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Optimization approach to depot location and trip selection in one-way carsharing systems

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ABSTRACT

In this paper, we present an optimization approach to depot location in one-way carsharing systems where vehicle stock imbalance issues are addressed under three trip selection schemes. The approach is based on mixed-integer programming models whose objective is to maximize the profits of a carsharing organization considering all the revenues and costs involved. The practical usefulness of the approach is illustrated with a case study involving the municipality of Lisbon, Portugal. The results we have obtained from this study provided a clear insight into the impact of depot location and trip selection schemes on the profitability of such systems.

1. Introduction

Carsharing systems are an alternative to private vehicle ownership. Instead of owning one or more vehicles, a household or business accesses a fleet of shared-use automobiles, benefiting from choosing the one that best fits its needs for a specific purpose (Shaheen et al., 1999).

The first experiments with carsharing systems were made in Europe, in the 1940s, mainly motivated by economic reasons. One of the best known is the Sefage cooperative, established in Zurich, Switzerland, in 1948 (Shaheen and Cohen, 2007). In the United States, these systems appeared much later, in 1983, within the Mobility Enterprise program. In contrast to Europe early users, US first adopters were motivated more by convenience than by affordability, possibly due to the lower costs of driving in the US (Lane, 2005). More recently there have been carsharing initiatives in Asia, primarily in Japan and Singapore. The main focus in Japan has been on business use, while household use has been dominant in Singapore. "This is likely because of limited vehicle licensing and high car-ownership costs in Singapore" (Barth et al., 2006). Today, carsharing systems exist in 18 countries and show enormous growth potential (Shaheen and Cohen, 2007).

Carsharing systems can be classified with respect to organization goals, geographic scope, depot location, and trip configuration. As regards the organization goals, some systems are run by volunteers and are non-profit making, while others are commercial ventures run by international companies. The geographic scope may span from a community-level operation with only one or two vehicles, to national organizations with many thousands of members in several major cities. In relation to depot location, there are systems with depots placed only at transit stations mainly to serve commuter trips, designated as station-car systems, and systems with depots scattered around a city independently of transit stations (Shaheen, 2001; Barth and Shaheen, 2002). Finally, with respect to trip configuration, it is usual to distinguish between one-way systems and round-trip (or two-way) systems. The former are by far the most common, being in particular adopted by the three largest organizations that account for 94% of the North America carsharing membership (Shaheen et al., 2006).

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The economic success of carsharing systems has been often associated with a number of city characteristics (Celsor and Millard-Ball, 2007; Nobis, 2006):

– Parking pressure: places where parking is scarce and/or expensive make carsharing a more attractive option.
– High density: high population density brings a large customer basis to carsharing.
– Mixed uses: business carsharing uses during the workday can be paired with residential uses in the evenings.

When these characteristics are not present, carsharing initiatives may have to be backed with support from public entities. Case studies illustrate a wide variety of ways these entities can help (Shaheen et al., 2004). Still, such help has been quite rare, in the main due to a lack of understanding of the documented positive impact that carsharing can have on mobility (Enoch and Taylor, 2006; Lane, 2005; Litman, 2000; Schuster et al., 2005), not only in large cities but also in small- and mid-size ones (Faghri et al., 2008). In a recent study by Sioui et al. (2010) describing a survey of the members of Communauto, a Montreal carsharing organization, it was concluded that a person who does not own a vehicle and resorts frequently to carsharing (5 days a week) makes 30% less trips than a person who owns a vehicle.

Despite the success of carsharing systems, there are still progresses to be made. In particular, these progresses can be accomplished along directions that were identified by the largest carsharing organizations in the US and Canada already in 2003: "facilitating more efficient land use (e.g., reducing the number of parking spaces needed); providing cost savings through pay per use (...); increasing mobility options and connectivity among transportation modes; and reducing pollution, if vehicles link to alternative travel modes (...) or if the fleet consists of clean-fuel vehicles" (Shaheen et al., 2004).

One-way carsharing is a very obvious way of allowing customers pay per use and a long time request from clients. This is well expressed in the following opinion of a user: "My only real complaint about carsharing is that it has one major, built-in, self-limiting inefficiency: because cars must be returned to the spot where you pick them up, you can't take one-way trips. That means that even if what I really want to do is drive from my house to the store and to a friend's party, from which I'll get a ride or return by bus, I'm forced to drive to the party and drive home. (This also automatically makes me the designated driver.)" (Barnett, 2010). Notwithstanding this, one-way carsharing has not been a priority for the major carsharing organizations in the world. In a survey reported in 2006, vendors who sell technology to carsharing programs in the US (e.g., reservations and billing, vehicle-access systems) believed "that carsharing operators are not likely to introduce innovative features (e.g., one-way rentals, ridesharing) because of added management complexities" (Shaheen et al., 2006). These complexities were responsible for the failure of Honda's Diracc system in Singapore, one of the best-known one-way carsharing experiments in the world, after 6 years of operation. The system offered one-way trips between any one of 21 depots with no reservation required and no need to pre-define a return time. The service had 2500 members with access to 100 vehicles. As membership grew, the system was not able to maintain the quality of service offered initially because everybody expected cars to be available at all times. But, in reality, this could not be guaranteed due to one-way trips leaving the system with significant imbalance in vehicle stocks (The Straits Times, 2008).

In this paper, we present an optimization approach to depot location in one-way carsharing systems where vehicle stock imbalance issues are addressed under three trip selection schemes. The objective is the maximization of the profits of a carsharing organization considering all the revenues and costs involved. To our best knowledge, vehicle stock imbalance issues have been dealt with from an optimization perspective only in two very recent papers. Fan et al. (2008) proposed a multistage stochastic mixed-integer programming model for dynamic vehicle allocation and trip selection where vehicle relocations are taken into account. Kek et al. (2009) went into more detail on the relocation operations, focusing on the problem of managing a team of people for moving the vehicles. The latter authors applied both optimization and simulation techniques for tuning relocation operations using data from the aforementioned Diracc system in Singapore. None of these papers, however, attend to the implications of depot location and different trip selection schemes upon the profit of a one-way car-sharing organization.

The remainder of the paper is organized in the following way. In the next section we provide a detailed description of the problem we want to address by means of an optimization approach. Afterward, we present three mixed-integer programming (MIP) models that have in common the depot location feature but vary with respect to the trip selection scheme. The differences between them are illustrated through a small-scale example. The models, which constitute the main contribution of the paper to the literature, are then tested on a case study involving the city of Lisbon. The paper finishes with the main conclusions withdrawn from our work and ideas for future research.

2. Problem description

The overarching problem we address in this paper is how to select sites for locating depots in order to maximize the profits of a one-way carsharing organization. The revenues are made by renting the vehicles against some price rate. The expenses are the cost of maintaining the vehicles and the depots, the cost of the depreciation of vehicles, and the cost of relocating vehicles at the end of the day. Relocation operations are not considered throughout the day, thus they are not seen as a means for providing vehicles to clients. This is significantly different from the approaches followed in previous research, where a great emphasis was put on these operations (Barth and Todd, 1999; Kek et al., 2009). Our view is more focused on selecting depot locations that 'naturally' circumvent the occurrence of vehicle stock imbalance situations. Vehicle relocation operations are only considered at the end of the day because vehicles need to be available for the next day at the original positions. These operations are very expensive and there is no empirical evidence, so far, that they are the solution to make one-way carsharing systems economically viable.

