriel et al. (1998), which is a dynamic slot-assignment scheme that terminated with a stowage plan, and to the best of our knowledge, is one of the leading heuristic algorithms for such stowage planning problem when there are many binary variables and constraints in the formulation.

Tabu search was employed to obtain an optimal solution in Wilson et al. (2001) and his model was feasible for it can change configuration step by step until each container was assigned to specific slot. Then, Bortfeldt et al. (2003) presented a parallel tabu search algorithm for the container loading problem with a single container to be loaded. According to an extensive comparative test also including heuristics from other authors, high utilizations of the container volume are already obtained with the sequential TSA. A slight improvement of these results could be achieved by the parallelization. The Ship Stowage Planning Problem have been addressed in Monaco et al. (2014). They have first provided a detailed description of the two phases in which the decision problem is usually decomposed, along with a classification scheme and a review of the most relevant related papers. They have proposed a Binary Integer Model for the minimization of the transportation times of the containers and the yard-shifts and they have developed a Tabu Search Algorithm for finding sub-optimal solutions to the problem and have solved the model by CPLEX.

Algorithm of evolutionary computation came from Darwins biologic theory of evolution. There are two or three branches at present, among which genetic algorithm is used widely. The presented hybrid GA for loading a single container in Bortfeldt & Gehring (2001) is particularly suitable for strongly heterogeneous containers stowage problems. The algorithm takes account of some constraints which are relevant in practice. Its good performance and superiority to comparative methods was verified in an extensive test. The use of a time limit kept the required computing times within acceptable limits. Dubrovsky et al. (2002) used a GA for solving the stowage planning problem of minimizing the number of container movements. Search space is significantly reduced by a compact and efficient encoding scheme. Ship stability criteria are reflected by appropriate constraints. Simulation runs demonstrate the efficiency and flexibility of the GA-based approach.

Kammarti et al. (2013) proposed an efficient genetic algorithm which consists on selecting two chromosomes (parent) from an initially constructed population using a roulette wheel technique. Then, the two parents are combined using a one point crossover operator. Finally, a mutation operator is performed. The variant tackled in Cohen et al. (2017) involved several constraints, inspired by real-life problems and application found in the literature. Given the complexity of the problem, which belongs to the class of NP-hard problems, a novel evolutionary metaheuristic algorithm is developed and designed. Considering the ability and flexibility of Genetic Algorithm (GA). The approach is based on a two-phase procedure, one for master planning and the other for allocation of the containers into slots. GA parameters are analyzed to achieve practical and best results. The system offers stowage allocation solutions for both phases, thus offering flexibility for a wide variety of vessels and route combinations.

Container ship stowage problem(CSSP) is combinatorial optimization problem with multiobjects and multiconstraints. In order to reduce computational difficulty, CSSP is decomposed two subproblems in Wei-Ying et al. (2005), and the first one is discussed detailed. CSSP is regarded as bin-packing problem. After being modeled£the algorithm of best fit for binpacking problem is used to solve the CSSP. A case shows that the algorithm is feasible for container ship stowage problem. A multiobjective ship stowage planning problem(SSPP) was investigated by Zhang & Lee (2016), which aims to optimize the ship stability and the number of rehandles simultaneously. They use metacentric height, list value, and trim value to measure the ship stability. Meanwhile, the number of rehandles is the sum of rehandles by yard cranes and quay cranes and all necessary rehandles at future ports. The algorithm can produce a set of nondominated solutions. Decision makers can then choose the most promising solution for practical implementation based on their experience and preferences. Extensive experiments are carried out on two groups of instances. The computational results demonstrate the effectiveness of the proposed algorithm compared to the NSGA-II and random weighted genetic algorithms, especially when it is applied in solving the six-objective SSPP.

Tierney et al. (2014) investigated the complexity of stowage planning problem and showed that the capacitated k-shift problem is solvable in polynomial time for any choice of stacks and stack capacities. And some scholars paid special attention to the process of loading and unloading, Malucelli et al. (2008) investigated stack reordering strategies aiming at minimizing the number of pop and push operations and focused on three versions of the problem in which reordering can take place in different phases: when unloading the stack, when loading it or in both phases. There are some extended problems in this field. Different problems arising in practice in the context of storage areas organized in stacks have been surveyed by Lehnfeld & Knust (2014). Different problem classes have been identified, namely loading, unloading, premarshalling and combined problems. To be able to categorize all these problems in a more abstract way, they developed a classification scheme which has been used to summarize complexity results and solution methods for all

problem classes. In conclusion, many problems have already been solved by exact or heuristic methods in all four considered problem classes. Nevertheless, due to the diversity of the problems, various other versions exist. For further research, a major challenge is the development of more efficient solution algorithms integrating additional constraints coming from practical applications. As we all known, the 3D Container ship Loading Plan Problem (CLPP) is an important problem that appears in seaport container terminal operations and is well known to be NP-hard. This problem consists of determining how to organize the containers in a ship in order to minimize the number of movements necessary to load and unload the container ship and the instability of the ship in each port. In Araujo et al. (2016), the hybrid method Pareto Clustering Search (PCS) is proposed to solve the CLPP and obtain a good approximation to the Pareto Front. The PCS aims to combine metaheuristics and local search heuristics, and the intensification is performed only in promising regions. Computational results considering instances available in the literature are presented to show that PCS provides better solutions for the CLPP than a mono-objective Simulated Annealing.

To the best of our knowledge, apart from the SSMP-ZR, only Avriel et al. (2000) and Jensen (2010) discussed the stowage stack minimization and its connection with the chromatic number of circle graphs when the stack height is unlimited. At present, the research tendency of the stowage planning problem is intelligent management system, which is a new stowage research direction. Intelligent containership stowage plan system is a technological innovation in contrast with traditional stowage method. It can improve the ability of decision-making management system of container shipping by means of artificial intelligence, information technology, communications and computer technology.

