

# AIM 5002 Computational Statistics and Probability

(Spring 2021)

## Assignment 1

Name: \_\_\_\_\_ Score: \_\_\_\_\_/4

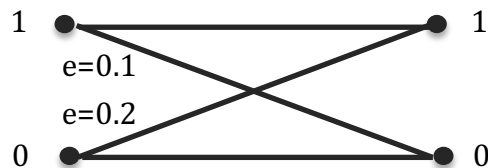
Submit your assignment at CANVAS by uploading your file.

**Due date: Monday, 8<sup>th</sup> of the February, 2021 by 11:59 PM**

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1. Out of the students in a class, 40% are geniuses, 70% love chocolate, and 30% fall into both categories, Determine the probability that a randomly selected students is neither a genius nor a chocolate lover (0.5 point).
  
2. A six-sided die is loaded in a way that each odd face is twice as likely as each even face. All odd faces are equally likely, as are all even faces. Construct a probabilistic model for a single roll of this die and find the probability that the outcome is greater than 4 (0.5 point).
  
3. We roll two fair 4-sided dice. Each one of the possible outcomes is assumed to be equally likely (1 points).
  - (a) Find the probability that doubles are rolled.
  
  - (b) Given that the roll results in a sum of 4 or less, find the conditional probability that doubles are rolled.
  
  - (c) Find the probability that at least one die roll is a 4.
  
  - (d) Given that the two dice land on different numbers, find the conditional probability that at least one die roll is a 4.

4. We are given three coins: one has heads in both faces, the second has tail in both faces, and the third has a head in one face and a tail in the other. We choose a coin at random, toss it, and the result is tails. What is the probability that the opposite face is heads (0.5 point)?
5. A source transmits a message (a string of symbols) through a noisy communication channel. Each symbol is 1 or 0 with the same probability  $\frac{1}{2}$  and is received incorrectly with probability 0.1 and 0.2 respectively. Errors in different symbol transmissions are independent (1.5 points + 0.5 point bonus).



- (a) What is the probability that the  $k$ th symbol is received correctly?
- (b) What is the probability that the string of symbols 10110 is received correctly?
- (c) In an effort to improve reliability, each symbol is transmitted three times and the received string is decoded by majority rule. In other words, a 0 (or 1) is transmitted as 000 (or 111, respectively), and it is decoded at the receiver as a 0 (or 1) if and only if the received three-symbol string contains at least two 0s (or 1s, respectively). What is the probability that a 0 is correctly decoded?
- (d) Suppose that the scheme of part (c) is used. What is the probability that a symbol was 0 given that the received string is 010 (bonus)?