With respect to the selection of trips (and the allocation of vehicles to these trips), we have considered three schemes of running the one-way carsharing system.

The first scheme assumes that the carsharing organization has total control over the selection of trips from a list of requests made by clients (S1 – controlled service). In this scheme, the organization is free to accept or reject trips in the period they are requested according to the profit-maximization objective (but cannot delay them for a later period). This means that a vehicle is only allocated to a specific trip if this is advantageous from the profit point of view. If this is not the case, the trip is not satisfied even when there are vehicles available in the pick-up depot. Such scheme can only be feasible if there is a central request management service which is able to compute profit and give instructions on which vehicles should be reserved for later trips. However, this may lead to great discontent as some clients may find themselves in a situation where they know of the existence of vehicles at a desired pick-up depot and still have their trips rejected. In addition to this, there is no guarantee that a decision to accept or reject a trip will lead to the maximization of expected profits given that demand is not deterministic.

The second scheme assumes that all trips requested by clients will be accepted (S2 – full service). Note that this is not the same as to say that all potential carsharing trips in a city will be met. Indeed, this scheme only guarantees that trips between existing depots will be satisfied, not trips which are missing a depot near their origin and/or destination.

Finally, the third scheme is a hybrid one, in which there is no obligation of satisfying all trips between existing depots, but rather, they can only be rejected if there are no vehicles available at the pick-up depot (S3 – conditional service). This means that, if there is a vehicle in a depot and a request is made for a trip starting at this depot, there is no possibility of rejecting that trip. This certainly makes sense in a situation where clients can act as walk-ins asking for a vehicle instantly at a depot.

3. Optimization models

In this section, we present the formulation of three MIP models aimed at determining the best number, location, and size for the depots of a one-way carsharing system, each one corresponding to a trip selection scheme described in the previous section.

3.1. Model S1 – controlled service

This model applies when trips can be freely selected from the total demand through a central request management service with the objective of maximizing profit.

Consider the following notation (sets, decision variables, and parameters are introduced in the order they appear in the model):

Sets

- $\boldsymbol{N} = \{1, \ldots, i \ldots N\}$: set of candidate sites for the location of depots.
- $\boldsymbol{T} = \{1, \ldots, t \ldots T\}$: set of time steps in the operation period (e.g., one working day).
- $\boldsymbol{X} = \{1_1, \ldots, i_{t-1}, i_t, i_{t+1}, \ldots, N_T\}$ where i_t represents candidate site i at time step t: set of the nodes of a time–space network combining the N candidate sites with the T time steps.
- $\boldsymbol{A}_1 = \left\{\ldots, a_1\left(i_t, i_t + \delta_{ij}^t\right), \ldots\right\}, i_t \in \boldsymbol{X}$: set of arcs that represent the movement of vehicles between depots i and $j, \forall i,j \in \boldsymbol{N}, i \neq j$, between time steps t and $t + \delta_{ij}^t$, where δ_{ij}^t is the travel time (in number of time steps) between depots i and j when the trip starts at time step t (this allows considering the effect of traffic congestion at certain periods during the day).
- $\boldsymbol{A}_2 = \{\ldots, a_2(i_t, i_{t+1}), \ldots\}, i_t \in \boldsymbol{X}$: set of arcs that represent vehicles stocked in depot $i, \forall i \in \boldsymbol{N}$, from time step t to time step $t + 1$.
- $\boldsymbol{A}_3 = \{\ldots, a_3(i_T, i_1), \ldots\}, i_T \in \boldsymbol{X}$: set of arcs that represent relocation operations from depot i to depot $j, \forall i,j \in \boldsymbol{N}, i \neq j$, at the end of the operation period (time step T).

Decision variables

- $D_{i_t j_{t+\delta_{ij}}}$: number of vehicles used between depots i and j from time step t to $t + \delta_{ij}, \forall (i_t, i_{t+t_{ij}}) \in \boldsymbol{A_1}$;
- R_{ij}: number of vehicles relocated between depot i and j after the operation period, $\forall (i,j) \in \boldsymbol{A_3}$;
- Z_i: size of depot $i, \forall i \in \boldsymbol{N}$;
- V_{i_t}: number of available vehicles at depot i in time step $t, \forall i_t \in \boldsymbol{X}$;
- $S_{i_t i_{t+1}}$: number of vehicles stocked at each depot i from time step t to $t + 1, \forall (i_t, i_{t+1}) \in \boldsymbol{A_2}$;
- $Y_i = 1$ if a depot is located at candidate site $i, \forall i \in \boldsymbol{N}$, otherwise $Y_i = 0$;

Parameters

- P: price rate per time step driven;

– C_{m1}: cost of maintaining one vehicle per time step driven;
– C_r: cost of relocating a vehicle per time step driven;
– δ_{ij}^t: travel time, in time steps, between depots i and j when departure time is $t, \forall i_t \in \boldsymbol{X}, j \in \boldsymbol{N}$;
– C_{m2}: cost of maintaining one parking space per day;
– C_v: cost of the depreciation of one vehicle per day;
– M: large number;
– $Q_{i_t j}^P$: demand of vehicles between depots i and j when departure time is t and the price rate is $P, \forall i_t \in \boldsymbol{X}, j \in \boldsymbol{N}$;
– N_{max}: maximum number of depots to create at the N candidate sites;
– Q_{min}: minimum share of demand to satisfy (in percentage of the total demand).

Using the notation above, the model corresponding to trip selection scheme S1 can be formulated as follows:

$$\text{Max } \Pi = (P - C_{m1}) \times \sum_{i_t j_{t+\delta_{ij}^t} \in \mathbf{A}_1} D_{i_t j_{t+\delta_{ij}^t}} - C_r \sum_{ij \in \mathbf{A}_3} \delta_{ij}^T R_{ij} - C_{m2} \sum_{i \in \boldsymbol{N}} Z_i - C_v \sum_{i \in \boldsymbol{N}} V_{i_1} \tag{1}$$

subject to,

$$V_{i_t} = V_{i_{t-1}} - \sum_{j_t \in \mathbf{X}} D_{i_{t-1} j_{t-1+\delta_{ij}^t}} + \sum_{j_t \in \mathbf{X}} D_{j_{t-\delta_{ji}^t} i_t} \quad \forall i_t \in \mathbf{X} \tag{2}$$

$$S_{i_t i_{t+1}} = V_{i_t} - \sum_{j_t \in \mathbf{X}} D_{i_t j_{t+\delta_{ij}^t}} \quad \forall i_t \in \mathbf{X} \tag{3}$$

$$V_{i_1} = V_{i_T} + \sum_{j \in \boldsymbol{N}} R_{ji} - \sum_{j \in \boldsymbol{N}} R_{ij} \quad \forall i \in \boldsymbol{N} \tag{4}$$

$$\sum_{j \in \boldsymbol{N}} R_{ij} \leqslant V_{i_T} \quad \forall i \in \boldsymbol{N} \tag{5}$$

$$Z_i \geqslant V_{i_t} \quad \forall i_t \in \mathbf{X} \tag{6}$$

$$V_{i_t} \leqslant M \times Y_i \quad \forall i_t \in V \tag{7}$$

$$Y_i \leqslant Z_i \quad \forall i \in \mathbf{N} \tag{8}$$

$$D_{i_t j_{t+\delta_{ij}}} \leqslant Q_{i_t j}^P \quad \forall i_t, j_t \in \mathbf{X} \tag{9}$$