3. Problem Description

A ship starts its journey at port 1 and sequentially visits port 2, 3, ..., P, 1, 2, 3, ... P, 1, ..., which forms a circular route. There are standard containers to be shipped along the journey. Some containers are shipped from i to j ($i < j$) and some containers are shipped from i to j ($i > j$) going across port 1. Hence, it is easy to understand that containers with $O(i) < D(i)$ and with $O(i) > D(i)$ both exist. Moreover, the ship is never empty once sets out due to the circular feature.

Supposing that the voyage terminates in the second circle, we can break the loop into a straight line. A ship starts its journey at port 1 and sequentially visits port 2, 3, ..., P, 1, 2, 3, ... P. There will be P kinds of different containers according to their origin ports. Containers at port 1 can be transported into port 2, 3, ..., $P-1$, P. Containers at port 2 can be transported into port 3, 4, ..., P, 1. The last kind of containers originates from port P. Containers at port P can be transported into port 1, 2, ..., $P-1$. However, we only work out the result under the condition that $O(i) < D(i)$ happens for the convenience of experiment design. Consequently, we need break the loop voyage into the line voyage. Actually it is not difficult, we can regard the port 1 in

the second circle as port $P + 1$, the next port in the second circle will be port $P + 2$ and so on. Eventually we only need work out the container stowage planning for the line voyage from port 1 to port $2P$ and it is equivalent to cope with the problem with a two-circle voyage. Similarly, we can use this method to deal with the circular route container stowage problem no matter how many circles they are. There is another approach we can think about to solve the circular route problem. After going across port 1 in each circle, we reset the origin port of containers remained in vessel as port 1. For example, containers from port 3 to the port 2 in the next circle will be containers from 1 to port 2 after going across port 1 in the next circle. Thus, it only exists the port from port 1 to port P. In addition, all the loading and discharging ports of N containers are known in advance. The "look ahead" mechanism, which can make use of all loading information available£is helpful for us to make detailed stowage plan.

At each port, the containers destined to the current port are discharged, and the containers stowed at the port are loaded to the ship. Each container i is characterized by its shipping leg $O(i) \rightarrow D(i)$, indicating that the container is loaded at port $O(i)$ and discharged at port $D(i)$. The containers remain unmoved when the ship voyages among the ports.

Shifts may occur in these ports, either necessary or voluntary or both. A container is called a j-container if it is destined for port j. 0bviously all j-containers are loaded before the ship visits port j and should be unloaded at port j. A j-container is called a i, j-container if it is originated from port i. A j-container in slot (r, c) is said to be blocked if there exists a j'-container in slot (r', c) with j' > j and r'<r, and the j'-container is called a blocking container with respect to the j-container. Occurrence of container blocking at any time means that shifts are no longer avoidable.

We assume that the ship has enough capacity to accommodate all the containers along the journey. The ship spaces are divided into several stacks. Each stack can hold at most H vertically piled containers, i.e., the height limit of every stack is H. When H is bounded, we call the SSMP-KR as the capacitated SSMP-KR (CSSMP-KR); when H is sufficiently large (e.g., $H \geq N$) or unbounded, we refer to it as the uncapacitated SSMP-KR (USSMP-KR). For the CSSMP-KR, if the number of containers in a non-empty stack is less than H, the stack is termed as a *partial* stack; otherwise it is a *full* stack. The objective of the SSMP-KR is to transport all the containers using the fewest number of stacks with K rehandles allowed. The USSMP-KR is far from the real situation in the stowage planning, since stack heights of ships are restricted by operation cranes. Hence, this paper give emphasis on the CSSMP-KR and for the convenience of description, we use SSMP-KR for short in this paper.

A feasible solution to the SSMP-KR is represented by a operation set, from which we can clearly know the slot changing of each container, even when there is an overstowage. The detailed introduction will be given in the below.

Considering that the complexity of working out the integer model for the SSMP-KR problem, we give the IP model for the SSMP-ZR problem. The capacitated SSMP-ZR can be mathematically formulated by the following integer programming (IP) model.

$$(IP) \quad \min \quad \sum_{s=1}^{S} y_s \tag{1}$$

$$\text{s.t.} \quad \sum_{s=1}^{S} x_{is} = 1, \ \forall 1 \le i \le N \tag{2}$$

$$\sum_{i=1}^{N} x_{is} \le M y_s, \ \forall 1 \le s \le S \tag{3}$$

$$\sum_{i:O(i)\le p\le D(i)-1} x_{is} \le H, \ \forall 1 \le s \le S, 1 \le p \le P \tag{4}$$

$$x_{is} + x_{js} \le 1, \ \forall 1 \le s \le S, 1 \le i, j \le N, O(j) < O(i) < D(j) < D(i) \tag{5}$$

$$x_{is}, y_s \in \{0, 1\}, \ \forall 1 \le s \le S, 1 \le i, j \le N \tag{6}$$

In the above model, M is a sufficiently large number and S is the number of stacks on the ship. x_{is} is a binary decision variable which is equal to 1 if container i is loaded to stack s, and 0 otherwise. y_s is a binary decision variable that is equal to 1 if stack s is used for stowage, and 0 otherwise. $O(i)$ and $D(i)$ indicate the shipping leg of container i. The objective (1) minimizes the number of stacks used. Constraints (2) ensure that each container i is loaded to only one stack. Constraints (3) guarantee that containers can only be loaded to used stacks. Constraints (4) assure that at each port p, the number of containers in stack s does not exceed H. Constraints (5) avoid overstowage requiring that container i is prohibited to block container j in the same stack when container j is retrieved.

It is straightforward that the resultant layout from such a loading order does not include overstowage.

We have also presented the simple IP model for the SSMP-KR to make our problem clear.

