1. Let $x$ be the number of points

| game | | $X_{first}$ | | |
|---|---|---|---|---|
| | P | 0 | 1 | 2 |
| $X_{second}$ | 0 | 0.12 | 0.14 | 0.14 |
| | 1 | 0.09 | 0.105 | 0.105 |
| | 2 | 0.09 | 0.105 | 0.105 |

$P_1(\text{not lose}) = 0.7$

$P_1(\text{win}) = P_1(\text{tie}) = 0.35$

$P_1(\text{lose}) = 0.3$

$P_2(\text{not lose}) = 0.6$

$P_2(\text{win}) = P_2(\text{tie}) = 0.3$

$P_2(\text{lose}) = 0.4$

Points : 0, 1, 2, 3, 4

$X = X_1 + X_2$

PMF: $$P_X(x) = \begin{cases} 0.12, & \text{if } x=0 \\ 0.23, & \text{if } x=1 \\ 0.335, & \text{if } x=2 \\ 0.21, & \text{if } x=3 \\ 0.105, & \text{if } x=4 \\ 0, & \text{otherwise} \end{cases}$$

2. The probability of Fischer win at round $i$ is $P_i = 0.3^{i-1} \times 0.4$

$$\therefore P_{\text{Fischer win}} = \sum_{i=1}^{10} P_i = \sum_{i=1}^{10} 0.3^{i-1} \times 0.4$$

$$\approx 1.42856 \times 0.4$$

$$\approx 0.571424$$

The probability of spassky win at round $i$ is $Q_i = 0.3^{i-1} \times 0.3$

For $i$ in $1 \sim 9$

$$P_{\text{either win}} = 0.3^{i-1} \times 0.4 + 0.3^{i-1} \times 0.3$$
$$= 0.3^{i-1} \times 0.7$$

For $i$ in 10

$$P_{\text{match}} = 1 - 0.3^{i-1} \times 0.7$$

$$\therefore \text{PMF: } P_{\text{match}} = \begin{cases} 0.3^{i-1} \times 0.7, & \text{if } x = 1 \text{ to } 9 \\ 1 - \sum_{i=1}^{9} 0.3^{i-1} \times 0.7, & \text{if } x = 10 \\ 0, & \text{otherwise} \end{cases}$$

3. (a) Let $x$ is number of modems

assume that $y$ is number of customers need to connect

PMF: When $y \leq 99$, means $x \leq 99$

$$P_{(0 \leq x \leq 99)} = \binom{10000}{x}(0.001)^x(1-0.001)^{10000-x}$$

When $y \geq 100$, means $x = 100$

$$P(x=100) = \sum_{x=100}^{10000} \binom{10000}{x}(0.001)^x(1-0.001)^{10000-x}$$

$$\therefore P_{(x)}^{x} \begin{cases} \binom{10000}{x}(0.001)^x(1-0.001)^{10000-x}, & \text{if } x=0,1,2,\cdots 99 \\ \sum_{x=100}^{10000} \binom{10000}{x}(0.001)^x(1-0.001)^{10000-x}, & \text{if } x=100 \\ 0, & \text{otherwise} \end{cases}$$

(b) For poisson method

$\lambda = n \cdot p = 10000 \times 0.001 = 10$

$P(0 \leq x \leq 99) = e^{-10}\frac{1}{x!}$ when $x \leq 99$

$P(x=100) = \sum_{x=100}^{10000} e^{-10}\frac{1}{x!}$ when $x=100$

$$\therefore \text{Poisson PMF: } P_{(x)}x = \begin{cases} e^{-10}\frac{1}{x!}, & \text{if } x=0,1,2\cdots 99 \\ \sum_{x=100}^{10000} e^{-10}\frac{1}{x!}, & \text{if } x=100 \\ 0, & \text{otherwise} \end{cases}$$

(c) if customers more than 100

it means $x = 101, 102 \cdots 10000$

$$\therefore \begin{cases} \text{Exact } P_{(x)} = \sum_{x=101}^{10000} \binom{10000}{x}(0.001)^x(1-0.001)^{10000-x} \\ \text{Poisson } P_{(x)} = \sum_{x=101}^{10000} e^{-10}\frac{1}{x!} \end{cases}$$

