0. Readme

The below derivations are based on the first term ($2\pi G\rho$*{part1}$|_{r'=r_1}^{r'=r_2}$) and second term ($2\pi G\rho$*(part 2)$|_{r'=r_1}^{r'=r_2}\begin{cases}+1 & r>r_2\\ -1 & r<r_1\end{cases}$ in Eq . (3) (c.f. Eq. (52) of Heck and Seitz 2007), where r3 represents r'.

1. Vz = Vzpart1 + Vzpart2 in Eq. (5)

In[1]:=

$$\text{Vzpart1} = \text{FullSimplify}\Big[\text{D}\Big[\Big(\frac{1}{3*r}*\left(\sqrt{r^2+r3^2-2*r*r3*\text{Cos}[\psi]}\right)^3+\frac{1}{2}*\sqrt{r^2+r3^2-2*r*r3*\text{Cos}[\psi]}*\text{Cos}[\psi]*(r3-r*\text{Cos}[\psi])+\frac{1}{2}*r^2*\text{Cos}[\psi]*(\text{Sin}[\psi])^2*\text{Log}\left[\sqrt{r^2+r3^2-2*r*r3*\text{Cos}[\psi]}+r3-r*\text{Cos}[\psi]\right]\Big), r\Big]\Big]\ /.\ \left\{\sqrt{r^2+r3^2-2\,r\,r3\,\text{Cos}[\psi]}\rightarrow l\right\}$$

Out[1]=

$$\frac{1}{6\,r^2}\left(l\left(r^2-2\,r3^2-3\,r^2\,\text{Cos}[2\,\psi]\right)+r\,\text{Cos}[\psi]\left(-2\,l\,r3+3\,r^2\,\left(1+2\,\text{Log}[l+r3-r\,\text{Cos}[\psi]]\right)\text{Sin}[\psi]^2\right)\right)$$

In[2]:=

$$\text{Vzpart2} = \text{FullSimplify}\left[\text{D}\left[\left(\frac{1}{3*r}*r3^3-\frac{1}{2}*r3^2\right), r\right]\right]$$

Out[2]=

$$-\frac{r3^3}{3\,r^2}$$

2. Vzz= Vzzpart1 + Vzzpart2 in Eq. (7)

In[3]:=

$$\text{Vzzpart1} = \text{FullSimplify}\left[\text{D}\left[\text{Vzpart1}\ /.\ \left\{l\rightarrow\sqrt{r^2+r3^2-2\,r\,r3\,\text{Cos}[\psi]}\right\}, r\right]\right]\ /.\ \left\{\sqrt{r^2+r3^2-2\,r\,r3\,\text{Cos}[\psi]}\rightarrow l\right\}$$

Out[3]=

$$\left(r^4+2\,r^2\,r3^2+4\,r3^4-r\left(2\,r\,r3^2\,\text{Cos}[\psi]^2+3\,r^3\,\text{Cos}[2\,\psi]+\text{Cos}[\psi]\left(r3\left(r^2+4\,r3^2\right)-3\,r^2\left(r3\,\text{Cos}[2\,\psi]+\left(3\,l-2\,r3+2\,l\,\text{Log}[l+r3-r\,\text{Cos}[\psi]]\right)\text{Sin}[\psi]^2\right)\right)\right)\right)\Big/\left(6\,r^3\sqrt{r^2+r3^2-2\,r\,r3\,\text{Cos}[\psi]}\right)$$

In[4]:=

$$\text{Vzzpart2} = \text{FullSimplify}\left[\text{D}\left[\left(\frac{1}{3*r}*r3^3-\frac{1}{2}*r3^2\right), \{r, 2\}\right]\right]$$

Out[4]=

$$\frac{2\,r3^3}{3\,r^3}$$

3. Vzzz= Vzzzpart1 + Vzzzpart2 in Eq. (9)

In[5]:=
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Vzzzpart1 =
 FullSimplify[FullSimplify[D[Vzzpart1 /. {l → Sqrt[r^2 + r3^2 - 2 r r3 Cos[ψ]]}, r]] /.
   {Sqrt[r^2 + r3^2 - 2 r r3 Cos[ψ]] → l}]
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Out[5]=

$$-\Big(\big(4\,r3\,(r^6+3\,r^4\,r3^2+15\,r^2\,r3^4+4\,r3^6)+l\,(r^6+3\,r^4\,r3^2+36\,r^2\,r3^4+16\,r3^6)-2\,r\,(r^6+2\,r^4\,r3\,(l+2\,r3)+8\,r3^5\,(3\,l+4\,r3)+3\,r^2\,r3^3\,(3\,l+10\,r3))\,\mathrm{Cos}[\psi]+r^2\,(4\,r3^3\,(2\,r^2+3\,r3\,(l+3\,r3))\,\mathrm{Cos}[2\,\psi]+r\,(2\,r^4+2\,(l-2\,r3)\,r3^3+3\,r^2\,r3\,(l+2\,r3))\,\mathrm{Cos}[3\,\psi]-r^2\,(4\,r3\,(r^2+r3^2)+l\,(r^2+3\,r3^2))\,\mathrm{Cos}[4\,\psi]+r^3\,r3\,(l+2\,r3)\,\mathrm{Cos}[5\,\psi])\big)\Big/\Big(8\,r^4\,(l+r3-r\,\mathrm{Cos}[\psi])\,(r^2+r3^2-2\,r\,r3\,\mathrm{Cos}[\psi])^{3/2}\Big)\Big)$$

In[6]:=
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Vzzzpart2 = FullSimplify[D[(1/(3*r) * r3^3 - 1/2 * r3^2), {r, 3}]]
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Out[6]=

$$-\frac{2\,r3^3}{r^4}$$

In[7]:=
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NotebookSave[EvaluationNotebook[]];
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