The SSMP-KR can be mathematically formulated by the following integer programming (IP) model.

$$(IP) \quad \min \quad \sum_{s=1}^{S} y_s \tag{7}$$

$$\text{s.t.} \sum_{s=1}^{S} x_{is} = 1, \ \forall 1 \le i \le N \tag{8}$$

$$\sum_{i=1}^{P} R_i \le K \tag{9}$$

$$\sum_{i=1}^{N} x_{is} \le H, \ \forall 1 \le s \le S \tag{10}$$

$$\max \quad \sum_{i=1}^{P} (L_i - U_i + R_i) \le S * H \tag{11}$$

In the above model, H is the limit height of each stack, N is the number of containers, P is the number of ports and S is the number of stacks on the ship. x_{is} is a binary decision variable which is equal to 1 if container i is loaded to stack s, and 0 otherwise. y_s is a binary decision variable that is equal to 1 if stack s is used for stowage, and 0 otherwise. L_i, U_i, R_i are the number of loading containers, discharging containers and rehandling containers, respectively. The objective (7) minimizes the number of stacks used. Constraints (8) ensure that each container i is loaded to only one stack. Constraints (9) guarantee that the total number of rehandles is no greater than than K, the value of allowed rehandles. Constraints (10) ensure that the total number of containers in each stack is no greater than than H. Constraints (11) assure that at each port p, the number of containers in vessel does not exceed $S * H$.

Note that the solution to the IP model only reflects the assignment of containers to stacks. It can be converted to a complete loading plan as follows. At each port p, load containers to their corresponding stacks (given by x_{is}) by the decreasing order of their destinations. Compared with the IP model for the capacitated SSMP-ZR, we can find that the resultant layout from such a loading constraints will be more flexible to design our algorithm and closer to the actual situation.

4. Methodology

In this section, we will talk about the heuristic algorithms to the SSMP-KR, as well as the basic performance guarantee of the algorithms.

4.1. Heuristic Algorithms for the SSMP-KR

Before the introduction of our algorithm, we give the related notations as followings in the table:

In our algorithm, two classes (collections of objects with the same characteristics and behavior in Java) are introduced in order to describe the containers' characteristic and record the operation for containers. They

Table 1: constants, sets, variables and methods in the algorithm

K, P, N, H	the meanings of these constants are the same as what described in the above
LB	the lower bound for each given instance
UB	the upper bound for each given instance
S_0	stack index set to store feasible stacks when there isn't an rehandle allowed
S_1	stack index set to store partial stacks without rehandles when K doesn't reach
S_2	stack index set to store partial stacks with rehandles when K doesn't reach
S_3	stack index set to store empty stacks when K doesn't reach
height	a one-dimension array, represents the current height of each stack
Layout	a two-dimension array, represents the layout of containers in vessel after each operation
Numofstack	the cumulative number of used stacks
lb	a one-dimension array, represents the lower bound of used stacks in each port
ub	a one-dimension array, represents the upper bound of used stacks in each port
ac	an arraylist to ordinally store the information of each container including the origin port and the destination port
Solution	an arraylist to ordinally store the operation of each container
Container	a Class to record the information of each container
Operation	a Class to record the information of each operation, including loading, unloading and rehandling.

are "Container" class and "Operation" class, respectively. For each container, we use arraylist (a type of data structure) "ac" to store its sequence number, origin port and destination port. For each operation, we use arraylist "Solution" to store its change. No matter what the operation is , we add all of operations into the arraylist "Solution" to record every container's slot changing from the shore to the chosen stack or from the previous stack to the shore. We also denote the dynamic *Layout* as the layout of the ship after different operations for containers. We even can output the layout after the operations for one container or after all the operations at one specific port through the algorithm proposed in this paper. Other important global variables are Count_rehandle and Numofstack. The former is no greater than the given K, and the latter no longer change until the end of the shipping. Methods used in our algorithm aren't exhibited in the below are read() and bound(). The method read() is used to read the parameters and the detailed containership information from each instance. As for the method bound(), we use it to give a upper bound for our algorithm and the rationality proved. Another important method "GetstackNum", which is used in both loading_rehandle_lessK method and loading_rehandle_equalK method, is used to get the sequence number of chosen stack.

If the allowed K doesn't reach, we use arraylist S_1 to store partial stacks without rehandles occuring, arraylist S_2 to store partial stacks with rehandles occuring, arraylist S_3 to store empty stacks whether there is a rehandle or not. The priority order of them is S_1, S_2, S_3. If there isn't an overstowage allowed, we use S_0 to store the feasible stacks. For ease of illustration, we need to define a one dimensional array called nearport to store the nearest port for every stack. For example, three containers are stored in the same stack, and their destination port are 4,5,6, respectively, then the value of nearport of this stack is 4. If a stack is empty, its value will be set to $P + 1$, P is the total number of the ports along the voyage.

We firstly talk about the simple condition that the allowed rehandles run out. When traversing all stacks to choose the optimal according to our rules for each container, we add non-full stacks whose nearest port is greater than or equal to the current loading container into arraylist S_0. For the stacks in S_0, we choose the

one with the nearest port. Next we talk about the complex condition that a rehandle is allowed and it is a little different from the simple condition. Under this condition, S_1 have the highest priority. If S_1 is null, we choose the stack from arraylist S_2 which is used to store partial stacks whose nearest port is smaller than the current loading container. Both in S_1 and S_2, we choose the stack whose nearest port is smallest. If S_2 is null, we choose the first stack we traverse in the S_3 which is used to store empty stacks. So it is the loading rules in the above and the following is the discharging rules.

When discharging containers, we just need to unload containers from top to bottom one by one when the container's destination port is the current port if there isn't an overstowage in the stack. when an overstowage happens, we have to unload the blocking containers, then we can unload those right ones. As for blocking containers, we reset their origin ports as current port and their destination ports remain unchanged. That's to say, the outbound containers at the current port increases.

We give out some parts of algorithms as followings: For all the ports along the voyage, we adopt the heuristic method to dispose containers for the given input. And we can transfer the input into the output by applying the heuristic method. So the below is the main process of our algorithm.

The simple condition when there isn't rehandle allowed.

The complex condition when there is allowed rehandles.

The following pseudo gives a simplistic explanation of heuristic method. The variables "height", "Layout" and "Numofstack" are one dimensional array, two-dimensional array and the number of accumulative used stack in the pseudo, respectively. *Unloading_at_port*(p) and *Loading_at_port*(p) methods provide the way how to unload the right containers and load the right containers at each port. Given a specific instance, we can get the correspondent value of "Numofstack".

