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## A MIP heuristic for multi port stowage planning

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### Abstract

In this paper we extend the problem of determining how to stow a given set of containers of different types into the available locations of a containership, that is, the so-called Master Bay Plan Problem (MBPP), to the Multi-Port Master Bay Plan Problem (MP-MBPP). In the MP-MBPP the whole route of the ship is investigated; in particular, at each port of the route different sets of containers must be loaded for being shipped to the next ports. Differently from MBPP, in MP-MBPP at each port the sequence of two handling operations affects the effectiveness of a stowing plan: first, the import containers must be unloaded from the ship, then the export containers can be loaded. Only few papers in the recent literature deal with the MP-MBPP. Here, we propose a Mixed Integer Programming (MIP) heuristic based on an exact MIP model for the MP-MBPP very recently proposed in the literature; the main aim is the minimization of the total berthing time of the ship. Unproductive movements are included in the analysis, as well as the workload of the quay cranes used in each port visited by the ship. As a novel issue the new proposed MIP heuristic deals with actual operative handling operations; in particular, the presence of hatches is taken into account for the final stowage plans and different types of containers are included in the analysis, that is 20' and 40' standard containers, reefer and open top ones. The proposed MIP heuristic permits to find good stowage solutions in a short amount of time and thus to include the model into an effective tool that can help the liner planner during the whole trip of the ship for defining the stowage planning in accordance with the updated transport demands. Computational tests, executed for ships with increasing capacity up to a very large ship with a capacity of 18000 TEUs, show the efficacy of the proposed method.

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## 1. Introduction and literature review

Nowadays, over 70% of the world trade by value is carried by sea. Since a lot of general cargos transported by ship is containerized, for being competitive shipping companies choose to serve the actual demand by even larger containerships. Further, a liner shipping network may consist of some tens of ship routes and ports, depending on the fleet capacity. As a consequence of this ever increasing maritime transport demand, the port activities and shipping lines can no longer be considered independent. Despite this, most of the recent management science literature on maritime transport is mainly focused either on the optimization techniques required to manage efficiently import / export containerized flows at maritime terminals (Stahlbock and Voss, 2008) or on the improvement of shipping services (Meng et al. (2014)). The relation between port activities and shipping lines is emphasized in Talley and Ng (2013). However, many authors recognized that loading / unloading operations and stowage planning strongly impact on both the efficiency of the terminals and the liner shipping choices (Imai et al. (2013), Rashidi and Tsang (2013), Pacino et al. (2011)). In particular, many recent papers are devoted to the optimization of the stowage plans for containerships.

In fact, the stowage planning is a problem faced by two different interacting decision makers having to solve the same problem with a different degree of detail and at different level (Steenken et al. 2004): the shipping line coordinator (SC) and the terminal planner (TP).

The SC has a view of the whole trip of the ship and considers aggregated information to take decisions at a planning level. In particular, the SC receives the *o-d* transport demands and verifies if it is possible to accept each transport demand. Then, the SC defines a stowage plan for each port of the ship trip and updates the plan accordingly whenever a new *o-d* demand for that ship is received and accepted. An efficient tool is usually needed to assist the SC in generating feasible stowage plans. Before the ship arrival in a port, the SC has to send a feasible plan to the TP giving general instructions about how to stow different groups of containers, depending on their destination, type, size and weight. In particular, for each group of containers the SC indicates a portion of the ship in which load it.

The TP takes more detailed decisions at an operational level. The TP work starts after having received the stowage instructions from the SC. Specifically, the TP has to define the exact location for each container to be loaded on board, following the pre-stowage instructions received from the SC. Then, the detailed stowage plan adopted by the terminal is communicated to the SC who updates the current cargo of the ship and starts a new planning phase merging the new cargo with the old and new *o-d* transport demands for the successive ports in the route ship.

The need to merge the ship cargo composition before the ship arrival in a port (that is the stowage plan sent by the SC to the terminal), with the actual cargo after the ship leaves the port (that is the stowage plan defined by the TP), is due to the different degree of detail of the two plans. In fact, the plan defined by the SC gives general instructions to the terminal, while the TP defines the exact position of each container to load on board. As an example, once the SC establishes that “*20 containers of class A1 - 20', standard, light weight, destination Port A - must be stowed in bay 10*”, the TP decides the row and tier indices for each of the 20 containers of class A1 to be stowed in bay 10.