$$\sum_{i \in N} Y_i \leqslant N_{\max} \tag{10}$$

$$\sum_{i_t j_{t+\delta_{ij}^t} \in \mathbf{A}_1} D_{i_t j_{t+\delta_{ij}^t}} \Bigg/ \sum_{i_t \in \mathbf{X}, j \in \mathbf{N}} Q_{i_t j}^P \geqslant Q_{\min} \tag{11}$$

$$D_{i_t j_{t+\delta_{ij}^t}} \geqslant 0 \quad \forall \left(i_t, j_{t+\delta_{ij}^t}\right) \in \boldsymbol{A}_1 \tag{12}$$

$$R_{ij} \geqslant 0 \quad \forall (i,j) \in \boldsymbol{A}_3 \tag{13}$$

$$Z_i \geqslant 0 \quad \forall i \in \mathbf{N} \tag{14}$$

$$V_{i_t} \geqslant 0 \quad \forall i_t \in \mathbf{X} \tag{15}$$

$$S_{i_t i_{t+1}} \geqslant 0 \quad \forall (i_t, i_{t+1}) \in \boldsymbol{A}_2 \tag{16}$$

$$Y_i = (0, 1) \quad \forall i \in \mathbf{N} \tag{17}$$

The objective function (1) of this mixed-integer optimization model maximizes the total daily profit (Π) of the one-way car-sharing organization, taking into consideration the revenues made from the trips paid by clients, the vehicle maintenance costs, the vehicle relocation costs, the depot maintenance costs, and the vehicle depreciation costs. No cost was considered for rejected demand as this is very difficult to quantify in monetary units, thus not contributing for a better understanding of the financial equilibrium of the one-way carsharing systems. Rejected demand is an output of the model that can be seen a performance indicator.

The model comprises ten sets of constraints. Constraints (2) ensure the conservation of vehicle flows at each node (i_t) of the time–space network and updates the number of vehicles available at each depot i at time step t taken into account the number of vehicles at time step $t-1$. Constraints (3) compute the vehicle stocks available at each depot i from t to $t+1$. Constraints (4) guarantee that relocation operations will make the number of vehicles at each depot i in time step 1 always the same, thus the same operations can be reproduced each and every day ("permanent regime" constraints). Constraints (5) impose that the number of vehicles being relocated from the depot located at site i to other depots must not exceed the number of vehicles available at that depot at time step T (the end of the day). Constraints (6) assure that the size of the depot of site i must be greater than the number of vehicles present there at each time step t. In practice, size will not exceed the highest value of V_{i_t} for the operation period because this would penalize the objective function through component: $C_{m2}\sum_{i\in N} Z_i$. Constraints (7) prevent vehicles to be stocked at sites where there are no depots while Constraints (8) specify that the size of any depot is at least one parking space (one vehicle). Constraints (9) ensure that the number of accepted trips between depots i and j at time step t will not exceed demand. Constraint (10) defines the maximum number of depots. If $N_{\max} = N$ then it is possible to locate a depot in each candidate site. Constraint (11) assures that the satisfied demand is above the minimum limit $Q_{\min}$. When $Q_{\min} = 1$ all demand must be attended.

Expressions (12)–(17) set the domain for the decision variables.

3.2. Model S2 – full service

The formulation of model S2, applicable when all demand between existing depots must be satisfied, is obtained from the previous one just by adding the following constraints:

$$D_{i_t j_{t+\delta^t_{ij}}} \geqslant Q^P_{i_t j} + M \times (Y_i + Y_j - 2) \quad \forall \left(i_t, j_{t+\delta^t_{ij}}\right) \in \boldsymbol{A_1} \tag{18}$$

According to these constraints, the trips accepted between two existing depots i and j ($y_i = y_j = 1$) will be at least the total demand, as ($y_i + y_j - 2$) will be equal to zero. In practice, this means that, for these depots, all demand will be satisfied, because Constraints (9) will obligate $D_{i_t j_{t+\delta_{ij}}} = Q^P_{i_t j}$.

3.3. Model S3 – conditional service

The formulation of model S3 is more complex, as it involves replacing Constraints (9) of model S1 with three new constraint sets and defining a new type of decision variables: $x_{i_t} = 1$ if the number of vehicles available in depot i at time step t is greater than the number of vehicles which are requested, otherwise $x_{i_t} = 0, \forall i_t \in \boldsymbol{X}$.

The new constraint sets are as follows:

$$x_{i_t} \leqslant \left(V_{i_t} + \sum_{j\in\mathbf{N}} Q_{i_t j} - \sum_{j\in\mathbf{N}} Q_{i_t j} \times Y_j\right) \Big/ \left(\sum_{j\in\mathbf{N}} Q_{i_t j}\right) \quad \forall i_t \in \mathbf{X}' \tag{19}$$

$$S_{i_t i_{t+1}} \leqslant M \times x_{i_t} \quad \forall i_t \in \mathbf{X}' \tag{20}$$

$$\sum_{j\in\mathbf{N}} D_{i_t j_{t+\delta^t_{ij}}} \geqslant \sum_{j\in\mathbf{N}} Q_{i_t j} \times (x_{i_t} - 1) + \sum_{j\in\mathbf{N}} Q_{i_t j} \times Y_j \quad \forall i_t \in \mathbf{X}' \tag{21}$$

where $\mathbf{X}' = \left\{(i_t, j) \in \mathbf{X} : \sum_{j\in\mathbf{N}} Q_{i_t j} \geqslant 0\right\}$.

Constraints (19) assure that when the total number of trips with origin in depot i is greater than the number of vehicles available there, V_{i_t}, the binary variable x_{i_t} is zero and when the opposite is true the variable can be either 1 or 0. Constraints (20) force the stock of vehicles $S_{i_t i_{t+1}}$ at depot i between time steps t and $t+1$ to be zero when demand is greater than the number of vehicles available there (that is, when $x_{i_t} = 0$). This way a trip will never be refused when there are vehicles available at the depot. When variable x_{i_t} is free to take value 0 or 1, meaning that V_{i_t} is greater than demand (according to Constraints (19)), x_{i_t} automatically becomes equal to 1. Indeed, this is the only way stock $S_{i_t i_{t+1}}$ is positive, which is mandatory given that there are more vehicles than trips and the conservation of vehicle flows must hold (2). Finally, Constraints (21) compel the total number of accepted trips in depot i at time step t to be equal or greater than the demand when x_{i_t} is 1. When x_{i_t} is 0, $\sum_{j\in N} D_{i_t j_{t+\delta^t_{ij}}}$ has to be greater than a negative number (that is, Constraints (21) become irrelevant).

Notice that this model is valid for all values of $V_{i_t} \forall i_t \in \mathbf{X}'$: when this value is zero for node i_t, x_{i_t} automatically becomes zero due to Constraints (19), which by their turn obligate $S_{i_t i_{t+1}}$ to be zero given that Constraints (20) must hold. Trips with origin in depot i in time step t will therefore be zero due to the vehicle flow conservation Constraints (2).

3.4. Model comparison

We will use a small scale example to illustrate the kind of results that can be obtained through the application of the three models (S1, S2, and S3). In the example, we consider trips between three possible locations for depots, 1, 2, and 3, and an operation period of 10 time steps. These trips were designed in such a way that some of them are self-balanced, which means that vehicles arriving at a depot will later be useful for other trips, and other are odd trips which do not have this property. In Fig. 1 we may observe the trips created for this example embedded in the time–space network. The trip duration between depot 2 and the other two depots is 2 time steps, while between depots 1 and 3 is 1 time step.