4. (1) $Y = X \bmod (2)$

Let $X$ be $0, 1, 2, \cdots 9$

$\therefore$ $Y = X \bmod (2)$ is 0 or 1

when $Y=0$, $P(0) + P(2) + P(4) + P(6) + P(8) = \frac{5}{10} = \frac{1}{2}$

whe $Y=1$, $P(1) + P(3) + P(5) + P(7) + P(9) = \frac{5}{10} = \frac{1}{2}$

PMF : $P_Y(y) = \begin{cases} \frac{1}{2}, & \text{if } y=0 \\ \frac{1}{2}, & \text{if } y=1 \\ 0, & \text{otherwise} \end{cases}$

(2) $Y = 7 \bmod (X+1)$

$\therefore$ when $X = \begin{cases} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{cases}$, $Y = \begin{cases} 0 \\ 1 \\ 1 \\ 3 \\ 2 \\ 1 \\ 0 \\ 7 \\ 7 \\ 7 \end{cases}$

PMF : $P_Y(y) = \begin{cases} \frac{2}{9}, & \text{if } y=0 \\ \frac{3}{9}, & \text{if } y=1 \\ \frac{1}{9}, & \text{if } y=2 \\ \frac{1}{9}, & \text{if } y=3 \\ \frac{1}{3}, & \text{if } y=7 \\ 0, & \text{other wise} \end{cases}$

5. (1) $\because -1 \leq X \leq 3$, there are 5 values

$0 \leq Y - X \leq 2$, there are 3 values of $Y-X$

$\therefore$ $5 \times 3 = 15$ possibility

Tabular Form :

| y \ x | -1 | 0 | 1 | 2 | 3 | |
|---|---|---|---|---|---|---|
| 5 | | | | | $\frac{1}{15}$ | $\to \frac{1}{15}$ |
| 4 | | | | $\frac{1}{15}$ | $\frac{1}{15}$ | $\to \frac{2}{15}$ |
| 3 | | | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ | $\to \frac{3}{15}$ |
| 2 | | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ | | $\to \frac{3}{15}$ |
| 1 | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ | | | $\to \frac{3}{15}$ |
| 0 | $\frac{1}{15}$ | $\frac{1}{15}$ | | | | $\to \frac{2}{15}$ |
| -1 | $\frac{1}{15}$ | | | | | $\to \frac{1}{15}$ |
| | $\downarrow \frac{3}{15}$ | $\downarrow \frac{3}{15}$ | $\downarrow \frac{3}{15}$ | $\downarrow \frac{3}{15}$ | $\downarrow \frac{3}{15}$ | |

$\therefore$ marginal PMF $P_X(x) = \begin{cases} \frac{1}{5}, & \text{if } x = -1, 0, 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$

$P_Y(y) = \begin{cases} \frac{1}{15}, & \text{if } y = -1, 5 \\ \frac{2}{15}, & \text{if } y = 0, 4 \\ \frac{3}{15}, & \text{if } y = 1, 2, 3 \\ 0, & \text{other wise} \end{cases}$

$E(x) = \frac{1}{5}(-1+0+1+2+3) = 1$

$E(y) = \frac{1}{15}(-1+5) + \frac{2}{15}(0+4) + \frac{3}{15}(1+2+3) = 2$

(2) Profit $= 200 E(x) + 500 E(y)$

$= 1200$

6. For $x$ or $y$ can be 0, 1, 2, 3, 4, 5 ; $0 \le x+y \le 5$

$P_{(1s)} = \frac{1}{6}$

In binomial,

mariginal $P_X(x) = \binom{5}{x}\left(\frac{1}{6}\right)^x\left(\frac{5}{6}\right)^{5-x}$

$\therefore$ For $y$, rolls can be $5-x$

values can be 2, 3, 4, 5, 6

for 6s value $P_y = \frac{1}{5}$

$\therefore P_{Y|X}(y|x) = \binom{5-x}{y}\left(\frac{1}{5}\right)^y\left(\frac{4}{5}\right)^{5-x-y}$

$\therefore P_{XY}(x,y) = P_X(x) \cdot P_{Y|X}(y|x)$

$= \binom{5}{x}\left(\frac{1}{6}\right)^x\left(\frac{5}{6}\right)^{5-x}\binom{5-x}{y}\left(\frac{1}{5}\right)^y\left(\frac{4}{5}\right)^{5-x-y}$