1. Let X be the number of points

game		X_{first}		
	P	0	1	2
X_{second}	0	0.12	0.14	0.14
	1	0.09	0.105	0.105
	2	0.09	0.105	0.105

$P_1(\text{not lose}) = 0.7$

$P_1(\text{win}) = P_1(\text{tie}) = 0.35$

$P_1(\text{lose}) = 0.3$

$P_2(\text{not lose}) = 0.6$

$P_2(\text{win}) = P_2(\text{tie}) = 0.3$

$P_2(\text{lose}) = 0.4$

Points : 0, 1, 2, 3, 4

$X = X_1 + X_2$

PMF: $$P_X(x) = \begin{cases} 0.12, & \text{if } x=0 \\ 0.23, & \text{if } x=1 \\ 0.335, & \text{if } x=2 \\ 0.21, & \text{if } x=3 \\ 0.105, & \text{if } x=4 \\ 0, & \text{otherwise} \end{cases}$$

2. The probability of Fischer win at round i

is $P_i = 0.3^{i-1} \times 0.4$

$$\therefore P_{\text{Fischer win}} = \sum_{i=1}^{10} P_i = \sum_{i=1}^{10} 0.3^{i-1} \times 0.4$$

$$\approx 1.42856 \times 0.4$$

$$\approx 0.571424$$

The probability of spassky win at round i

is $Q_i = 0.3^{i-1} \times 0.3$

For i in $1 \sim 9$

$$P_{\text{either win}} = 0.3^{i-1} \times 0.4 + 0.3^{i-1} \times 0.3$$

$$= 0.3^{i-1} \times 0.7$$

For i in 10

$$P_{\text{match}} = 1 - 0.3^{i-1} \times 0.7$$

$$\therefore \text{PMF: } P_{\text{match}} = \begin{cases} 0.3^{i-1} \times 0.7, & \text{if } x = 1 \text{ to } 9 \\ 1 - \sum_{i=1}^{9} 0.3^{i-1} \times 0.7, & \text{if } x = 10 \\ 0, & \text{otherwise} \end{cases}$$

3. (a) Let x is number of modems
assume that y is number of customers need to connect

PMF: When $y \leq 99$, means $x \leq 99$

$$P_{(0 \leq x \leq 99)} = \binom{10000}{x}(0.001)^x(1-0.001)^{10000-x}$$

When $y \geq 100$, means $x = 100$

$$P(x=100) = \sum_{x=100}^{10000} \binom{10000}{x}(0.001)^x(1-0.001)^{10000-x}$$

$$\therefore P_{(x)}x \begin{cases} \binom{10000}{x}(0.001)^x(1-0.001)^{10000-x}, & \text{if } x=0,1,2,\cdots 99 \\ \sum_{x=100}^{10000} \binom{10000}{x}(0.001)^x(1-0.001)^{10000-x}, & \text{if } x=100 \\ 0, & \text{otherwise} \end{cases}$$

(b) For poisson method

$\lambda = n \cdot P = 10000 \times 0.001 = 10$

$P(0 \leq x \leq 99) = e^{-10}\frac{1}{x!}$ when $x \leq 99$

$P(x=100) = \sum_{x=100}^{10000} e^{-10}\frac{1}{x!}$ when $x = 100$

$\therefore$ Poisson PMF:

$$P_{(x)}x = \begin{cases} e^{-10}\frac{1}{x!}, & \text{if } x=0,1,2\cdots 99 \\ \sum_{x=100}^{10000} e^{-10}\frac{1}{x!}, & \text{if } x=100 \\ 0, & \text{otherwise} \end{cases}$$

(c) if customers more than 100 it means $x = 101, 102 \cdots 10000$

$$\therefore \begin{cases} \text{Exact } P_{(x)} = \sum_{x=101}^{10000} \binom{10000}{x}(0.001)^x(1-0.001)^{10000-x} \\ \text{Poisson } P_{(x)} = \sum_{x=101}^{10000} e^{-10}\frac{1}{x!} \end{cases}$$