The following unloading_at_port() method is used to decide which container should be discharged at port p. Then we need substitute the related information of chosen container into the Unloading_for_container(s,t,p) method.

Considering the length of pseudo code, some details are omitted. We will give a explanatory about the following pseudo code. We first define a one-dimensional array nearport which consists of "UB+1" integer number and it is used to store the nearest destination among all containers located in each stack. If the stack is empty, then the value will be set as "P+1". The ac is an arraylist to store all containers to store all the containers before the shipping. If one container is to be unloaded and there isn't overstowage, then it will be removed from the arraylist. For each blocking container, its origin port will be reset as current port after it is removed from its location, then it will be added into ac again.

As a result of the existence of rehandle, we need sort ac before we start loading containers at port p. The sorting rules are shown as followings. We compare origin port of containers in the first place, the number smaller, the location ahead. If the origin port is identical, compare the destination port, the number larger, the location ahead. If ac is not empty, we just need to choose the first element *cn* to load. We use loading_for_container method to choose the optimal stack. After loading it into the stack *st*. *st*'s height and nearport will be updated. At last, record this operation and add the operation into the arraylist Solution. Hence, we can eventually get all the operation information.

The following Loading_for_container() method provides us the method how to load containers and two different sub-methods are given to return the stack where the loading container belongs. The specific procedure of two sub-methods is described in the above content.

The below pseudo code of $unloading\_for\_container()$ method shows us how to unload containers one by one whether there is overstowage or not. Before the unloading operation,we need input the current port, the stack and the tier where the unloading container is. If there is overstowage, we invoke "$Relocate(s,t)$" method. After processing blocking container(s), we set out to unload the right container. We firstly record the unloading operation and add it into the arraylist Solution. They remove the container from its previous location and reduce the height of stack where it is located.

In a word, a lot of variables and methods of the concrete Unloading and Loading strategies have been involved, we don't give the correspondent pseudo here for the sake of context length. What is identical for the two procedures is that they choose the optimal operation for each container during the unloading and loading process according to the greedy rule made by our algorithm. As is explained in the heading of this section, we divide the loading method into two parts on the basis of the number of allowed rehandles. And similarly, the unloading method are divided into two conditions whether there are rehandles. Hence, specific steps are a little different from each other between the two circumstances.

Our algorithm can give the detailed location changes of each container and output the container status at arbitrary time. That's to say, we can grasp the container stowage process before the voyage as long as we know some information in advance. Therefore, our algorithm makes a little difference and make some sense

in the reality application.

4.2. Performance Guarantee of the Algorithms

In order to simplify the problem, we show that the heuristic algorithms have a performance guarantee whether the value of K is zero or not and we regard the port number P as a fixed number. For ease of exposition, we first give some related notations.

- $\mathcal{U}_k^*$ & C_k^*: the optimal solution to the USSMP-KR and the SSMP-KR instances, respectively.
- $\mathcal{U}_k$ & C_k: the solution to the USSMP-KR and the SSMP-KR instances by the algorithm, respectively.
- $\mathcal{U}_p$ & C_p: the number of stacks used at port p by the algorithm.
- N_p: the number of containers on the ship before its departure from port p.
- V_p: the number of loading ports that the ship has visited before it departs from port p. A port is called a loading port if there exists at least one container to be loaded. Clearly, $V_p \leq p, \forall p = 1, \ldots, P$.

Proposition 1. *For the USSMP-KR instance, the heuristic approach generates a solution in which at most V_p stacks are used before the ship departs from port p, i.e. $\mathcal{U}_p \leq V_p, \forall p = 1, \ldots, P$.*

Proof. We use an inductive proof.

Basis: For $p = 1$, without loss of generality, we assume that port 1 is a loading port. The heuristic piles up containers in one stack in a way that the containers with later destinations are placed in lower tiers. We have $\mathcal{U}_1 = V_1 = 1$, and therefore $\mathcal{U}_1 \leq V_1$ holds.

Inductive step: Suppose that before the ship departs from port p, there are at most V_p stacks used, i.e., $\mathcal{U}_p \leq V_p$. Upon the ship arrives at port $p + 1$, it first unloads the containers from the ship. This process does not increase the number of stacks utilized. If there are containers to be loaded at port $p+1$, then $V_{p+1} = V_p + 1$, otherwise $V_{p+1} = V_p$.

(1) If there exist some containers to be loaded, according to our heuristic, they are placed on either the extant stacks or a new blank stack (with the containers of later destinations placed lower). Hence, $\mathcal{U}_{p+1} \leq \mathcal{U}_p + 1 \leq V_p + 1 = V_{p+1}$.

(2) If no container is loaded, $\mathcal{U}_{p+1} \leq \mathcal{U}_p \leq V_p = V_{p+1}$.

Based on the above two steps, we have $\mathcal{U}_p \leq V_p, \forall p = 1, \ldots, P$. Furthermore, we have

$$\mathcal{U}_k = \max_{p=1,\ldots,P} \mathcal{U}_p \leq \max_{p=1,\ldots,P} V_p = V_P \leq P$$

□

Proposition 2. *The SSMP-KR instance has a lower bound:*

$$\max_{p=1,\dots,P}(\lceil\frac{N_p}{H}\rceil) \leq C_k^*$$

Proof. $\lceil\frac{N_p}{H}\rceil$ is the least number of stacks to accommodate all the containers at port p, and thus, $\max_p(\lceil\frac{N_p}{H}\rceil)$ is the least number of stacks needed throughout the journey. □

The lower bound can help to check the optimality of solutions obtained by our heuristic. If the solution is equal to the lower bound, we can conclude that the solution is optimal.

Proposition 3. *For the heuristic solution to the SSMP-KR instance, it holds*

$$C_k^* \leq C_k \leq \max_{p=1,\dots,P}(\lfloor\frac{N_p}{H}\rfloor + V_p) \leq C_k^* + V_P$$

Proof. The first inequality obviously holds.