Studies on the stowage plans start from the single port problem, i.e. the master bay planning problem (MBPP), that has been proven to be a NP-hard problem (Avriel et al. 2000). A detailed description of MBPP is given in Ambrosino et al. (2004). A recent research by Monaco et al. (2014) tries to include the yard management in MBPP.

The problem of determining a stowage plan for each port in the route of the ship, i.e., the planning problem concerning the shipping line operator, is known as the Multi Port-MBPP (MP-MBPP). Papers dealing with MP-MBPP generally propose decomposition approaches. All these papers originate from the works by Wilson and Roach (2000) and Wilson et al. (2001), where a methodology for generating stowage plans for a containership on a multi-port journey is presented and the decision process is decomposed into two planning sub-processes: a strategic and a tactical one. Many authors proposed decomposition approaches too; among others, Kang and Kim (2002), Zhang et al. (2005), Pacino et al. (2011), Delgado et al. (2012). Pacino et al. (2012) and Ambrosino et al. (2015a), (2015) focused only on MP-MBPP at a strategic level. In Pacino et al. (2012) a linear model with ballast tanks for generating master plans that includes the main stability and stress moments calculations is proposed for handling variable displacements. In Ambrosino et al. (2015a) the authors proposed two MIP models for solving MP-MBPP

for taking into account the hatches and the irregular keels of a containership. Finally, in Ambrosino et al. (2015b) the authors extend the problem studied in Ambrosino et al. (2015a) in order to include different type of containers (i.e. standard cargo, reefers and open top) and to guarantee that all containers be loaded on board, even if some re-handles are necessary. Note that re-handles are permitted either for loading other containers in the hold or for unloading containers from the hold.

In the present paper, a new solution method based on the MIP model proposed in Ambrosino et al. (2015b) for MP-MBPP is presented. In particular, this work presents and evaluates a new MIP heuristic for MP-MBPP that exploits a *relax-and-fix* principle (Pochet and Wolsey 2006) as iteratively solves relaxations of the MIP model progressively fixing subset of binary variables. An experimental campaign is detailed in order to show the capability of the proposed method to be included in an effective tool able to define stowage plans for the SC. The paper is organized as follows: the problem under investigation is presented in details in Section 2. After a brief reminder of the underlying MIP model for the problem, Section 3 presents the main steps of the proposed MIP Heuristic. Computational results are given in Section 4, while Section 5 draws the main conclusions and outlines for future works.

## 2. Problem under investigation

Given a ship with its structural characteristics, its route, described by a circular sequence of ports to be visited, and its current cargo, the problem consists in defining the stowage plan for a given set of containers that differ for size, type and weight and for the loading and destination port, so that all the containers are loaded on board, while the structural and operative constraints are satisfied and the time spent by the ship at the ports for loading/unloading operations is minimized.

In more details, the MP-MBPP is defined considering:

- a ship with a particular structure characterized by the sets  $I$ ,  $J$ ,  $K$  and  $H$  respectively of bays, rows, tiers and hatches of the ship (see Figure 1);
- the load capacity, i.e.,  $CTeuH_h$  and  $CTeuD_h$ , corresponding to the number of TEUs available in the locations respectively under hatch  $h$  in the hold (*hold hatch locations*) and over  $h$  on deck (*deck hatch locations*), further detailed in capacity for loading reefer and open top containers;
- some requests concerning the ship stability, expressed in terms of maximum cross equilibrium tolerance  $Q_1$  and maximum horizontal equilibrium tolerance  $Q_2$ ;
- the set  $P$  of ports included in the circular route of the ship and the set  $D$  of *o-d* shippings in the considered trip of the vessel;
- the demand of transport  $N_{od}^{sig}$  expressed as number of containers of size  $s$ , type  $t$ , weight class  $g$  to be loaded at port  $o$  with destination port  $d$ .

Then, the MP-MBPP consists in defining how to allocate each group of containers in the locations above/under the hatches of the ship. A group is a set of homogeneous containers for that concerns size (20' or 40' containers), type (general cargo (STD), reefer (R) and open top (OT)), weight class and the loading port ( $o$ ) and the unloading port ( $d$ ).

