AIM 5002 Computational Statistics and Probability

(Spring 2021)

Assignment 3

Name:______________________________ Score:________/5

Submit your assignment at CANVAS by uploading your file.
Due date: Tuesday, 2nd of the March, 2021 by 11:59 PM

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1. In the bank, when you deposit your money, you equally likely find 0 or 1 customer ahead of you. The service time of the customer ahead, if present, is exponentially distributed with parameter λ. Find the CDF of your waiting time (0.5 point).

2. Let X and Y be normal random variables with means 0 and 1, respectively, and variances 1 and 9, respectively (0.5 point)
   (a) Find $P(X \leq 1.5)$ and $P(X \leq -1)$
   (b) Find $P(-5 \leq Y \leq 1)$

3. A city's temperature is modeled as a normal random variable with mean and standard deviation equal to 3 degree Celsius and 7 degree Celsius respectively. What is the probability that the temperature at a randomly chosen time will be less than or equal to 50 degrees Fahrenheit (Fahrenheit = 1.8 * Celsius + 32) (0.3 points)?

4. A point is chosen at random (according to a uniform PDF) within a quarter circle of the form $\{(x, y) \mid x^2 + y^2 \leq r^2, x \geq 0, y \geq 0\}$, for some given $r > 0$ (0.6 points).
   (a) Find the joint PDF of the coordinates X and Y of the chosen point
   (b) Find the marginal PDF of Y

5. Let X be a random variable with PDF (1.5 points)

$$f_X(x) = \begin{cases} x/4, & if\ 1 < x \leq 3 \\ 0, & otherwise \end{cases}$$

   And let A be the event $\{X \geq 2\}$.
   (a) Find E[X]
   (b) Find P(A)
   (c) Find $f_{X|A}(x)$
   (d) Find E[X|A]
   (e) Let $Y = X^2$, Find E[Y] and var(Y)

6. Let the random variables X and Y have a joint PDF which is uniform over the triangle with vertices at (0, 0), (0, 2), and (2, 0) (1.5 points).
   (a) Find the joint PDF of X and Y
   (b) Find the marginal PDF of Y
   (c) Find the conditional PDF of X given Y
   (d) Find $E[X|Y = y]$, and use the total expectation theorem to find $E[X]$ in terms of $E[Y]$
   (e) Use the symmetry of the problem to find the value of $E[X]$