For the second inequality, $C_k = \max_p C_p$. Remind that C_p is the number of stacks in port p result from our algorithm. We divide the stack used in the port into two parts: full stacks and partial stacks. For the full stacks, the number of full stacks in port p is no greater than $\lfloor\frac{N_p}{H}\rfloor$. For the partial stacks, we define the number of partial stacks in port p as $P(p)$.

Proposition 4. *The partial stacks in port p is no greater than V_p:*

$$P(p) \leq V_p$$

Proof. We use an inductive proof. *Basis*: For $p = 1$, without loss of generality, we assume that port 1 is a loading port, and therefore $V_(p = 1)$ =1. In addition, P(1)=0 or 1. Hence, $P(p) \leq V_p$ holds. *Inductive step*: Suppose that the partial stacks in port p is no greater than V_p. Upon the ship arrives at port $p + 1$, it first unloads the containers from the ship. This process does not increase the number of stacks utilized.

(1)If there are containers to be loaded at port $p + 1$, then $V_{p+1} = V_p + 1$, $P(p+1) \leq P(p) + 1$ whether we allow the rehandle or not in port $p + 1$.

(2)If there is no containers to be loaded at port $p + 1$, $V_{p+1} = V_p$, $P(p+1) = P(p)$. Hence, $P(p+1) \leq V_{p+1}$. Based on the above two steps, we have $P(p) \leq V_p, \forall p = 1, \dots, P$. □

Therefore, the second inequality holds.

For the third inequality, $\lfloor \frac{N_p}{H} \rfloor \leq \lceil \frac{N_p}{H} \rceil$, and $\lceil \frac{N_p}{H} \rceil$ is the lower bound of the number of stacks used at port p by Proposition 2. Thus, $\lfloor \frac{N_p}{H} \rfloor \leq C_p^*$. It then holds $\max_p(\lfloor \frac{N_p}{H} \rfloor + V_p) \leq \max_p(C_k^* + V_p) \leq \max_p(C_k^* + V_p) = C^* + V_P$. □

The above proposition indicates that our approximation algorithm has a constant performance guarantee if we regard the number of ports as fixed under the condition that the number of allowed rehandle is zero. In practice, the number of ports is generally much smaller than the number of stacks used along the journey. Hence, the gap between the optimal solution and our heuristic solution is relatively small, which demonstrates that our heuristic can generate promising solutions.

Due to page limit, we omit the deatils here. Interested readers are welcome to contact the authors for details.

5. Experiments and Analysis

This paper uses the test data from the existing literature and we add an extra parameters K into the primary instance in order to meet the demands of research. K means the number of allowed rehandles and it is selected from {0,10,20,50,100}. Particular£$K = 0$ represents the benchmark and comparable condition. In fact, the problem becomes SSMP-ZR if $K = 0$. There are 2400 sets of instances in total, which are categorized by three parameters: the number of ports P, which is selected from {5, 10, 20,30}; the number of containers N, which is selected from {50,100,200,500,1000,5000}, and the result isn't shown in the table when $N = 5000$; the height limit H selected from {4,7, 8, 12}. Each set consists of 5 instances generated by different random seeds. In each instance, the origin $O(i)$ and the destination $D(i)$ of a container i are generated from a uniform distribution on the integers $1, 2, \ldots, P$ satisfying that $O(i) < D(i)$. For the convenience of comparison, we only show the average "Numofstack" when $K = 0$, $K = 10$, $K = 20$, $K = 50$, $K = 100$, respectively. And this is applicable to the heuristic algorithm as well as the deuterogenic $P\&H$ algorithm, the simple introduced will be introduced in the next content.

In this section, we will showcase the performance of our heuristic algorithms on a number of test instances. The heuristic was implemented in Java 8 and the experiments were conducted on a computer with Intel Core processor clocked at 2.30 GHz and 8 GB RAM. The operating system of the computer is Windows 10.

The experimental results are shown in Table 2 and Table 3.

Table 2 summarizes the results of those easy instances with small number of ports, i.e. $P \in \{5, 10\}$. For each given P, N and H, we work out the value of UB, the average numbers of $Numofstack$ in different instances when K selects different values. The UB is used to test the rationality and the $Numofstack$ is used to compare with the result when $K = 0$, respectively. Table 3 summarizes that the results of those difficult

Table 2: Results of the instances with $P \in \{5, 10\}$

P	N	H	LB	UB	K=0	K=10	K=20	K=50	K=100
5	50	4	8	9.6	8	8	8	8	8
5	50	7	5	6.8	5	5	5	5	5
5	50	8	4	6	4.6	4	4	4	4
5	50	12	3	5	3.2	3	3	3	3
5	100	4	15	16.8	15	15	15	15	15
5	100	7	8.8	10.4	9	8.8	8.8	8.8	8.8
5	100	8	7.6	9.6	7.6	7.6	7.6	7.6	7.6
5	100	12	5.4	7.6	5.4	5.6	5.4	5.4	5.4
5	200	4	30.6	32.2	30.6	30.6	30.6	30.6	30.6
5	200	7	17.6	19.4	17.6	17.6	17.6	17.6	17.6
5	200	8	15.6	17.4	15.6	15.6	15.6	15.6	15.6
5	200	12	10.4	12.4	10.4	10.4	10.4	10.4	10.4
5	500	4	76.6	78	76.6	76.6	76.6	76.6	76.6
5	500	7	44.2	45.6	44.2	44.2	44.2	44.2	44.2
5	500	8	38.6	40	38.6	38.6	38.6	38.6	38.6
5	500	12	26	40	27.4	26	26	26	26
5	1000	4	152.2	153.8	152.2	152.2	152.2	152.2	152.2
5	1000	7	87.2	89	87.2	87.2	87.2	87.2	87.2
5	1000	8	76.6	78.2	76.6	76.6	76.6	76.6	76.6
5	1000	12	51	52.8	51	51	51	51	51
10	50	4	7.8	12.6	9	8	7.8	7.8	7.8
10	50	7	4.8	10	6.4	6.2	5.6	5.6	5.6
10	50	8	4	10	6	6	5	5	5
10	50	12	3	9.4	5.6	4.6	4.4	4.4	4.4
10	100	4	14.8	19.6	15.8	15.6	15	14.8	14.8
10	100	7	8.6	13.6	10.2	10.4	10.2	8.8	8.6
10	100	8	7.6	12.6	9.2	9.8	9.6	7.8	7.6
10	100	12	5.2	10.4	7	7	7.2	6.4	5.2
10	200	4	28.6	33.2	29.6	29.6	28.6	28.6	28.6
10	200	7	16.4	21.2	17.6	17.8	18	16.4	16.4
10	200	8	14.4	19.2	16.4	16.6	16.2	14.8	14.4
10	200	12	9.8	15	12	12	11.8	11.4	9.8
10	500	4	69.8	74.2	69.8	69.8	69.8	69.8	69.8
10	500	7	40	44.4	40.6	40.4	40.2	40	40
10	500	8	35	39.4	35.6	35	35.2	35	35
10	500	12	23.6	28	25.2	24.8	24.8	24	23.6
10	1000	4	137.6	141.8	137.6	137.6	137.6	137.6	137.6
10	1000	7	79	83.2	79	79	79	79	79
10	1000	8	69	73.2	69	69	69	69	69
10	1000	12	46.2	50.4	46.8	46.8	46.6	46.2	46.6