The main goal when defining the allocation of containers is to serve the *o-d* transport demand, while satisfying the capacity of the ship for the different types of containers and minimizing the time spent by the ship at the ports for loading/unloading operations. Note that the time at the ports is minimized by minimizing in turn the number of re-handles (i.e., unproductive movements) and the unbalance among the loading and unloading operations performed by the cranes working in parallel at each port. This latter objective is justified by the fact that the time at a port depends on the time needed by the crane performing the largest number of handling operations. To this end, the set  $Y_p$  of quay cranes serving the ship in parallel at port  $p$  and the subset  $H_{pc}$  of hatches served by crane  $c$  at port  $p$  are assumed known.

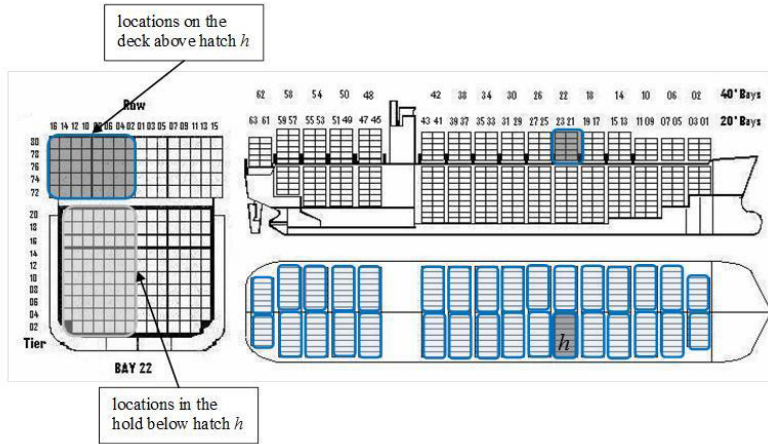


Fig1: The structure of a containership

The main decisions, expressed by the variables reported below as defined in the model proposed in Ambrosino et al. (2015b), are described in the following together with the main constraints they are subjected:

- the number of containers of a given group (having fixed size  $s$ , type  $t$ , weight class  $g$ , origin port  $o$  and destination port  $d$ ) assigned in the locations of hold/deck hatch  $h$  of the ship, represented by the integer variables  $xh_{odh}^{stg} \in \mathbb{Z}_+$ ,  $xd_{odh}^{stg} \in \mathbb{Z}_+$   $\forall s \in S, t \in T, g \in G, h \in H, (o, d) \in D$ , in such a way to satisfy the transport demand for each group of containers  $N_{od}^{stg}$  (i.e. one of the objectives).

The assignment of containers to the hatch locations must respect the capacity conditions derived by the ship structure, these capacity conditions refer to all containers loaded on board but also to the different types of containers (standard, open top and reefer containers). Moreover, also stability conditions must be respected by the assignment; in particular, the horizontal and the cross equilibrium must be satisfied.

- the assignment at a port  $p$  of the hold/deck hatch locations  $h$  to a destination  $d$ , expressed by the binary variables  $yh_{pdh} \in \{0,1\}$ ,  $yd_{pdh} \in \{0,1\}$ ,  $\forall p \in P, d \in P: d \neq p, h \in H$ .

This assignment is performed at each port and it can change along the trip but at each port departure only containers with the same destination are loaded into each hatch location.

At port  $p$  re-handles can occur in locations on the deck above a hatch  $h$  for both unloading containers bound for port  $p$  and stowed in locations in the hold below hatch  $h$ , and for loading containers departing from  $p$  in the free locations in the hold below hatch  $h$  (see Figure 2). Note that the number of re-handles to perform at port  $p$  over hatch  $h$  is equal to the number of containers in the cargo when the ship arrives at port  $p$  that are stowed in the deck hatch locations  $h$  and not bound for  $p$ , i.e. the containers loaded in origins  $o < p$  with destinations  $(d > p) \vee (d < o)$ .

The need to perform re-handles in deck hatch locations  $h$  is modeled through binary auxiliary variables  $f_{ph} \in \{0,1\}$ ,  $\forall p \in P, h \in H$  and the exact number of re-handles executed at each port  $p$  on the deck hatch locations  $h$ , is computed by integer variables  $r_{ph} \in \mathbb{Z}_+$ ,  $\forall p \in P, h \in H$ .

