1. $g \rightarrow$ geniuses ; $c \rightarrow$ Chocolate

$P(g) = 0.4$;

$P(c) = 0.7$;

$P(g \cap c) = 0.3$;

$\because P(g) + P(c) - P(g \cap c) + P(g^c) + P(c^c) = 1$

$\therefore P(g^c) + P(c^c) = 1 - (0.4 + 0.7 - 0.3)$

$= 1 - 0.8$

$= 0.2$

2. $P(odd) = \frac{2}{3}$;

$P(even) = \frac{1}{3}$;

outcome greater than 4 : 5 ; 6

$P(odd_i) = \frac{1}{3}$ → probability of each number in odd numbers

$P(even_i) = \frac{1}{3}$ → probability of even number in even numbers

$\because P(odd) \times P(odd_i) = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$,

$P(even) \times P(even_i) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

$\therefore P(5) + P(6) = \frac{2}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$

3.(a). $P(1^{st}_i) = \frac{1}{4}$ → Probability of each number in first die

$P(2^{nd}_i) = \frac{1}{4}$ → Probability of each number in second die

double → (1,1) (2,2) (3,3) (4,4)

$\therefore P(double) = \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4}$

$= \frac{1}{16} \times 4$

$= \frac{1}{4}$

(b). sum of less than 4 → (1,1) (1,2) (1,3) (2,1) (2,2) (3,1)

$P(less4) = \frac{6}{16}$

double → (1,1) (2,2) (3,3) (4,4)

$P(double) = \frac{4}{16}$

$P(less4 \mid double) = \frac{2}{4}$

$P(double \mid less4) = \frac{P(less4 \mid double) P(double)}{P(less4)}$

$= \frac{\frac{2}{4} \times \frac{4}{16}}{\frac{6}{16}} = \frac{\frac{2}{16}}{\frac{6}{16}} = \frac{1}{3}$

(c) At least one die roll is 4 → (1,4) (4,1) (2,4) (4,2) (3,4) (4,3) (4,4)

2 die4 → both die are 4

1st die4 → first die is 4

2nd die4 → second die is 4

$P(2die4) + P(1^{st}die4) + P(2^{nd}die4)$

$= \frac{1}{16} + \frac{3}{16} + \frac{3}{16} = \frac{7}{16}$

(d) $P(1^{st} \text{ die } 4) = \frac{4}{16} \begin{cases} (4,1)\ (4,3) \\ (4,2)\ (4,4) \end{cases}$

$P(2^{nd} \text{ die } 4) = \frac{4}{16} \begin{cases} (1,4)\ (2,4) \\ (3,4)\ (4,4) \end{cases}$

two Die diff : (1,2) (1,3) (1,4)
(2,1) (2,3) (2,4)
(3,1) (3,2) (3,4)
(4,1) (4,2) (4,3)

$P(\text{two diff}) = \frac{12}{16}$

$P(1^{st} \text{ die } 4 \mid \text{two diff}) + P(2^{nd} \text{ die } 4 \mid \text{two diff})$

$= \dfrac{P(\text{two diff} \mid 1^{st} \text{ die } 4)P(1^{st} \text{ die } 4)}{P(\text{two diff})} + \dfrac{P(\text{two diff} \mid 2^{nd} \text{ die } 4)P(2^{nd} \text{ die } 4)}{P(\text{two diff})}$

$= \dfrac{\frac{3}{4} \times \frac{4}{16}}{\frac{12}{16}} + \dfrac{\frac{3}{4} \times \frac{4}{16}}{\frac{12}{16}} = \dfrac{\frac{3}{16} + \frac{3}{16}}{\frac{12}{16}} = \frac{6}{12} = \frac{1}{2}$

4. 3^{rd} : the opposite is head when it was tolled tail

$P(\text{tail}) = \frac{3}{6}$, $P(\text{tail} \mid 3^{rd}) = \frac{1}{2}$

$P(3^{rd}) = 1/3$

$P(3^{rd} \mid \text{tail}) = \dfrac{P(\text{tail} \mid 3^{rd})P(3^{rd})}{P(\text{tail})} = \dfrac{\frac{1}{2} \times \frac{1}{3}}{\frac{3}{6}} = \frac{1}{3}$

5. (a) k^{th} is either 0 or 1

$P(0 \text{ correct}) = \frac{1}{2} \times (1-0.2) = \frac{1}{2} \times 0.8 = 0.4$

$P(1 \text{ correct}) = \frac{1}{2} \times (1-0.1) = \frac{1}{2} \times 0.9 = 0.45$

$P(k^{th} \text{ correct}) = P(0 \text{ correct}) + P(1 \text{ correct}) = 0.85$

(b) $P(1c) = 0.9$; $P(0c) = 0.8$

$P(10110 \text{ correct}) = P(1c) \cap P(0c) \cap P(1c) \cap P(1c) \cap P(0c)$

$= 0.9 \times 0.8 \times 0.9 \times 0.9 \times 0.8$

$= 0.46656$

(c) three symbol string : (0, 0, 1)
(0, 1, 0)
For only two symbol are detected correct, (1, 0, 0)
(0, 0, 0)

$P(\text{two } 0c) = 0.8 \times 0.8 \times 0.2 = 0.128$

$P(\text{three } 0c) = 0.8 \times 0.8 \times 0.8 = 0.512$

$P(c) = P(\text{two } 0c) \times 3 + P(\text{three } 0c)$

$= 0.384 + 0.512$

$= 0.896$

5. (d) $P(0|010) = \dfrac{P(010|0) \cdot P(0)}{P(010)}$

$= \dfrac{P(010|0) \cdot P(0)}{P(010|0)P(0) + P(010|1) \cdot P(1)}$

$\because P(0) = \frac{1}{2}, \ P(1) = \frac{1}{2}$

$\therefore = \dfrac{(0.8 \times 0.2 \times 0.8) \cdot (\frac{1}{2})}{(0.8 \times 0.2 \times 0.8)\frac{1}{2} + (0.1 \times 0.9 \times 0.1)\frac{1}{2}}$

$= \dfrac{0.064}{0.064 + 0.0045}$

$= \dfrac{0.064}{0.0685}$

$= 0.9343$