Solutions depend not just on the models, but also on the parameters. In order to test the models avoiding excessive (and unnecessary) difficulties, only the price rate per time step, P, the cost of maintaining one parking space per day, C_{m2}, and the cost of the depreciation of one vehicle per day, C_v, were considered to be different from zero. The values used for these parameters were the following: $P = 20$, $C_{m2} = 5$, and $C_v = 20$.

Graphical results for the accepted trips in the optimum solution $(D_{i_t j_{t+\delta_{ij}^t}})$ and the number of vehicles present at each depot in each time step (V_{i_t}) are displayed in Fig. 2. Results are according to expected, given the formulations. In Fig. 2a we see that model S1 has placed depots at the three possible locations. Since trips could be freely chosen to maximize profit, no trips were accepted between depots 1 and 3 at time step $t = 4$. The reason was because these are very unbalanced trips, which could only be satisfied with a larger fleet of vehicles that would stay idle for the rest of the operation period. The value of the objective function for this model was 300. In Fig. 2b, we show that model S2 also placed depots in all possible locations. In this case the objective function decreased to 210 since trips between existing depots cannot be rejected. Finally, in Fig. 2c, we observe the results from applying model S3, where trips originating at some depot must be satisfied when there are vehicles at the depot. In this case, only eight of the 10 trips between depots 1 and 3 were satisfied. The two additional vehicles needed to cope with these trips are not made available because this would be non-optimal given the time they would be inactive. The value of the objective function for model S3 is 250, greater than the value for model S2 but lower than the value for model S1.

4. Lisbon case study

Lisbon is the capital city of Portugal and the center of the Lisbon Metropolitan Area (LMA), which has 18 municipalities, an area of 2962.6 km^2, and a population of 2.8 million (roughly 25% of the Portuguese total). The municipality of Lisbon has a population of 565,000, and is, by far, the most relevant trip generator in the LMA.

The municipality of Lisbon has been facing the classic mobility problems of the major urban areas around the world: traffic congestion and parking shortage, aggravated by the organic genesis of part of the city. ‘‘In the last decade of the 20th century, there was an increase of 60% in the total number of vehicles entering the city diurnally. This increase was due to the combined effect of different factors such as the expansion of the road network, the rapid increase in car ownership, higher family incomes and the proliferation of urban expansion areas on the metropolitan area periphery without public transport’’ (Oliveira and Pinho, 2010). The result of this growing demand has been an increasing congestion in the main road network of Lisbon during morning and evening peaks.

The improvements in the public transport network, namely the subway extensions to the north outskirts, the suburban train to the south bank of the river Tagus, and the improved trains and regularity of service in the main railway suburban lines (municipalities of Sintra and Cascais), were not able to restrain the growth of the private car modal share for commuter trips. Thus other alternatives should be considered in the scope of an efficient Transportation Demand Management (TDM) program. Carsharing is one of the available TDM measures, and Lisbon would benefit from having such system running in order to decrease private car dependency for morning and evening commutes. This makes the municipality of Lisbon an interesting test bed for applying the optimization models introduced above and assessing their practical usefulness in a real world setting.

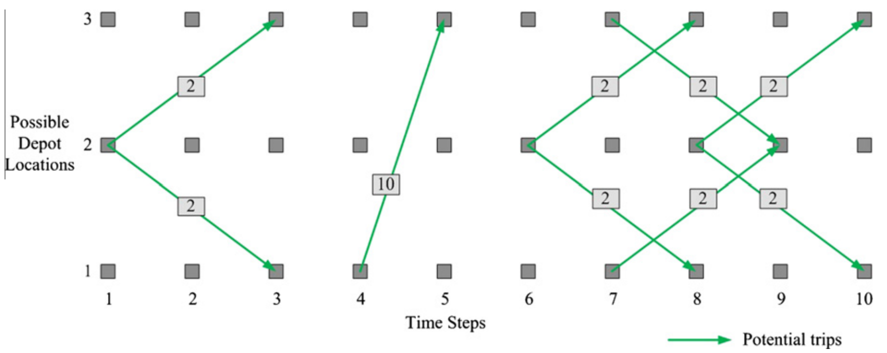

Fig. 1. Data for the small scale example.

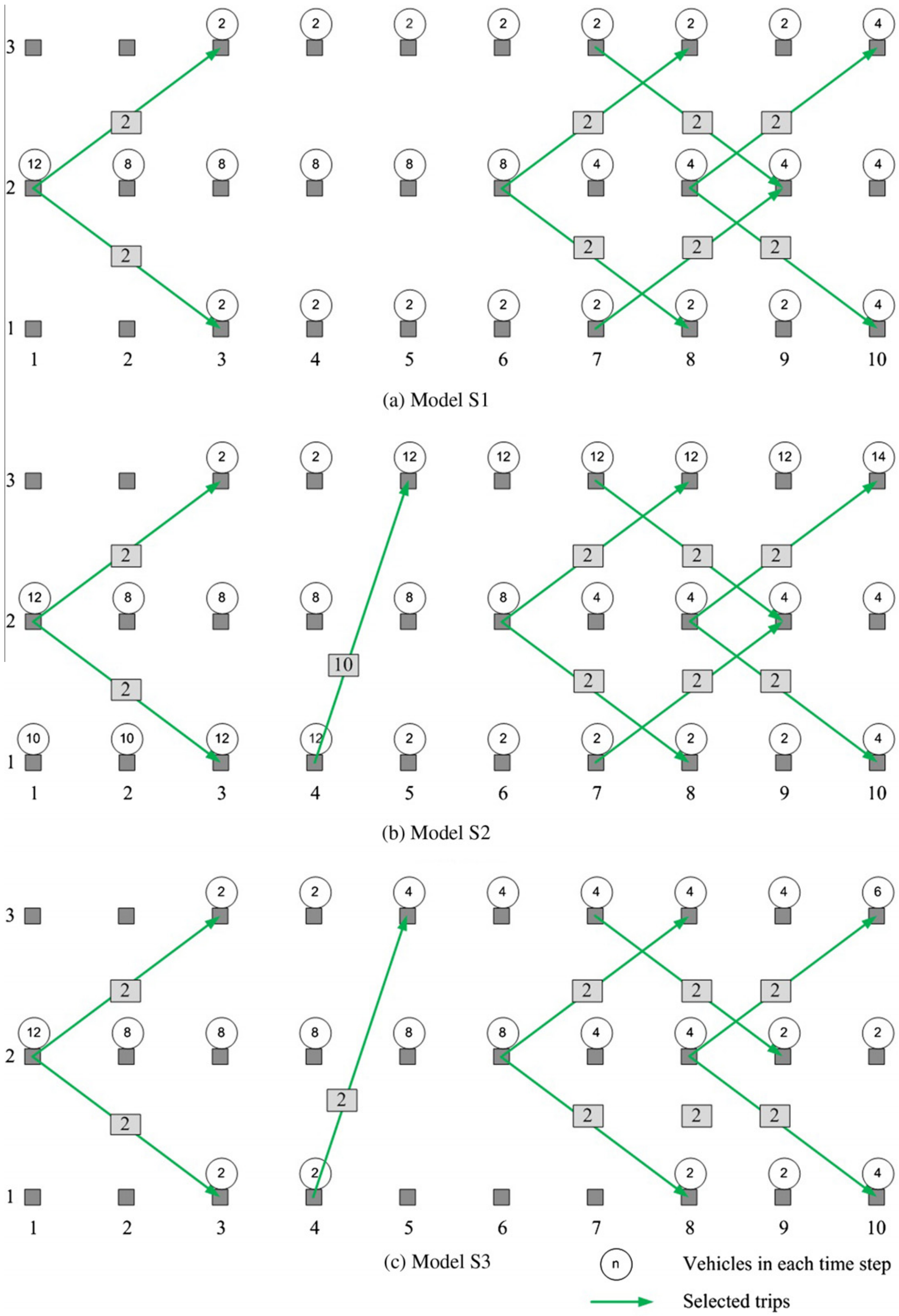

Fig. 2. Results for the small scale example.