As already said, the second objective perceived is the minimization of the time spent by the ship at the ports, and this is related also to the work crane balance. Thus it is computed the total number of loading and unloading operations of each crane  $c$  at each port  $p$ , by variables  $op_{pc} \in \mathbb{R}$   $\forall p \in P, c \in Y_p$ ; the absolute difference between the number of unloading and loading operations executed by each pair of cranes at each port  $p$ , by variables  $\Delta_{pce} \in \mathbb{R}$   $\forall p \in P, c, e \in Y_p: c \neq e$ ; the maximum difference between the number of handling operations performed by the cranes for each pair of cranes in each port by variable  $\Delta_{max}$  that represents the maximum crane operations unbalance in the route of the ship.

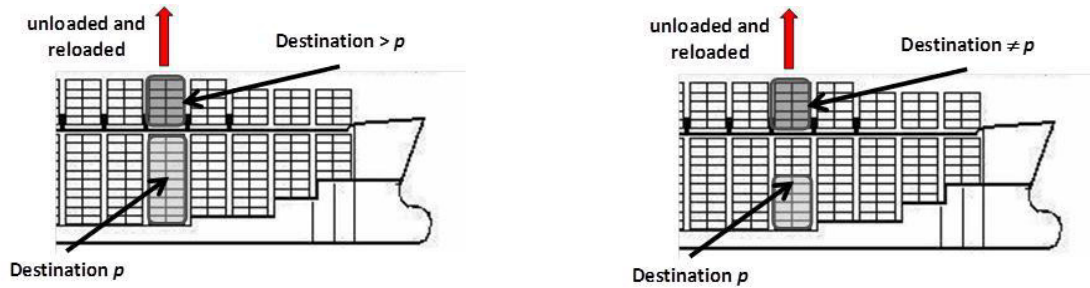


Fig. 2 (a) Re-handles at port  $p$  for unloading hold locations

(b) Re-handles at port  $p'$  for loading in hold locations containers with destination port  $p$

### 3. The MIP Heuristic for MP-MBPP

In this section a two-level Progressive Random Fixing Procedure (PRFP) for solving the MP-MBPP is described. This procedure is based on the MIP model proposed in Ambrosino et al. (2015b) whose variables and main constraints have been briefly described in the previous section. Let  $M$  be the MIP model for MP-MBPP.

The PRFP operates in a similar way as a relax-and-fix heuristic. In particular, the MP-MBPP is characterized by two main types of decisions: the assignment of locations to destinations, modelled by the binary  $y$  variables, and the assignment of containers to the locations, modelled by the integer  $x$  variables whose value depends on the one of the  $y$  variables. Thus, the PRFP considers the  $y$  variables as the most important ones and then, in a first step, determines a value for them by optimally solving an apparently simpler MIP problem where all the  $x$  variables are linearly relaxed. Then, the heuristic proceeds trying to find a good feasible solution by solving the original problem where all the location decisions, corresponding to the  $y$  variables, are fixed to the values obtained in the first step. However, this second step could be still quite time consuming and so it is divided into a sequence of simpler steps following a sort of dive-and-fix strategy: since, as we observed, for the MP-MBPP many of the  $x$  variables assume an integer value in the solution of the first step, such variables are tentatively fixed in the following steps. Note that, in general, such fixing could not produce a feasible solution or it could produce a feasible solution that is not good enough, i.e., its gap from the lower bound obtained in the first step is too large. When this happens, the  $x$  variable fixing is considered too constraining and it is gradually removed for a subset of variables that is randomly selected. This strategy leads to solve a sequence of increasingly harder MIP problems that terminates either when a feasible and good enough solution is found, or when the MIP problem, where only the  $y$  variables are fixed, is solved.

More formally, the PRFP initially solves a partial linear relaxation ( $M_{PR}$ ) of the model  $M$  in order to obtain a starting solution and some bounds exploited during the search for integer feasible solutions. The partial relaxation is obtained by linearly relaxing the variables  $xh_{odh}^{sig}$  and  $xd_{odh}^{sig}$  that define the number of containers of a given group assigned, respectively, to hold and deck hatch locations. Note that also  $r_{ph}$  variables, in the model linked to the  $xd_{odh}^{sig}$ , are linearly relaxed.