Table 3: Results of the instances with $P \in \{20, 30\}$

P	N	H	LB	UB	K=0	K=10	K=20	K=50	K=100
20	50	4	7.2	19.6	9	9	8	7,2	7.2
20	50	7	4.4	19.2	7.8	7	6.6	5.2	4.4
20	50	8	3.8	19	7.8	7	6.6	5.4	3.8
20	50	12	2.8	19	7.8	6.6	5.6	4	2.8
20	100	4	14.2	25	16	16	16	14.2	14.2
20	100	7	8.4	20.4	11.6	11.4	11.2	10.6	8.4
20	100	8	7.4	19.6	10.8	11	10.8	10.2	7.6
20	100	12	5	19.2	10.2	9.8	9	9	7.6
20	200	4	27.6	37.6	29.8	30.2	29.8	28.8	27.6
20	200	7	16.2	26.8	19.2	20	19.4	18.8	18.2
20	200	8	14.2	25.2	17.6	18	18	17.4	17.4
20	200	12	9.4	21.6	13.8	13	13.8	13.2	13.4
20	500	4	67.6	77.2	70.2	70.4	70	70.4	67.6
20	500	7	39	48.6	41.8	42.4	42.2	42.2	41.8
20	500	8	34	43.6	38	37.6	37.8	38	38
20	500	12	22.8	33.2	27	27.2	27.2	27.2	27.2
20	1000	4	134.4	143.6	135.8	135.8	136.2	136	134.4
20	1000	7	77.2	86.4	80.4	80.6	80.6	80.6	80.4
20	1000	8	67.6	76.8	70.6	71	70.4	70.4	70.6
20	1000	12	45.2	54.4	48.8	49.8	49.2	49.2	48.8
30	50	4	8	29.8	10	9.8	9.4	8	8
30	50	7	4.8	29	8.4	7.4	7.8	5.8	4.8
30	50	8	4.2	29	8.2	7.8	7.6	5.2	4.2
30	50	12	3	29	8	7.4	6.8	5.2	3.4
30	100	4	13.8	33.4	16.8	16.6	16.2	15.6	13.8
30	100	7	8.2	29.8	12.2	12	11.6	11.6	9.4
30	100	8	7.2	29.6	12	11.4	11.8	10.8	9.8
30	100	12	5	29	10.8	10.4	10.2	9.8	8.2
30	200	4	27.6	43.8	30.4	31	31.6	31.2	28
30	200	7	16	33.8	20	19.8	19.4	20.2	20
30	200	8	14	32.2	18.6	18.8	19	19.4	19.8
30	200	12	9.6	29.8	17	16.6	16.2	16.2	16.2
30	500	4	64.8	80.4	69.6	69.6	69.6	69.4	69.2
30	500	7	37.6	53.4	42.4	42	42.8	42.6	42.8
30	500	8	32.6	49	38.4	38.6	38.6	38.2	37.8
30	500	12	22.2	39.2	29	28.6	28.6	28.6	28.8
30	1000	4	128.8	144	133.6	133.6	133.6	133.8	133.6
30	1000	7	74	89.6	79.8	79.8	79.8	80	79.4
30	1000	8	64.8	80.4	71.4	70.6	71	70.6	70.6
30	1000	12	43.2	59.4	50.8	50.6	50.8	50.8	50.8

instances with large number of ports, i.e. $P \in \{20, 30\}$. The columns in this table are similarly defined as those of Table 1 and so is the conclusion we can get from Table 1.

Most of instances provide the evidence that it works well when K isn't equal to zero comparing the condition where K is equal to zero. However, the results between the easy instances and difficult instances have some differences. Comparing with the significant influence of different K in the difficult instances, the influence of different K in the easy instances isn't obvious. The same conclusion can be drawn if we focus on one specific variable (or factor) while keeping other variables unchanged. As far as we are concerned, we think it is the density and sparseness that causes this consequence. In other words, the smaller value of each variable is, the same kind of containers that have the identical origin port and destination port have more chance to be placed in the same stack and less rehandles will arise. We call this stack with high density, and the high sparseness in contrast.