4.1. Study data

In order to apply the models for studying the one-way carsharing problem in Lisbon, we first needed to obtain several types of data, namely:

– Potential trip matrix.
– Candidate depot locations.
– Driving and walking travel times.
– Costs of running the system.

The trip matrix was provided by a consulting company (TIS.pt) that conducted a geo-coded survey for the LMA in the mid-1990s when the subway company was deciding where and when to expand the existing system. That survey contains very detailed information on the mobility patterns of Lisbon at that time. Despite some changes in the economic structure, land use, and transport conditions in the municipality, the survey still provides a good representation of mobility patterns in Lisbon.

Because our objective was to study the effect of carsharing depot location in a system implemented only in Lisbon, the matrix was filtered just to encompass trips with origin and destination within the municipality limits. All trips in a whole daily tour were recorded in that survey. Thus, we are considering the trips of all people who, despite not living in Lisbon, have made a trip inside this municipality. This could be the case of a car trip for providing some service to a company.

Other filtering features were:

– Trip done by car.
– Euclidean distance between origin and destination greater than 1 km.
– Trip duration greater than 10 min.
– Beginning and end of a trip after 6 a.m. and before 12 p.m.
– Age between 18 and 55 years old. These should be more inclined to carsharing according to previous research (Burkhardt and Millard-Ball, 2006).

The filtering resulted in 1776 trips, which, having in consideration the survey sample coefficients, represent 39,389 trips made inside Lisbon obeying the above conditions. Thus by considering the 1776 survey trips we assumed that carsharing would not surpass a 4% mode split of all trips starting and ending in Lisbon.

The candidate depot locations were defined simply by considering a grid of squared cells with 1000 m side over Lisbon and associating one location with the center of each cell (Fig. 3). This certainly is a rough simplification. For a real-world application, a detailed study of appropriate locations would be necessary.

All cells with no trip origins or destinations were eliminated from the original set, which resulted in a total of 75 possible depot locations. This is a key figure with respect to model size, as most of the decision variables are indexed to the set of candidate locations.

Driving travel times were computed using the transportation modeling software VISUM (PTV), as we had available a detailed map of the Lisbon road network and the mobility survey for the LMA. This was necessary in order to compute realistic travel times between all candidate depot locations. To take into account variations during the day, different travel times were computed for the morning peak period (8:00–10:00 a.m.), the evening peak period (6:00–8:00 p.m.), and the off-peak period. The travel times were then expressed in time steps of 10 min. Since we have dealt with an operation period of 18 h (between 6:00 a.m. and 12:00 p.m.), 108 time steps were considered.

Walking travel times were also computed to associate each trip origin or destination with the closest carsharing depot, i.e., the squared cells were not used as pick-up or drop-off areas, they worked only as references for placing candidate depot locations. Thus it is possible, and this actually happened, to assign a trip origin or destination inside a certain cell to a depot located in a different cell due to network effects.

The cost values considered were as follows:

– C_{m1} (cost of maintaining a vehicle): 0.07 euros per 10 min.
– C_r (cost of relocating a vehicle): 12 euros per hour, the average hourly wage in Portugal.
– C_{m2} (cost of maintaining a parking space): 5 euros per day, the parking fee in a low price area in Lisbon.
– C_v (cost of the depreciation of one vehicle): 17 euros per day, calculated for a city car.

The price rate, P, was considered as a sensitivity analysis parameter, together with $N_{\max}$ and $Q_{\min}$ that is, the maximum number of depots to locate and the minimum percentage of demand to satisfy, respectively. These three parameters define the experiments ran to understand the influence of depot locations and trip selection schemes upon the profitability of the carsharing system.

For each model S1, S2, and S3, first we varied P between 2, 3 and 4 euros per 10 min (the time step), assuming demand to be inelastic with the price rate, as we had no information on how carsharing clients would react to its variations (and did not want to guess on this issue). Then, $N_{\max}$ and $Q_{\min}$ define three scenarios: one in which only 10 depots can be located

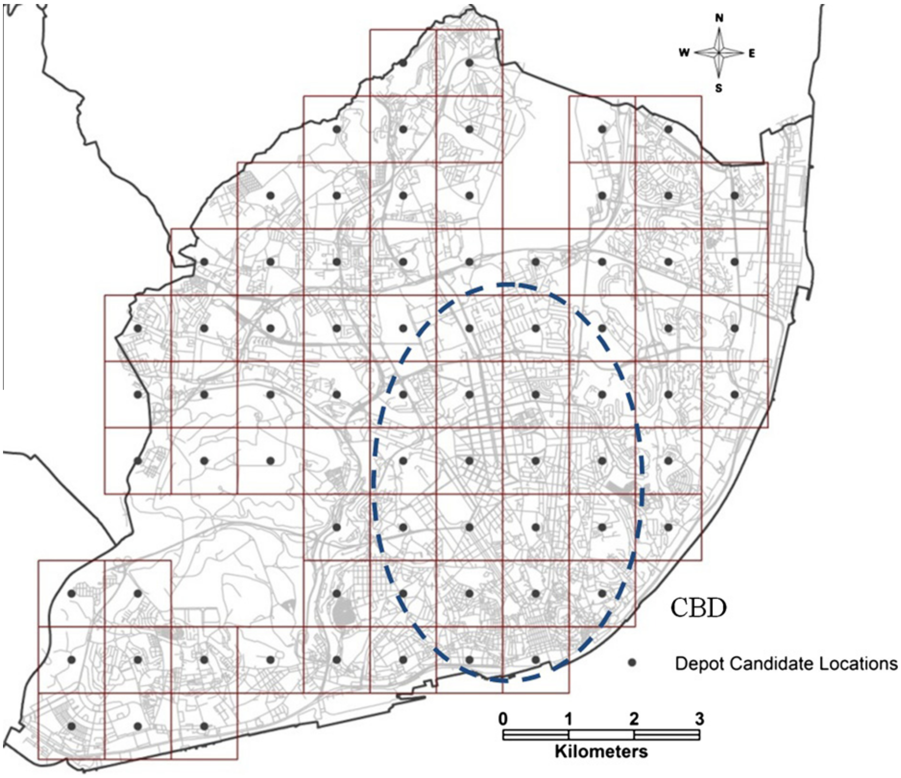

Fig. 3. Depot candidate locations.

(N_{max} = 10), leaving free the minimum demand to meet; another where there is no maximum number of depots to locate nor minimum percentage of demand to satisfy, referred to as "unconstrained scenario"; and a third one, designated as "100% demand scenario", where all demand in the sample, 1776 trips, must be attended, i.e., Q_{min} = 1 (100%).