Note that, due to such relaxations, a solution of  $M_{PR}$  may not be a feasible solution for  $M$  if some  $x$  variable assumes a non-integer value. On the other hand, in the solution of  $M_{PR}$  a (usually large) number of  $xh_{odh}^{sig}$  and  $xd_{odh}^{sig}$  assumes integer values. Then, the proposed heuristic iterates the solution of model  $M$ , whose integer variables are partially fixed to the integer values found  $M_{PR}$ , as follows: at each iteration, all the binary variables  $yh_{pdh}$  and  $yd_{pdh}$  are fixed to the values found solving  $M_{PR}$ , as well as a given percentage *PercFix* of the subset of variables  $xh_{odh}^{sig}$  and  $xd_{odh}^{sig}$  whose value in the solution of  $M_{PR}$  is integer. The value of *PercFix* is initialized equal to 100% and is decreased during the iterations.

The procedure is called “progressive random fixing” since, during the iterations, it tries to solve model  $M$  having fixed a randomly chosen subset of integer variables whose cardinality is progressively reduced. Moreover, before solving model  $M$  for a given *PercFix*, an evaluation of the possibility of obtaining good quality solutions is performed considering the lower bound associated with the first integer solution found. The PRFP adopts as termination condition either a maximum time limit, fixed equal to 3600 seconds, or the satisfaction of a given

tolerance value. Note that, if finding a high quality solution turns out to be too difficult, i.e., when *PercFix* approaches to zero, the tolerance value is increased, so relaxing one of the termination conditions.

However, if the considered instance is feasible, the final solution returned by the heuristic is always a feasible solution for model M and consequently for the MP-MBPP.

Let us summarize the notation useful for describing the proposed procedure:

- $M_{PR}$  linear relaxation of the MIP model M with respect to variables  $xh_{odh}^{sig}$  and  $xd_{odh}^{sig}$  and  $r_{ph}$ ;
- $S_{PR}$  solution set of the relaxed model  $M_{PR}$ ;
- $I_{PR}$  subset of variables  $xh_{odh}^{sig}$  and  $xd_{odh}^{sig}$  whose value in solution  $S_{PR}$  is integer;
- $LB_{PR}$  lower bound obtained by solving  $M_{PR}$ ;
- $Obj_{PR}$  objective value obtained by solving  $M_{PR}$ ;
- $M_{PF}$  partially fixed model M;
- $FLB_{PF}$  lower bound obtained by solving  $M_{PF}$  associated with the first feasible solution;
- $LB_{PF}$  lower bound obtained by solving  $M_{PF}$ ;
- $Obj_{PF}$  objective value obtained by solving  $M_{PF}$ ;
- PercFix* percentage of the subset  $I_{PR}$  of integer variables to fix when solving  $M_{PF}$ ;
- IterToRelaxCond* maximum number of iterations before relaxing the termination conditions;
- Iter* counter of MIP solver calls;
- Ntries* counter of attempts of random fixing the variables in  $I_{PR}$  for a given value of *PercFix*;
- Tol* optimality tolerance for accepting solution;
- TolInc* increase of tolerance;
- MinPercFix* minimum allowed value for *PercFix*;
- MaxRndTries* maximum number of attempts of random variable fixing for a given percentage.

The following steps describe in detail the heuristic procedure PRPF:

1. derive  $M_{PR}$  from M by continuously relaxing variables  $xh_{odh}^{sig}$ ,  $xd_{odh}^{sig}$  and  $r_{ph}$ ;
2. solve  $M_{PR}$  by a MIP solver with a given optimality tolerance *Tol*, producing solution  $S_{PR}$ , objective function value  $Obj_{PR}$  and lower bound  $LB_{PR}$ ;
3. build subset  $I_{PR}$  of integer variables  $xh_{odh}^{sig}$ ,  $xd_{odh}^{sig}$  whose values in  $S_{PR}$  are integer;
4. initialize: *Iter* = 0; *Ntries* = 0; *PercFix* = 1.0;
5. derive  $M_{PF}$  from model M by fixing  $yh_{pdh}$  and  $yd_{pdh}$  to the values in solution  $S_{PR}$  and by randomly fixing *PercFix* percent of the variables in  $I_{PR}$  to the (integer) values in solution  $S_{PR}$ ;
6. start solving  $M_{PF}$ ; if  $|FLB_{PF} - Obj_{PR}| > Tol$ , then stop the MIP solver; *PercFix* = *PercFix* - 0.05; go to 5;
7. solve  $M_{PF}$  finding solution  $S_{PF}$ , objective function  $Obj_{PF}$  and lower bound  $LB_{PF}$ ; *Iter* = *Iter* + 1;
8. terminate returning solution  $S_{PF}$  if one of the following conditions is satisfied:
  - a.  $|Obj_{PF} - Obj_{PR}| \leq Tol$
  - b. *Iter* > *IterToRelaxCond* And  $|LB_{PF} - LB_{PR}| \leq Tol$
  - c. *PercFix* = 0
9. if *Ntries* < *MaxRndTries* update *Ntries* = *Ntries* + 1; otherwise, set *Ntries* = 0 and *PercFix* = *PercFix* - 0.05;
10. if *PercFix* < *MinPercFix* update *Tol* = *Tol* + *TolInc* and set *PercFix* = 1.0;
11. iterate at step 5.

