1. (a) $std_{x_i} = 1\,m$ $\quad Var_{x_i} = 1^2 = 1\,m$

$Var(M_n) = \frac{\sigma^2}{n} = \frac{1}{n}$

$\because std(M_n) = \sqrt{Var(M_n)} = \frac{1}{\sqrt{n}} \le 0.01\,m$

$\therefore n \geqslant 10000$

(b) $P(|M_n - h| \le 0.05) \geqslant 0.99$

$\because h = \mu = E[M_n]$, $var(M_n) = \frac{1}{n}$

$\therefore P(|M_n - E[M_n]| \geqslant 0.05) \le \frac{\frac{1}{n}}{(0.05)^2}$

$0.99 \leftarrow \quad \rightarrow \frac{\frac{1}{n}}{(0.05)^2}$

0.05

$\therefore \frac{\frac{1}{n}}{(0.05)^2} + 0.99 = 1$

$n = 40000$

$\therefore n \geqslant 40000$

2. (a) $S_n = X_1 + X_2 \cdots + X_n$, $\mu = 5$, $\sigma^2 = 9$

$E[S_n] = n \cdot 5$

$Var(S_n) = n \cdot 9$

$\sigma_n = \sqrt{var(S_n)} = 3\sqrt{n}$

$P(S_{100} < 440) = P(S_{100} \le 439)$

$= P\left(\frac{S_{100} - 500}{3\sqrt{100}} \le \frac{439 - 500}{3\sqrt{100}}\right)$

$\approx \Phi\left(\frac{439-500}{30}\right) = \Phi(-2.03)$

$= 1 - \Phi(2.03) = 1 - 0.97882 = 0.02118$

(b) $P(S_n \geqslant 200 + 5n) = P\left(\frac{S_n - 5n}{3\sqrt{n}} \geqslant \frac{200 + 5n - 5n}{3\sqrt{n}}\right) \le 0.05$

$= \left(1 - \Phi\left(\frac{200}{3\sqrt{n}}\right)\right) \le 0.05$

$\Phi\left(\frac{200}{3\sqrt{n}}\right) \geqslant 0.95 \approx \Phi(1.65)$

$\therefore \frac{200}{3\sqrt{n}} \geqslant 1.65$

$n \le 1632.48$

$\therefore$ The largest n can be 1632

(c)

$N = 220$ is the first day exceeds 1000

$\therefore S_{219} \le 1000$

$P(N \geqslant 220) = P(S_{219} \le 1000)$

$= P\left(\frac{S_{219} \times 5 \times 219}{3\sqrt{219}} \le \frac{1000 - 5 \times 219}{3\sqrt{219}}\right)$

$= \Phi(-2.14) = 1 - \Phi(2.14)$

$= 1 - 0.98382 = 0.01618$

3.

M: During 8 am to 10 am : $\lambda = 6$, $\tau = 2$

N: During 10 am to 12 pm : $\lambda = 2$, $\tau = 2$

M + N : $\lambda\tau = 6 \times 2 + 2 \times 2 = 16$

$\therefore$ Possion PMF : $P = e^{-16} \frac{(16)^k}{k!}$, $k = 0, 1, \dots$

4. (a) 0 fish during 0-3 hours

$P(0,3) = e^{-0.6\times 3} \frac{(0.6 \times 3)^0}{0!} = e^{-1.8} = 0.165$

(b) 0 fish during 0-3 hours

at least 1 fish during 3-5 hours

$P(0,3) \cdot (1 - P(0,2))$

$= e^{-1.8} \cdot (1 - e^{-1.2}) = 0.165 \times 0.7 = 0.1155$

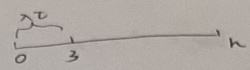
(c) $1 - (P(0,3) + P(1,3)) = 1 - (e^{-1.8} + e^{-1.8} \cdot 1.8)$

$= 1 - (0.165 + 0.165 \times 1.8)$

$= 1 - 0.462$

$= 0.538$

At least catch 2, means the total fishing time is 3 hours

(d) 

During 0 ~ 3 . $E[n\text{-fish}] = \lambda\tau = 0.6 \times 3 = 1.8$

During 3 ~ ∞ . $E(n\text{-fish}) = P(0,3) = 0.165$

$P(n\text{-fish}) = 1.8 + 0.165 = 1.965$

(e) $E(1\ \text{fish}) = \frac{1}{\lambda} = \frac{1}{0.6} = 1.\bar{6}$ hours

$5 + 1.\bar{6} = 6.\bar{6}$ hours

5. (a)

type A : $\lambda = 3$; type B : $\lambda = 4$

$\lambda\tau = (3+4) \times 3 = 21$

$\therefore E[N_\tau] = 21$; $Var[N_\tau] = 21$

PMF. $P_{(k,3)} = e^{-21} \frac{21^k}{k!}$, $k = 0, 1, \dots$

(b) $\frac{\lambda_A}{\lambda_A + \lambda_B} = \frac{3}{3+4} = \frac{3}{7}$

Binomial: $\binom{10}{3} \left(\frac{3}{7}\right)^3 \cdot \left(\frac{4}{7}\right)^7$

$= 120 \cdot 0.0787 \cdot 0.01989$

$= 0.188$