4.2. Model solving

The optimization models were run in an i7 processor @ 2.67 GHz, 6 Gb RAM computer under a Windows 7 64 bit operation system using Xpress, an optimization tool that uses advanced branch-and-cut algorithms for solving MIP problems (FICO, 2008).

In Table 1 we may observe several model solving indicators. We display the value of the objective function (the daily profit obtained by the carsharing organization), the best bound, the gap between both, and the time it took to compute the optimal solution. It can be seen that, for most scenarios, the optimal solution was found (0% gap), often very quickly. However, for model S3 only the 100% demand scenario converged to a 0% gap.

The decision to stop searching for an optimal solution was made when the gap stabilized in the same value. When the search stopped for model S3, the gap was still greater than 100% in one scenario (P = 2, N_{max} = 10, Q_{min} = 0). This is due to the complexity brought by the additional integer variables and constraints of this model. Nevertheless we may observe that the upper bound in that scenario is lower than the profit for the same scenario under model S1, which allows concluding that the profit will be lower for model S3.

4.3. Study results

The results obtained for the three trip selection schemes are presented in Table 2. The table contains the values for several performance indicators in order to allow discussing their implications.

The analysis of the table reveals that the scheme with higher profits when considering the same price rate charged to the client is the one where the carsharing organization has full control over trip selection (scheme S1). This was expected since it is the scheme with the higher degree of freedom. That is not to say that this scheme always leads to positive profits: when the total demand is coped with, it results in exactly the same losses as the other two schemes, as one may notice in Table 2 by comparing the results obtained when all trips must be accepted (for the same price rate). The value of the objective function and remaining indicators are in fact equal across the three models, as there is no flexibility when trips cannot be

Table 1
Model solving indicators.

Model	Scenarios			Objective function (Euros)	Best bound (Euros)	Gap (%)	Time (min)
	Price rate (P)	N_{max}	Q_{min}				
S1	2	10	0	91	91	0	27
		Unconstrained		622	622	0	<1
		75	1	−7007	−7007	0	<1
	3	10	0	261	261	0	183
		Unconstrained		1731	1731	0	<1
		75	1	−4776	−4776	0	<1
	4	10	0	470	470	0	<1
		Unconstrained		3079	3079	0	<1
		75	1	−2545	−2545	0	<1
S2	2	No depots located – profit is always negative					
	3	No depots located – profit is always negative					
	4	10	0	95	95	0	340
		Unconstrained		289	289	0	59
		75	1	−2545	−2545	0	<1
S3	2	10	0	33	72	110	1893
		Unconstrained		260	456	76	1412
		75	1	−7007	−7007	0	<1
	3	10	0	160	278	74	1630
		Unconstrained		955	1418	48	1290
		75	1	−4776	−4776	0	<1
	4	10	0	337	486	44	2072
		Unconstrained		2192	2663	21	656
		75	1	−2545	−2545	0	<1

rejected. The second highest profit was obtained for scheme S3, which allows refusing trips due to vehicle shortage. This scheme systematically leads to lower profits than scheme S1 but higher than scheme S2.

When one compares the different scenarios for a given price rate, it can be seen that, as expected, the unconstrained optimal solution is always characterized with a profit higher than the one obtained by locating just 10 depots. However, if we relate that profit to the number of depots or the number of vehicles, we see that there is not much difference between these two scenarios, and there are even situations where the 10-depot solution is the best one. This occurs for schemes S1 and S3 when the price rate is 3 euros. This is pointing to similar returns on investment for both scenarios, i.e., the efficiency of each euro invested would be similar.

Another important result to notice is that it is not advisable to attend the total demand, because this does not lead to a positive profit. Although we have found an optimal solution with positive profit which allows capturing all demand at existing depots (scheme S2), this occurs for the price rate of 4 euros – a rate that could put carsharing in competition with taxicabs given the current fares. Satisfying all demand requires a large number of vehicles to face odd trips, since there are no vehicle relocation operations. We may observe that, in the three schemes, the number of vehicles needed per 100 trips is much larger in the 100% demand scenario than in the other two. It is interesting to notice that the 22.7 vehicles per 100 trips needed in the 100% demand scenario (1776 trips) falls in the interval of 18–24 vehicles per 100 trips indicated by Barth and Todd (1999) as being the one leading to minimum relocation operations. Thus our study is confirming a key finding of a simulation study carried out in another urban region, which strengthens this reference value as being trustworthy.

Table 2 also shows the percentage of time vehicles are used during the operation period. We may see there that the maximum percentage reached is very low, 31%, meaning that there is a large stock of vehicles accumulated in the depots along the day which are not being used. The lowest percentage (5%) is attained when total demand must be met because attending all trips requires having many vehicles idle at the depots. The difference is smaller for scheme S2 given that, in this case, all trips between existing depots must be satisfied in any scenario (demand between existing depots is always 100%). In order to gain deeper insight into vehicle usage, it is interesting to analyze in detail the vehicle stock at the depots along the day. The charts in Fig. 4 display two types of data for scheme S1 and the price rate of 2 euros. In the primary (left) y-axis we show the percentage of the vehicle stock kept at each depot represented by a unique color band (the sum has to be 100% for every time step) and in the secondary y-axis we show the total number of vehicles accumulated at the depots represented with a white broken line. Fig. 4a refers to the unconstrained scenario whereas Fig. 4b refers to the 100% demand scenario.

It is interesting to see that the color bands have a very different shape in both scenarios. While for the unconstrained scenario there is a clear tendency for loading and unloading different sets of depots, producing a wavy pattern, in the case where all trips must be met there is a great predominance of a smaller set of depots during the busiest part of the day. This is due to the fact that vehicles are transferred in the beginning of the day to the CBD and stay idle there for most of the time before returning home in the evening. When we compare the total number of vehicles stocked in both figures, we see a maximum of 35 vehicles for the unconstrained scenario and 403 for the 100% demand scenario. The pattern of trips associated with the

Table 2
System performance indicators.

Scheme	Scenarios			Daily profit (Euros)	Total demand satisfied (Trips)	Total demand satisfied (%)	Satisfied demand between existing depots (%)	Vehicles	Veh./100 trips	Veh. use (%)	Depots	Mean depot size	Max. depot size	Profit per depot	Profit per vehicle
	Price rate (P)	N_{max}	Q_{min}												
S1	2	10	0	91	120	7	40	6	5	28	10	1.3	2	9.1	15.2
		Unconstrained		622	713	40	44	37	5.2	31	55	1.5	4	11.3	16.8
		75	1	−7007	1776	100	100	403	22.7	5	75	10.3	39	−93.4	−17.4
	3	10	0	261	165	9	57	11	6.7	20	10	2.2	3	26.1	23.7
		Unconstrained		1731	950	53	55	60	6.3	24	65	2.1	6	26.6	28.9
		75	1	−4776	1776	100	100	403	22.7	5	75	10.3	39	−63.7	−11.9
	4	10	0	470	192	11	66	15	7.8	16	10	3	4	47	31.3
		Unconstrained		3079	1081	61	61	79	7.3	20	71	2.6	7	43.4	39
		75	1	−2545	1776	100	100	403	22.7	5	75	10.3	39	−33.9	−6.3
S2	2	No depots located – profit is always negative													
	3	No depots located – profit is always negative													
	4	10	0	95	63	4	100	11	17.4	10	10	2.4	4	9.5	8.6
		Unconstrained		289	707	40	100	110	15.6	8	34	6.8	16	8.5	2.6
		75	1	−2545	1776	100	100	403	22.7	5	75	10.3	39	−33.9	−6.3
S3	2	10	0	33	94	5	28	5	5.3	25	10	1.5	2	3.3	6.6
		Unconstrained		260	457	26	35	23	5.0	31	42	1.8	4	6.2	11.3
		75	1	−7007	1776	100	100	403	22.7	5	75	10.3	39	−93.4	−17.4
	3	10	0	160	128	7	46	8	6.3	20	10	2.2	3	16	20
		Unconstrained		955	764	43	51	54	7.1	20	54	2.9	7	17.7	17.7
		75	1	−4776	1776	100	100	403	22.7	5	75	10.3	39	−63.7	−11.9
	4	10	0	337	193	11	62	16	8.3	14	10	4	7	33.7	21.1
		Unconstrained		2192	1014	57	60	84	8.3	16	63	3.7	9	34.8	26.1
		75	1	−2545	1776	100	100	403	22.7	5	75	10.3	39	−33.9	−6.3