Note that, at each iteration, step 6 heuristically evaluates if the current partially fixed model  $M_{PF}$ , obtained fixing the integer values for a given percentage of the *x* variables, is *explorable*, i.e., it worth solving it. This is done by checking if the absolute difference between the lower bound associated with the first solution found for  $M_{PF}$  and the objective value of the solution of  $M_{PR}$  is lower than the given tolerance. Such a test is introduced to speed up the computation stopping the solution of a partially fixed model whenever there is an indication that the quality of the produced solution could not satisfy the termination conditions. In such cases, the percentage *PercFix* is reduced and the whole evaluation process is repeated. Step 8 implements the termination conditions; specifically, the PRPF stops if one of the following three conditions succeeds:



- a. the quality of the obtained solution  $S_{PF}$  is comparable to the one of  $S_{PR}$ , i.e., the objective function value  $Obj_{PF}$  is within the tolerance  $Tol$  from the  $Obj_{PR}$ ;
- b. after a maximum number of iterations spent solving explorable  $M_{PF}$ , the termination condition (a) is relaxed; the new relaxed termination condition consists in verifying if the lower bound  $LB_{PF}$  is within the tolerance  $Tol$  from the lower bound  $LB_{PR}$ ;
- c. the percentage of integer variables  $x$  to fix is zero ( $PercFix = 0$ ); this indicates that in model  $M_{PF}$  solved last only the binary variables  $y$  were fixed (note that this condition allows a termination if  $MinPercFix$  is set to zero).

Since the  $x$  variable to be fixed are randomly chosen, the role of step 9 is that of allowing a certain number of attempts,  $MaxRndTries$ , before reducing  $PercFix$  whenever an explorable  $M_{PF}$  is generated for such a percentage. Finally, if a minimum positive threshold  $MinPercFix$  is set for  $PercFix$ , step 10 resets  $PercFix$  to 1.0 and increase the tolerance value whenever  $PercFix$  decreases below  $MinPercFix$ .

#### 4. Computational results

The PRPF heuristic described in previous section was implemented in C#, using as MIP solver Cplex 12.5. All the tests ran on a 2.4GHz Intel Core 2 Duo E6600 computer with 4GB RAM.

The computational tests are based on random generated instances of the MP-MBPP that represent real size scenarios. Three containerhips with a capacity of 7800, 10000 and 18032 TEUs are used in these tests. Each containerhip has a given capacity for stowing reefer containers and open top containers. These capacities range from 800 to 1680 for reefer containers and from 1560 to 2576 for open top containers. The most common three kinds of weight classes are considered for the containers: light, medium and heavy class. The weight limit of each class is different for 20' and 40' containers and consists of 20' containers up to 7, 14 and 21 tons and 40' containers up to 10, 20 and 30 tons, respectively

Each ship travels on a circular route with 6 ports and the transport demand is randomly generated in such a way that for each origin  $o$  in the route of a ship, there can be a positive demand for standard, reefer and open top containers for the three successive ports of the route. For example when planning the stowage for port 5, the ship has on board a cargo deriving from loading operations executed at ports 2, 3 and 4. At port 5, after that the unloading operations are executed, the loading process regards containers bound for port 6, 1', 2', where 1' and 2' denote the port 1 and 2 reached during the second round of the ship. For each containerhip four instances have been generated; each instance differs from the others for the transport demand to satisfy. Anyway, all instances have been generated in such a way to stress the capability of the heuristic approach to obtain feasible and effectiveness solutions in a short amount of time. Table 1 reports for each of the considered ship (i.e., small ship (SS), medium ship (MS) and large ship (LS)) the percentage occupancy level of the ship (Occ.) when leaving the ports of the considered route and the percentage of the required capacity of the ship (Req. Cap.) associated with the demand to load at each port. Note that the occupancy level of the ship is given by the ratio between the TEUs on board and the TEUs capacity of the ship, and the required capacity of the ship is computed as the ratio between the TEUs to load and the TEUs capacity of the ship. These two parameters give an idea about the difficulty of instances to solve: given a certain percentage of the required capacity of the ship, the higher is the percentage occupancy level of the ship the harder are the instances to be solved.

