AIM 5002 Computational Statistics and Probability

(Spring 2021)

Assignment 2

Name:______________________________ Score:_________/5

Submit your assignment at CANVAS by uploading your file.
Due date: Wednesday, 17th of the February, 2021 by 11:59 PM

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1. Yeshiva soccer team has 2 games scheduled for one weekend. It has a 0.7 probability of not losing the first game, and a 0.6 probability of not losing the second game, independent of the first. If it does not lose a particular game, the team is equally likely to win or tie, independent of what happens in the other game. The Yeshiva team will receive 2 points for a win, 1 for a tie, and 0 for a loss. Find the PMF of the number of points that the team earns over the weekend (0.5 point).

2. Fischer and Spassky play a chess match in which the first player to win a game wins the match. After 10 successive draws, the match is declared drawn. Each game is won by Fischer with probability 0.4 is won by Spassky with probability 0.3 and is a draw with probability 0.3 independent of previous games (1 point).
   (A) What is the probability that Fischer wins the match?
   (B) What is the PMF of the duration of the match?

3. An internet service provider uses 100 modems to serve the needs of 10,000 customers. It is estimated that at a given time, each customer will need a connection with probability 0.001, independent of the other customers (1.5 points).
   (A) What is the PMF of the number of modems in use at the given time?
   (B) Repeat (A) by approximating the PMF of the number of customers that need a connection with a Poisson PMF
   (C) What is the probability that there are more customers needing a connection than there are modems? Provide an exact as well as an approximate formula based on the Poisson approximation of (B)

4. Let X be a random variable that takes values from 0 to 9 with equal probability 1/10 (1 point)
   (A) Find the PMF of the random variable Y = X mod(2)
   (B) Find the PMF of the random variable Y = 7 mod(X+1)

5. A stock market trader buys 200 shares of stock A and 500 shares of stock B. Let X and Y be the price changes of A and B respectively over a certain time period and assume that the joint PMF of X and Y is uniform over the set of integers x and y satisfying

$$-1 \leq x \leq 3, \qquad 0 \leq y - x \leq 2 \qquad \text{(1 point)}$$

   (A) Find the marginal PMFs and the means of X and Y
   (B) Find the mean of the trader's profit

6. Consider five independent rolls of a 6-sided die. Let X be the number of 1s and let Y be the number of 6s obtained. What is the joint PMF of X and Y? (Bonus)