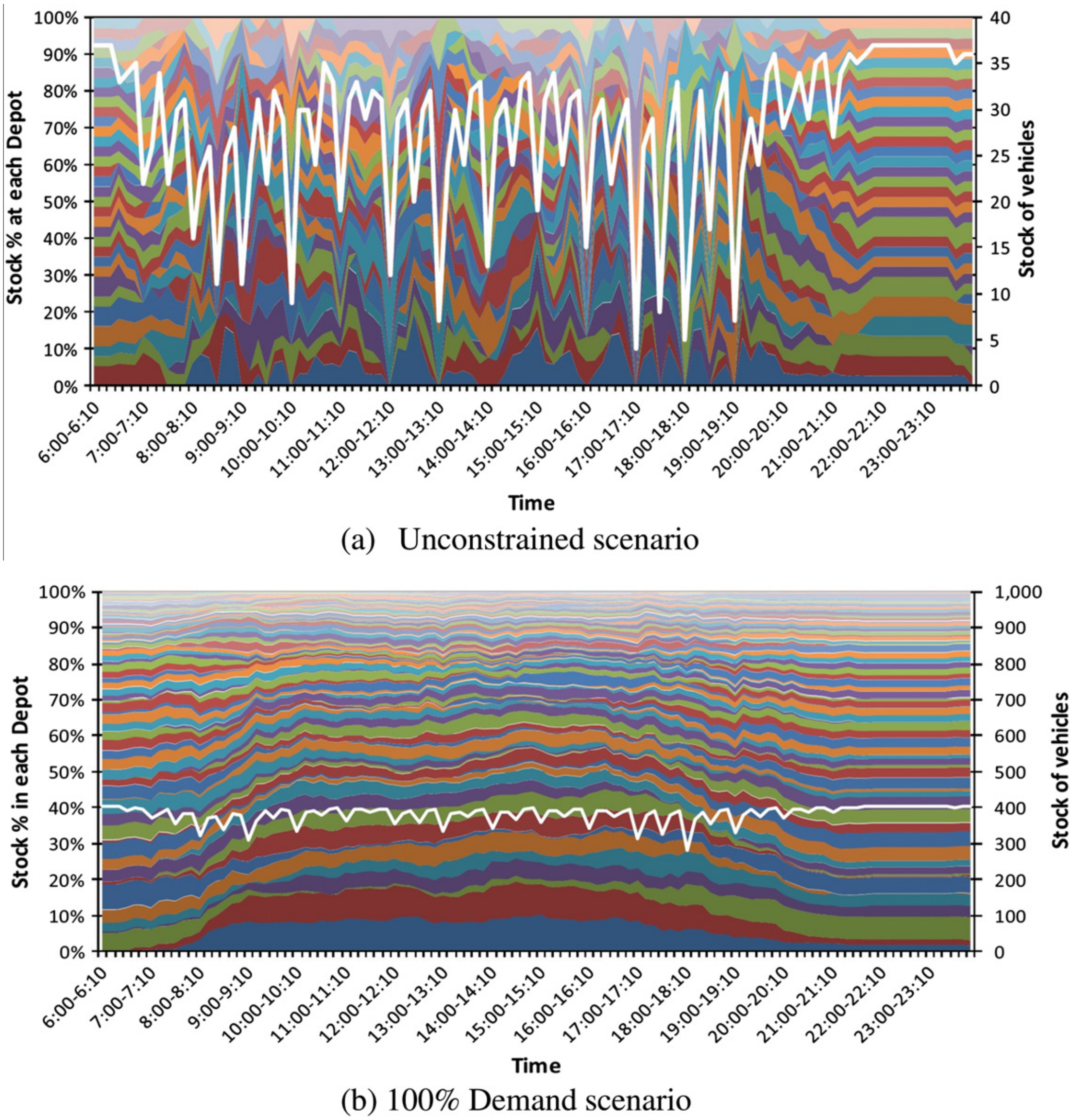

Fig. 4. Vehicle stocks along the day for a 2 euros price rate.

100% demand scenario leads to a much higher number of vehicles stocked at the depots, something that makes us believe that it is difficult to create synergies between commuter trips and more self-balanced trips in the CBD.

With respect to the number and size of depots, results indicate that they play an important role in trip selection. We can notice, in Table 2, that the optimal solution for the three schemes under the 10-depot and the unconstrained demand scenarios never consists in locating 75 depots (one in each potential location). The closer to that are the 71 depots located for scheme S1 when the price rate is 4 euros. In this situation the average and the maximum depot size (2.6 and 7 vehicles) are very low as compared with the values for the 100% demand scenario (10.3 and 39 vehicles).

The optimum location of depots varies considerably across scenarios, as shown in Fig. 4 for the 10-depot and the unconstrained scenarios under trip selection schemes S1 and S2 (the most and less profitable schemes) when the price rate is 4 euros (we used this price rate because it is the only one which leads to positive profit for scheme S2). In the case of the 10-depot scenario, when the trip selection scheme is S1, depots are primarily placed in the CBD of Lisbon, where the main office buildings are located (Fig. 5a). Clearly, this configuration cannot capture commuter trips because the neighborhoods where most people live are located outside the catchment area of the 10 depots. The carsharing business is concentrated in the CBD because this permits encompassing more balanced trips. The location of depots when the trip selection scheme is S2 is not so intuitive, and there is not a straightforward logic for explaining them (Fig. 5b). A possible reason is because the CBD depots, despite having many self-balanced trips, also have many unbalanced ones that cannot be rejected. In the case of the