4. (1) $Y = X \bmod (2)$

Let X be $0, 1, 2, \cdots 9$

$\therefore$ $Y = X \bmod (2)$ is 0 or 1

when $Y=0$, $P(0) + P(2) + P(4) + P(6) + P(8) = \frac{5}{10} = \frac{1}{2}$

whe $Y=1$, $P(1) + P(3) + P(5) + P(7) + P(9) = \frac{5}{10} = \frac{1}{2}$

$$\text{PMF}: P_Y(y) = \begin{cases} \frac{1}{2}, & \text{if } y=0 \\ \frac{1}{2}, & \text{if } y=1 \\ 0, & \text{otherwise} \end{cases}$$

(2) $Y = 7 \bmod (X+1)$

$\therefore$ when $X = \begin{cases} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{cases}$, $Y = \begin{cases} 0 \\ 1 \\ 1 \\ 3 \\ 2 \\ 1 \\ 0 \\ 7 \\ 7 \\ 7 \end{cases}$

$$\text{PMF}: P_Y(y) = \begin{cases} \frac{2}{9}, & \text{if } y=0 \\ \frac{3}{9}, & \text{if } y=1 \\ \frac{1}{9}, & \text{if } y=2 \\ \frac{1}{9}, & \text{if } y=3 \\ \frac{1}{3}, & \text{if } y=7 \\ 0, & \text{otherwise} \end{cases}$$

5. (1) $\because -1 \le X \le 3$, there are 5 values

$0 \le Y - X \le 2$, there are 3 values of $Y-X$

$\therefore$ $5 \times 3 = 15$ possibility

Tabular Form :

y \ x	-1	0	1	2	3	
5					$\frac{1}{15}$	$\to \frac{1}{15}$
4				$\frac{1}{15}$	$\frac{1}{15}$	$\to \frac{2}{15}$
3			$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\to \frac{3}{15}$
2		$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$		$\to \frac{3}{15}$
1	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$			$\to \frac{3}{15}$
0	$\frac{1}{15}$	$\frac{1}{15}$				$\to \frac{2}{15}$
-1	$\frac{1}{15}$					$\to \frac{1}{15}$
	$\downarrow \frac{3}{15}$	$\downarrow \frac{3}{15}$	$\downarrow \frac{3}{15}$	$\downarrow \frac{3}{15}$	$\downarrow \frac{3}{15}$	

$\therefore$ marginal PMF $P_X(x) = \begin{cases} \frac{1}{5}, & \text{if } x = -1, 0, 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$

$$P_Y(y) = \begin{cases} \frac{1}{15}, & \text{if } y = -1, 5 \\ \frac{2}{15}, & \text{if } y = 0, 4 \\ \frac{3}{15}, & \text{if } y = 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$$

$E(X) = \frac{1}{5}(-1+0+1+2+3) = 1$

$E(Y) = \frac{1}{15}(-1+5) + \frac{2}{15}(0+4) + \frac{3}{15}(1+2+3) = 2$

(2) Profit $= 200E(X) + 500E(Y)$

$= 1200$

6. For x or y can be $0,1,2,3,4,5$; $0 \le x+y \le 5$

$P_{(1s)} = \frac{1}{6}$

In binomial,

mariginal $P_X(x) = \binom{5}{x} (\frac{1}{6})^x (\frac{5}{6})^{5-x}$

$\therefore$ For y, rolls can be $5-x$

values can be $2,3,4,5,6$

for 6s value $P_y = \frac{1}{5}$

$\therefore P_{Y|X}(y|x) = \binom{5-x}{y} (\frac{1}{5})^y (\frac{4}{5})^{5-x-y}$

$\therefore P_{XY}(x,y) = P_X(x) \cdot P_{Y|X}(y|x)$

$= \binom{5}{x} (\frac{1}{6})^x (\frac{5}{6})^{5-x} \binom{5-x}{y} (\frac{1}{5})^y (\frac{4}{5})^{5-x-y}$