There are few exceptions when we analyze the result through a large amount of experiments. The number of needed stack is even larger when the allowed rehandle K isn't equal to zero, though the deviation is not great. For the instance with the same P, N and H, we call it benchmark instance when K is equal to zero. If the number of used stack is larger than the benchmark instance when K isn't equal to zero, we call it exceptional instance. We then find out those instances and compare the stowage planning with the same instances under the zero rehandle constraint, and we think it's the distribution status of previous containers to be blamed. The allowed rehandles delay the production of new stack but not to reduce it, that's why this problem happens. Actually, it's the unloading strategy for blocking containers that causes this exceptional condition. Because we reset the origin port of blocking containers, the loading containers in current port increases and meanwhile the allowed rehandles run out. In the end, the new stack is needed under the coincidence. In order to prove it, we output the layout of the exceptional instances and the corresponding benchmark instances. By comparing the exceptional instances with the corresponding benchmark instances, we get the conclusion that both the distribution of containers and the algorithm are the reason for this kind of abnormal condition. In our heuristic algorithm, we reset the blocking containers' origin port as the current port, and it enlarges the number of containers to be loaded. Meanwhile, the allowed rehandles run out and thus a new stack is needed to stow those containers.

However, it doesn't mean that our algorithm isn't universal because the probability of exceptional instance is very low and it hardly happens in our practical operations. Therefore, our algorithm is feasible for most of the instances and has a good performance as well.

Table 4 gives the average results when the K has different values. It isn't difficult to conclude that the value of $Numofstack$ becomes smaller if several rehandle is allowed and the more rehandle, the less stacks used. That's to say, our problem based on zero rehandle is meaningful and our algorithm is practical.

Table 4: average results of the instances with heuristic algorithm

K	0	10	20	50	100
$Numofstack$	97.6854	97.61042	97.4875	97.15	96.77708

In fact, we give another related algorithm named $P\&H$ algorithm based on our basic algorithm to get the stowage planning. This method is actually a process of parameter tuning, our original intention is attempting to find out better solution to stow containers. Nevertheless, the result through this method isn't as good as the heuristic algorithm and it doesn't work well as we expected. From another point of view, it just shows the good performance of our heuristic algorithm. We give some restrictions on the height and nearport of alternative stacks when choosing one stack for each container. The parameters in $P\&H$ values between 0 and 1. The former parameter represents the dispersion degree between the loading container's destination port and alternative stack's nearport. If their values are equal, the value of parameter is 1. The latter parameter means the ratio of stack's height and the limited height H. The concrete content of "P&H" method isn't described in this paper but the computational results will be listed in the following tables.

Table 5: average results of the instances with $P\&H$ algorithm

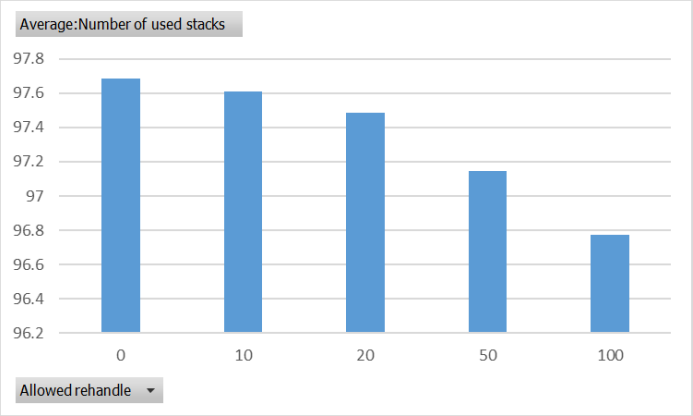
$P\&H$	0	10	20	50	100
0.75&0.75	97.725	97.6375	97.49583	97.12708	96.72083
0.75&0.80	97.725	97.6375	97.49583	97.12708	96.72083
0.80&0.75	97.69583	97.61667	97.47083	97.11667	96.74583
0.80&0.80	97.69583	97.61667	97.47083	97.11667	96.74583

Table 5 shows the result with "P&H" method, which is firstly designed to improve the original algorithm and eventually it is used to prove the good performance our original algorithm. Both the two parameters of the method are selected form {0.75,0.8}. We can draw a conclusion that the first parameter has a significant influence on the result while the second parameter makes no difference. I suppose that the it happens as a result of the value set of H. Comparing with the results shown in Table 3, we could find that original algorithm we propose works better when the value of K is small while the $P\&H$ method works better when the value of K is large. So maybe we can use the better one after working out the result when K is certain.

We also use some pictures to make our result more visual and the pictures are made by Excel.

As what is shown in Table 4, Figure 4 indicates visually the relationship between used stacks and allowed rehandle. It's not hard to draw the conclusion that the more allowed rehandle, the less used stacks. So it intuitively reflect the meaning of this research topic and it show the feasibility of our algorithm. Comparing with zero rehandle, some rehandle are allowed to reduce the partial stacks and improve the utilization of

Figure 4: Relationship between allowed rehandle and used stacks



space.

Figure 5: Relationship between containers and used stacks

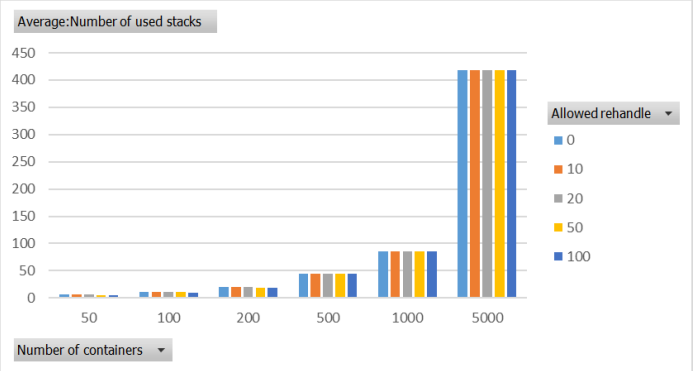


Figure 5 shows us the relationship between containers and used stacks. We can conclude that the more containers are, the more stacks will be used, which is undoubted in practice.

