1. $\frac{650}{10000} = 6.5\%$, $1 - 6.5\% = 93.5\%$

$\mu = 65$, $\sigma = 5$

According to Normal Central Limit Theorem

$Z = \frac{c-\mu}{\sigma} = \frac{85-65}{5} = 4$

$\Phi(4) = 0.99997 \rightarrow 99.997\%$ more than 93.5%

Thus the applicant who received 85 score can get into the K school.

2. (a) $P(k, \tau) = P(4, 2)$

$= e^{-\lambda\tau} \frac{(\lambda\tau)^k}{k!}$

$= e^{-6} \frac{6^4}{4!}$

$= 0.00248 \cdot \frac{1296}{24}$

$= 0.13392 = 13.39\%$

(b) $\sum_{k=2}^{\infty} P(k, 2) = 1 - (P(0,2) + P(1,2))$

$= 1 - (e^{-6} + e^{-6} \cdot 6)$

$= 1 - (7e^{-6})$

$= 1 - 0.01735$

$= 0.98265 = 98.265\%$

2.(c) Since the Memorylessness and Independence properties of Poisson process. The $P(4,2) = 13.29\%$

3. $P(h) = 0.55$, $P(t) = 0.45$

Based on Bernoulli process,

$P_S(5650) = \binom{10000}{5650} 0.55^{5650} \cdot 0.45^{(10000-5650)}$

$P_S(5400) = \binom{10000}{5400} 0.55^{5400} \cdot 0.45^{(10000-5400)}$

$P_S(5400) - P_S(5650) = 0.001064 - 0.00008458$

$\approx 0.000979 \approx 0.0979\%$

4. $P(c) = \frac{\pi}{4}$, Let X_i shoot in circle as event A whenever A occurs, X_i is 1 and 0 otherwise

$E[X_i] = P \quad X_i = \begin{cases} 1, & \text{if in center} \\ 0, & \text{otherwise} \end{cases}$

Since it's bernoulli, $\sigma^2 = P(1-P) \leq \frac{1}{4}$

Thus $P(|M_n - p| \geq 0.01) \leq \frac{1}{4 \cdot n (0.01)^2} \leq (1 - 95\%)$

$\therefore \frac{1}{n \cdot 0.0004} \leq 0.05$

$50000 \leq n$

Thus. we need 50000 shootings