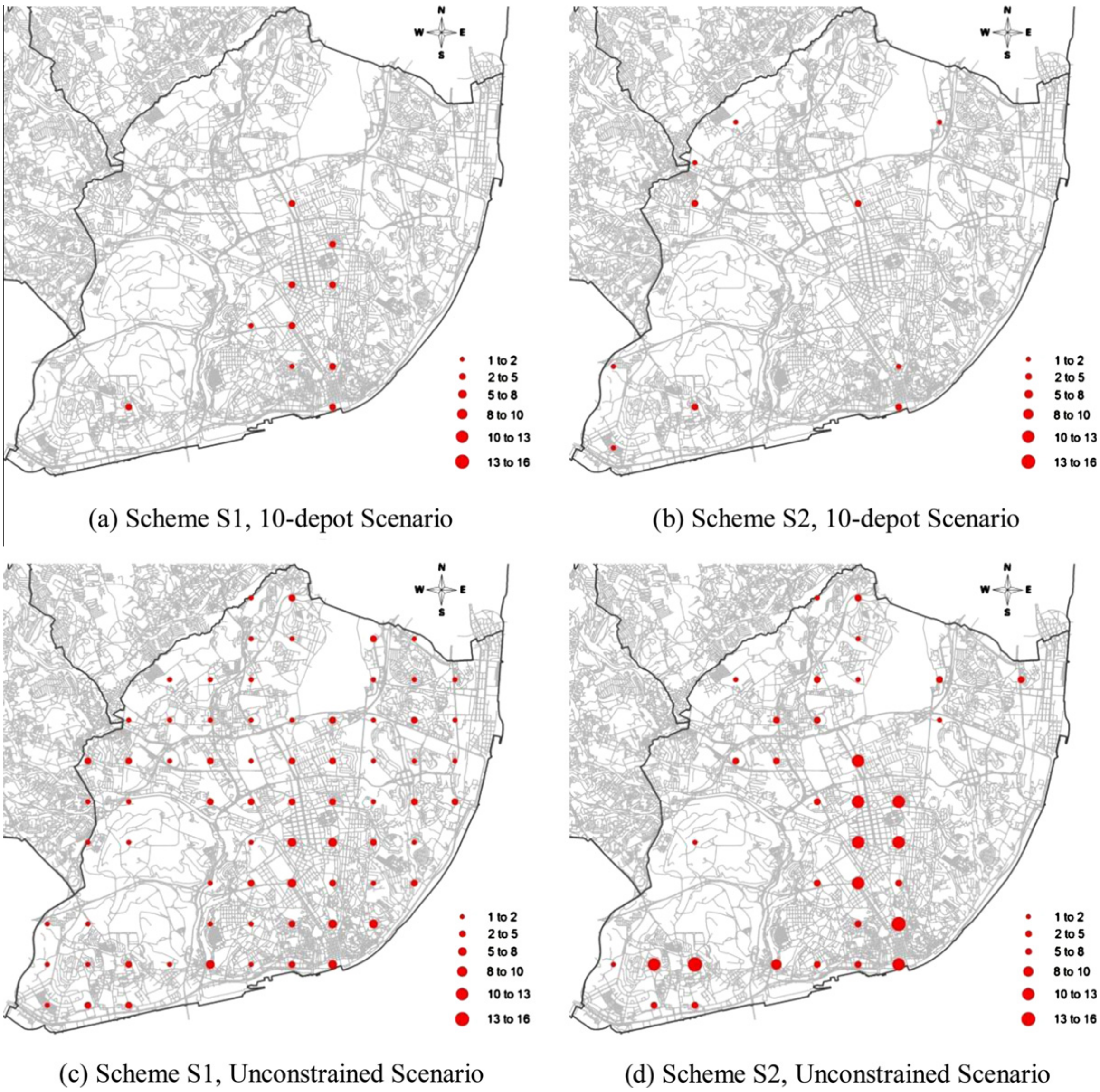

Fig. 5. Depot locations for a 4 euros price rate.

unconstrained scenario, scheme S1 leads to a large number of small depots scattered around the municipality (71 depots in Fig. 5c). The great number and dispersion of depots in this scenario is because there are system balancing trips to be captured in any district of the city. Scheme S2 leads to a smaller number of depots (34 depots in Fig. 5d), with a wide range of sizes. The reason is because, when the number of depots is allowed to increase, many of the new depots are located in the outskirts at residential neighborhoods, which causes the need to increase the size of the CBD depots, as these will have to stock the commuter trip vehicles. This can also be concluded from the fact that vehicle usage for scheme S2 is 8% against 20% for scheme S1 (Table 2), denoting the idleness of vehicles under the S2 trip selection scheme.

The joint analysis of the performance indicators in Table 2, the stocks' behavior in Fig. 4, and the depot locations in Fig. 5, reveals that not having a balance between vehicle stocks at the depots conduces to the worst results in efficiency and profit of the one-way carsharing system. Having to accept all trips in a city means serving commuter trips and other purpose trips under the same system, and results show that no clear synergy effects between these trip patterns exist, as one might think at the outset.

5. Conclusion

The vast majority of carsharing systems currently available in the world are round-trip. However, many clients of these systems have stated their desire to use a one-way system where they can pay per use, not having to return the vehicle to the original depot. This is important as many of the trips that we do in a city are not naturally even, and many times we do not need to use a vehicle in both ways of a tour. Despite this, the few experiments with one-way carsharing systems made in the past were not particularly successful, and certainly were not a match for the highly profitable and fast growing round-trip systems (Shaheen and Cohen, 2007).

The main problems encountered by one-way carsharing systems are related with vehicle stock imbalance issues. Some research has been conducted from an optimization perspective to devise ways of overcoming them, focusing on vehicle relocation operations (Fan et al., 2008; Kek et al., 2009). The contribution of this paper to the literature is to approach those problems at a strategic level, also from an optimization perspective, by considering the location of the carsharing depots and three different schemes for allocating vehicles to trips, avoiding relocation operations other than those needed at the end of the day for replacing initial vehicle stocks.

The three MIP models upon which our approach is based were tested on a case study involving the municipality of Lisbon where realistic data was used. This has provided us with a clear insight into the effect that depot location and trip selection decisions can have on the economic viability of this transportation alternative. One of the main conclusions we have drawn from this study is that the trip imbalance situation that characterizes Lisbon (and many other cities in the world) would lead to severe financial losses in a scenario where all demand for carsharing trips would be satisfied even for a very high price charged to the client. Indeed, a large fleet of vehicles would be necessary for coping with that demand, the ratio of 22.7 vehicles per 100 trips was found as a reference, which confirms previous research (Barth and Todd, 1999), most of which would stay idle during a large part of the day. Hence, it seems that synergies linking commuter trips to other trips may be difficult to make, something that goes against an intuitive idea about one-way carsharing. Another important conclusion is that financial losses could be reduced through appropriate choices with respect to the number, location, and size of the depots, but positive profits could only be achieved if carsharing trips were optimally selected from the total demand, either by previous reservation or by rejecting trips when there are no vehicles available at the depots. A final key conclusion is that a system comprised of only a few depots (10 depots in our experiment) could lead to values for return on investment indicators (profit per vehicle and profit per depot) similar to larger networks, provided that the depots are located in the CBD rather than in the outskirts and that undesirable trips are rejected. This plays in favor of the gradual development of a one-way carsharing system.

Despite we believe that our approach to depot location and trip selection in one-way carsharing systems can be of practical interest as is, there are a number of improvements that may increase its usefulness. In particular, we intend to take into account the risk issues inherent to this kind of systems. We have ignored them up to now, but, especially with regard to demand variations, these issues certainly are of great importance given the risk-averse behavior that characterizes most organizations. Also, we aim to consider the effects of price rate discrimination. Our assumption was that the same price rate would apply to all carsharing trips, but it would be interesting to assess the effect of a variable price policy established to prop up (or drop down) trips that balance (or unbalance) the system could contribute to more profitable operations. As before, we plan to test these improvements in the municipality of Lisbon, taking advantage of the additional information on carsharing demand that will be made available through a stated preference survey currently underway.

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