Figure 6: Relationship between ports and used stacks

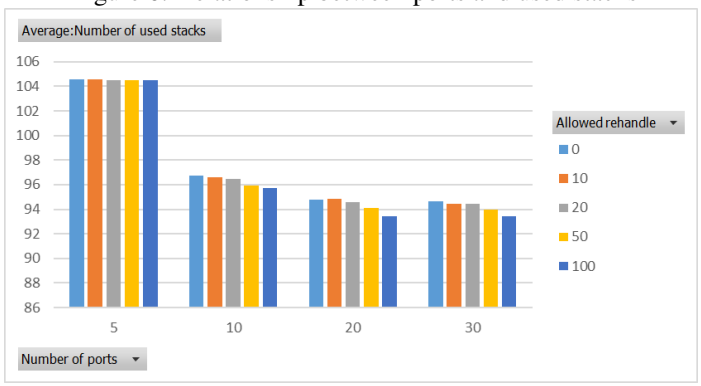


Figure 6 provides us the relationship between ports and used stack. In general, the more ports are among the shipping, the more stacks will be used. Another related factor to be taken into consideration is the sparse distribution situation of containers in each port.

Figure 7: Relationship between limit height and used stacks

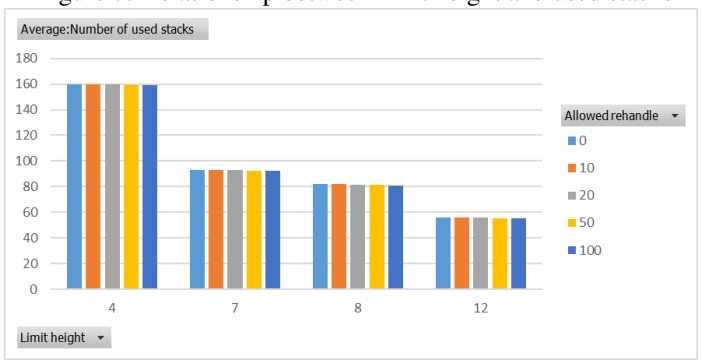
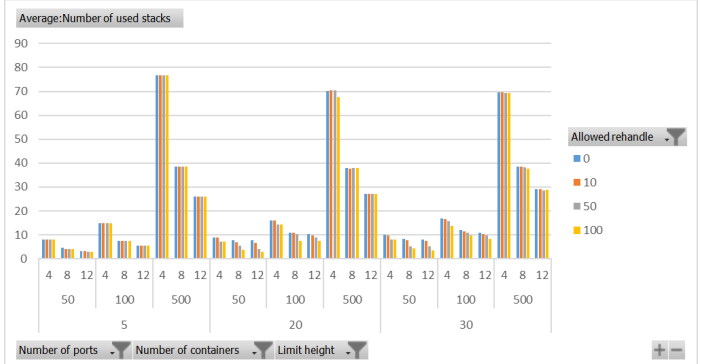


Figure 7 shows us that the greater the number of limit height is, the less stacks will be used, which is definitely right.

Figure 8 shows us that the comprehensive impact on the result we work out for all the basic factors considered in this paper. And we can choose the factor we want to analyze and it is convenient for us to work out how different factor influences our final result. For the convenience of observation, we only give the result with filtered numbers of each factor.

Figure 8: Relationship between comprehensive factors and used stacks



6. Conclusion

The stowage stack minimization problem with K-rehandle constraint (SSMP-KR) is aimed to find out a stowage plan that fewest stacks on a containership are required to accommodate given containers throughout a voyage by subject to K-rehandles.

In this paper, we analyze the structure of this two-dimensional bin-packing optimization problem and its relationship with the SSMP-ZR problem proposed in Wang et al. (2014).

We can regard the stowage stack minimization problem with zero rehandle as an particular case when we choose the value of K as zero. Hence, the problem proposed in this paper is a generalized and extended problem considering the existence of K-rehandle.

On the one hand, the stowage planning problems in cyclic navigation path is converted into linear navigation path using the specific method. On the other hand, the result is equivalent to each other whether we convert or not.

In this paper, we presented the integer programming model for the SSMP-ZR and presented a simple IP model for the SSMP-KR.

A heuristic approach is proposed to construct near-optimal solutions to the SSMP-KR problem in a very short computational time. Since there are different status in the process of handling containers, different methods in our algorithm are thought to cope with the corresponding conditions.

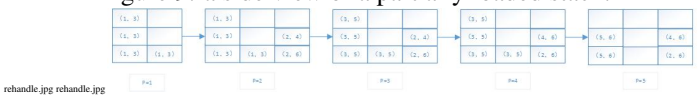
The experimental results show that our heuristic approaches generate very promising solutions on a variety of instances. Comparing with the stowage stack minimum problem with zero constraint, our problem with K rehandle constraint acts as a buffer and offers great flexibility for the actual problem. The results shows the problem we put forward is of practical significance and it can improve the utilization ratio of vessels with certain flexibility. Our algorithm uses greedy rules to find the best slot for each container, hence the results may be the near-optimal rather than global optimal. As we all know, it is difficult to get the global optimal for this kind of problem. Applying this algorithm into the real shipping management can enhance the hull space utilization and reduce the unnecessary spending in the case of meeting other constraints. The novelty and practicality show the importance of this paper and it can give a reference to the future study.

There is a simple example to elaborate why our algorithm can improve the utilization of containership. Assuming that there are thirteen containers to be transported along the voyage with six ports. We let C(i,j) denote index container whose origin port is i and destination port j. The following is the way to express those containers: four C(1,3), one C(2,4), one C(2,6), four C(3,5), one C(4,6), two C(5,6). If they are loaded without any rehandle, the loading sequence and layout will be the following figure:

If they are loaded with rehandles, the loading sequence and layout will be the following figure:

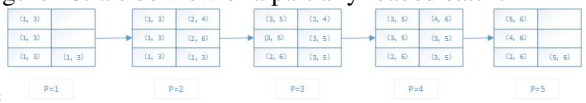
From the above figures, we can draw the conclusion that allowing a certain number of rehandles will

Figure 9: a side view of a partially loaded stack.



rehandle.jpg rehandle.jpg

Figure 10: a side view of a partially loaded stack.



rehandle.jpg rehandle.jpg

reduce the number of stacks used to store containers in a vessel compared with zero rehandle.

Furthermore, the rehandles are unavoidable in the actual port operation, which means the problem we propose in this paper does make some sense and the solution we work out using our heuristic algorithm can provide a preliminary assessment for forwarders to arrange linear shipping company for transportation.

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