In the generated instances, on average, at each port it is necessary to load a cargo that represents the 43% of the capacity of the ship. 75% of the containers to load at each port are 20' containers. Moreover, on average 80% of the cargo is related to standard containers, while the 10% of the cargo is reefer and 10% are open top containers. Finally, the percentage of occupancy, on average, is more than 85%.

Model M applied to the considered instances has a number of variables ranging from 93312, in the case of the smallest SS ship, to 259301 for the largest LS one, and a number of constraints ranging from 26708 to 72628, accordingly.

Table 1: The average occupancy characteristics of the considered containerships

Port	SS		MS		LS	
	Occ.	Req. Cap.	Occ.	Req. Cap.	Occ.	Req. Cap.
1	80.79%	44.14%	84.91%	49.94%	84.67%	45.50%
2	86.06%	37.46%	83.19%	32.86%	83.12%	33.13%
3	87.11%	47.23%	83.73%	45.48%	82.88%	45.74%
4	84.44%	40.63%	83.61%	42.53%	81.47%	39.87%
5	82.36%	39.42%	83.53%	40.40%	85.75%	43.76%
6	86.12%	46.44%	83.56%	42.66%	84.68%	42.13%

In order to evaluate the effectiveness of the proposed heuristic (PRFP), the parameters  $Q_1$  and  $Q_2$ , related to the stability conditions, were fixed to different and stronger values, ranging from 0.1% to 0.01% of the total weight on board of the ship. More precisely, stability is really important when defining stowage plans; in the present study stability conditions correspond to the horizontal and cross equilibrium that are computed on the cargo that is on board when the ship leaves each port of its route. Note that a stability values fixed to 0.01% of the total weight can be apparently regarded as a quite unrealistic scenario, as, for example, the maximal stability tolerance allowed for the largest ship corresponds alternatively to a single 20' or 40' heavy container, two 20' or one 40' light container. However, such very restrictive tolerance value was included just for the need of stressing the capability of the proposed MIP heuristic.

Table 2. Results obtained by the heuristic procedure PRFP

Ist	Solution of $M_{PR}$					Solution of $M_{PF}$					Time %dev
	Q1-Q2	Obj	RH	MCU	CPU	Obj	RH	MCU	CPU	Tot CPU	
SS	0.100%	0	0	0	132.42	1	0	1	13.00	145.42	-70.8%
	0.075%	0	0	0	83.31	1	0	1	12.53	95.83	-91.5%
	0.070%	0	0	0	95.84	1	0	1	11.99	107.84	-61.2%
	0.050%	0	0	0	149.78	1	0	1	16.01	165.80	-89.1%
	0.030%	0	0	0	133.22	1	0	1	10.29	143.51	-85.4%
	0.010%	0	0	0	139.07	1	0	1	53.00	192.07	-
MS	0.100%	0	0	0	160.26	1	0	1	18.64	178.91	-82.8%
	0.075%	0	0	0	159.56	1	0	1	15.25	174.81	-81.7%
	0.070%	0	0	0	151.52	1	0	1	13.64	165.16	-81.7%
	0.050%	0	0	0	153.40	1	0	1	22.31	175.71	-74.5%
	0.030%	0	0	0	227.20	1	0	1	23.21	250.41	-79.7%
	0.010%	0	0	0	176.80	1	0	1	88.76	265.56	-
LS	0.100%	0	0	0	770.55	1	0	1	34.88	805.43	-58.0%
	0.075%	0	0	0	937.21	1	0	1	29.40	966.61	-37.7%
	0.070%	0	0	0	1628.13	1	0	1	40.36	1668.49	-31.0%
	0.050%	0	0	0	1209.53	1	0	1	30.50	1240.03	-46.9%
	0.030%	0	0	0	769.12	1	0	1	26.45	795.56	-68.5%
	0.010%	0	0	0	2523.78	1	0	1	56.74	2580.52	-20.1%

The tests performed compare the performance of the PRFP heuristic with the ones yielded by the MIP solver. For both the methods a maximum time limit of 3600 seconds was fixed. The weights in the objective function were



fixed in such a way to consider re-handles ten time more undesirable than crane unbalances. The tolerance imposed for the heuristic corresponds to accept as optimal a solution with an absolute optimality gap equal to 1 (i.e., a maximum crane unbalance of one operation).

