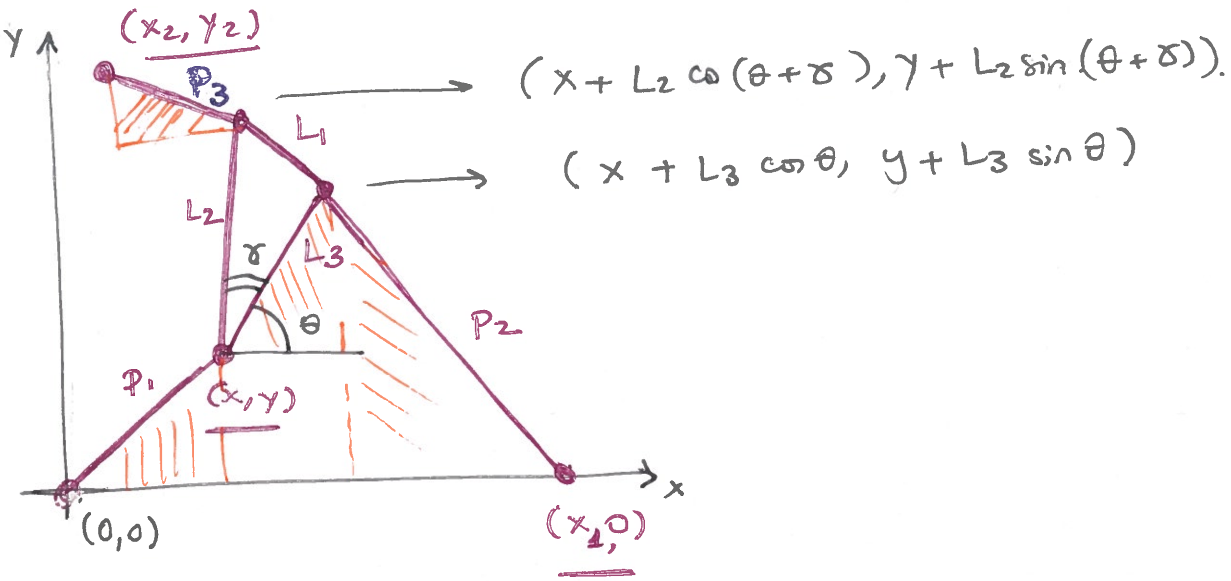
PROJECT 1 : ROBOTIC ARM



P_1, P_2, P_3 = variable lenght

L_1, L_2, L_3 = fixed lenght

$(x_1, 0), (x_2, y_2)$ = known.

Problem: Compute (x, y, θ) for each given P_1, P_2, P_3

(*)

- $P_1^2 = x^2 + y^2.$
- $P_2^2 = (\underbrace{x - x_1 + L_3\cos\theta}_{A_2})^2 + (\underbrace{y + L_3 \sin\theta}_{B_2})^2.$
- $P_3^2 = (\underbrace{x - x_2 + L_2 \cos(\theta+\gamma)}_{A_3})^2 + (\underbrace{y - y_2 + L_2 \sin(\theta+\gamma)}_{B_3})^2$

$$= (x - x_2 + L_2 \cos\theta \cos\gamma - L_2 \sin\theta \sin\gamma)^2$$

$$+ (y - y_2 + L_2 \sin\theta \cos\gamma + L_2 \cos\theta \sin\gamma)^2$$

With the notation

$$A_2 = -x_1 + L_3 \cos\theta$$

$$B_2 = L_3 \sin\theta$$

$$A_3 = -x_2 + L_2 \cos\theta \cos\gamma - L_2 \sin\theta \sin\gamma$$

$$B_3 = -y_2 + L_2 \sin\theta \cos\gamma + L_2 \cos\theta \sin\gamma .$$

$$\begin{aligned}
p_2^2 &= (x + A_2)^2 + (y + B_2)^2 . \\
&= x^2 + y^2 + 2A_2 x + 2B_2 y + A_2^2 + B_2^2 . \\
&= p_1^2 + 2A_2 x + 2B_2 y + A_2^2 + B_2^2
\end{aligned}$$

$$\begin{aligned}
p_3^2 &= (x + A_3)^2 + (y + B_3)^2 . \\
&= x^2 + y^2 + 2A_3 x + 2B_3 y + A_3^2 + B_3^2 \\
&= p_1^2 + 2A_3 x + 2B_3 y + A_3^2 + B_3^2
\end{aligned}$$

$$\begin{aligned}
2A_2 x + 2B_2 y &= p_2^2 - p_1^2 - A_2^2 - B_2^2 . \\
2A_3 x + 2B_3 y &= p_3^2 - p_1^2 - A_3^2 - B_3^2 .
\end{aligned}$$

SOLVE FOR x, y (LINEAR 2×2 SYSTEM, CRAMER'S RULE).

$$x = \frac{N_1}{D} = \frac{B_3(p_2^2 - p_1^2 - A_2^2 - B_2^2) - B_2(p_3^2 - p_1^2 - A_3^2 - B_3^2).}{2(A_2 B_3 - A_3 B_2).} \qquad **$$

$$y = \frac{N}{D} = \frac{-A_3(p_2^2 - p_1^2 - A_2^2 - B_2^2) + A_2(p_3^2 - p_1^2 - A_3^2 - B_3^2)}{2(A_2 B_3 - A_3 B_2).} \qquad ***$$

Provided $D = 2(A_2 B_3 - A_3 B_2) \neq 0$

SUBSTITUTE x and y in ,

$$p_1^2 = \frac{N_1^2}{D^2} + \frac{N_2^2}{D^2}$$

$$\Rightarrow \quad N_1^2 + N_2^2 - p_1^2 D^2 = 0$$

In A_2, A_3, B_2, B_3 and consequently in N_1, N_2, D the only unknown is θ.

$$f(\theta) = N_1^2 + N_2^2 - p_1^2 D^2$$

is a trigonometric polynomial in θ

more precisely in $\sin\theta$, $\cos\theta$.

Project:

1) Write a function whose input is $L_1, L_2, L_3, x_1, y_1, x_2, y_2, \gamma$ and whose output is $f(\theta)$.

Test your function for some parameter values.

$L_1 = 2$, $L_2 = L_3 = \sqrt{2}$; $\gamma = \frac{\pi}{2}$; $p_1 = p_2 = p_3 = \sqrt{5}$.

2) Plot $f(\theta)$ for θ in $[-\pi, \pi]$.

Approximately localize the roots of $f(\theta) = 0$.

3) Solve for θ using one of the equation solvers from Chapter 1. Use values for $L_1, L_2, L_3, x_1, y_1, x_2, y_2, \gamma$ as in the textbook.
(There can be at most six roots for θ).
Then solve for x and y using. **, ***