AIM 5002 Computational Statistics and Probability

(Spring 2021)

Assignment 4

Name:_____   Score:_____/5

Submit your assignment at CANVAS by uploading your file.
Due date: Tuesday, 23rd of the March, 2021 by 11:59 PM

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1. If X is a random variable that is uniformly distributed between -1 and 1, find the PDF of e^X.

2. Suppose that a random variable X satisfies

$$E[X] = 0, E[X^2] = 1, E[X^3] = 1, E[X^4] = 0$$

   and let
$$Y = a + bX + cX^2$$
   Find the correlation coefficient $\sigma(X, Y)$.

3. A retired professor comes to the office at a time which is uniformly distributed between 9 a.m. and 1 p.m., performs a single task, and leaves when the task is completed. The duration of the task is exponentially distributed with parameter $\lambda(y) = 1/(5-y)$, where y is the length of the time interval between 9 a.m. and the time of his arrival.
   (a) What is the expected amount of time that the professor devotes to the task?

(b) What is the expected time at which the task is completed?

4. Let X be a random variable that takes the values 0, 1, 3, 5:

   P(X=0) = 1/3, P(X=1) = 1/5, P(X= 3) = 1/5, P(X=5) = 4/15.

   Find the transform associated with X and use it to obtain the mean and variance.

5. A pizza parlor serves n different types of pizza, and is visited by a number of K of customers in a given period of time, where K is a nonnegative integer random variable with a known associated transform $M_K(s) = E[e^{sK}]$. Each customer orders a single pizza with all types of pizza being equally likely, independent of the number of other customers and the types of pizza they order. Give a formula, in terms of $M_K(*)$ for the expected number of different types of pizzas ordered.