Table 2 reports the results obtained by PRFP, showing for the three types of containerships and for the different values of the stability tolerances ( $Q_1$ - $Q_2$ ) the value for the overall objective function ( $Obj$ ), the number of re-handles ( $RH$ ), the maximum crane operation unbalance ( $MCU$ ) and the computation time ( $CPU$ ). Each row of Table 2 reports the average value of the four solved instances. In particular, the solutions produced by the partially relaxed ( $M_{PR}$ ) and partially fixed ( $M_{PF}$ ) models are highlighted, together with the total computation time of the PRFP ( $Tot\ CPU$ ).

The comparison with the MIP solver can be done observing the last column in Table 2 ( $Time\ \%dev$ ), which reports the percentage deviation of the CPU time obtained by PRFP with respect to the one needed by the MIP solver to produce exactly the same results. The cells in the  $Time\ \%dev$  column showing a dash denote the instances for which the MIP solver was not able to find a feasible solution within one hour of computation. In case of grey numbers (same cases of LS instances) the data is the average of only those instances solved by the model within the time limit of 3600 seconds (i.e. 3 instances of 4). We can observe the high computational performance of PRFP that was able to find for all the instances the optimal solution (the optimality can be proven also for the instances that were not solved by the MIP solver since the final lower bound was strictly greater than zero). PRFP needed a computation time that is lower than 270 seconds for all SS and MS instances, while the LS instances turned out to be more difficult to be solved with an average CPU time of 1342 seconds, but the overall CPU time needed by the heuristic, 562 seconds, is anyway quite acceptable and it decreases to 261 seconds excluding the harder four cases.

Table 3 reports the results obtained by solving Model M within a time limit of 3600 seconds. For the three types of containerships and for two different values of the stability tolerances ( $Q_1$ - $Q_2 = 0.100\%$  and  $0.030\%$ ), as in Table 2, the value for the overall objective function ( $Obj$ ), the number of re-handles ( $RH$ ), the maximum crane operation unbalance ( $MCU$ ) and the computation time ( $CPU$ ) are shown.

Table 3. Results obtained by solving model M

Ist	0. 1%				0. 3%			
	Obj	RH	MCU	CPU	Obj	RH	MCU	CPU
SS1	2	0	2	1164.9	1	0	1	1796.7
SS2	1	0	1	251.9	1	0	1	670.3
SS3	1	0	1	296.87	1	0	1	260.41
SS4	1	0	1	279.59	1	0	1	1198.75
MS1	1	0	1	536.89	1	0	1	465.63
MS2	1	0	1	2082.83	2	0	2	2437.52
MS3	1	0	1	307.34	1	0	1	472.97
MS4	1	0	1	1222.13	1	0	1	1555.83
LS1	1	0	1	1286.18	1	0	1	2183.3
LS2	1	0	1	1340.05	-			3600.28
LS3	1	0	1	2410.12	1	0	1	1789.05
LS4	1	0	1	2634.95	1	0	1	3600

## 5. Conclusions

This paper focused on the Multi Port Master Bay Plan Problem (MP-MBPP), which is a very crucial optimization problem in maritime logistics, particularly for the efficiency of maritime terminals and port operations. MP-MBPP involves two decision makers, the ship coordinator and the terminal planner, with different points of view and information, both requiring fast and good solutions. To give a support to these decisional processes, a MIP heuristic has been proposed aimed at determining stowage plans in circular routes for container ships, where containers of

different type, size and destination have to be loaded for being shipped to their destination. The minimization of the berthing time of the ships is the objective function. Structural and operative constraints both of the ship and the terminals involved in the routes are considered. The proposed heuristic relies on a very efficient MIP model, recently proposed in the literature, and allows to get very good feasible solutions for up to 18000 TEU containerships in a very short computational time. On the basis of the reported computational results we believe that the implementation of this MIP heuristic in an appropriate tool can really efficiently support and improve the definition of stowage plans.

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