```
In[]:= Clear["Global`*"];
```

1. Readme

1. (r, λ, z) and (r3, λ3, z3) (i.e. (r', λ', z')) are the cylindrical coordinates of the computation and integration points.
2. The integral kernels are focused, i.e. $\int_{z1}^{z2}\int_{\lambda 1}^{\lambda 2}\int_{r1}^{r2}\square\, dr'\, d\lambda'\, dz'$ should be added for the below expressions.
3. The kernels' formulas of the GV and GGT in Table D5 are represented as the following PhyVr, PhyVλ, PhyVz, PhyVrr, PhyVrλ, PhyVrz, PhyVλλ, PhyVλz, and PhyVzz.
4. Regarding the notations in Asgharzadeh et al. (2018) for the following expressions, the computation point (ρ_o, ϕ_o, Z_o) is used as (r, λ, z) and the integration point (ρ_s, ϕ_s, Z_s) is used as (r3, λ3, z3). k = G $\sigma_s \rho_s$ is used as G*ρ*r3. R_{os} is used as L.
5. The GV and GGT for the formulas in Eqs. (4a), (4b), (4c), (5), (6), and Tables 1-2 of Asgharzadeh et al. (2018) are denoted as A2018gr, A2018gλ, A2018gz for the GV and A2018grr, A2018grλ, A2018grz, A2018gλr, A2018gλλ, A2018gλz, A2018gzr, A2018gzλ, A2018gzz for the GGT.
6. After running the below codes, the differences of the GV and GGT between the expressions in Table 7 and the formulas in Asgharzadeh et al. (2018) are all equal to zero.

2. Formulas of the GV and GGT in Table D5 in this study

```
In[]:= l = Sqrt[r^2 + r3^2 - 2 * r * r3 * Cos[λ - λ3] + (z - z3)^2];

In[]:= PhyVr = -G * ρ * r3 * (r - r3 * Cos[λ - λ3]) / l^3;

In[]:= PhyVλ = -G * ρ * r3^2 * Sin[λ - λ3] / l^3;

In[]:= PhyVz = -G * ρ * r3 * (z - z3) / l^3;

In[]:= PhyVrr = G * ρ * r3 *
         (4 * r^2 + r3^2 - 2 * (z - z3)^2 - 8 * r * r3 * Cos[λ - λ3] + 3 * r3^2 * Cos[2 * (λ - λ3)]) / (2 * l^5);

In[]:= PhyVrλ = 3 * G * ρ * r3^2 * Sin[λ - λ3] * (r - r3 * Cos[λ - λ3]) / l^5;

In[]:= PhyVrz = 3 * G * ρ * r3 * (z - z3) * (r - r3 * Cos[λ - λ3]) / l^5;

In[]:= PhyVλλ = G * ρ * r3 * (-2 * r^2 + r3^2 - 2 * (z - z3)^2 +
            4 * r * r3 * Cos[λ - λ3] - 3 * r3^2 * Cos[2 * (λ - λ3)]) / (2 * l^5);

In[]:= PhyVλz = 3 * G * ρ * r3^2 * Sin[λ - λ3] * (z - z3) / l^5;

In[]:= PhyVzz = -G * ρ * r3 * (r^2 + r3^2 - 2 * (z - z3)^2 - 2 * r * r3 * Cos[λ - λ3]) / l^5;
```

3. Formulas of the GV and GGT in Eqs. (4a), (4b), (4c), (5), (6), and Tables 1-2 of Asgharzadeh et al. (2018)

```
In[]:= L = Sqrt[r^2 + r3^2 - 2 * r * r3 * Cos[λ - λ3] + (z - z3)^2];

In[]:= A2018gr = -G * ρ * r3 * (r - r3 * Cos[λ - λ3]) / L^3; (*Eq. (4a) of Asgharzadeh et al.(2018)*)
```

```
In[∘]:= A2018gλ = -G * ρ * r3 * (r3 * Sin[λ - λ3])/L^3;(*Eq. (4b) of Asgharzadeh et al.(2018)*)

In[∘]:= A2018gz = -G * ρ * r3 * (z - z3)/L^3;(*Eq. (4c) of Asgharzadeh et al.(2018)*)

In[∘]:= A2018grr = -G * ρ * r3 * (L^2 - 3 * (r - r3 * Cos[λ - λ3])^2)/L^5;
        (*Tables 1-2 of Asgharzadeh et al.(2018)*)

In[∘]:= A2018grλ =
          1/r * (-G * ρ * r3 * (r3 * L^2 * Sin[λ - λ3] - 3 * r * r3 * Sin[λ - λ3] * (r - r3 * Cos[λ - λ3]))/L^5 -
            A2018gλ);(*Tables 1-2 of Asgharzadeh et al.(2018)*)

In[∘]:= A2018grz = 3 * G * ρ * r3 * ((z - z3) * (r - r3 * Cos[λ - λ3]))/L^5;
        (*Tables 1-2 of Asgharzadeh et al.(2018)*)

In[∘]:= A2018gλr = 3 * G * ρ * r3 * r3 * (Sin[λ - λ3] * (r - r3 * Cos[λ - λ3]))/L^5;
        (*Tables 1-2 of Asgharzadeh et al.(2018)*)

In[∘]:= A2018gλλ = 1/r * (-G * ρ * r3 * r3 * (L^2 * Cos[λ - λ3] - 3 * r * r3 * (Sin[λ - λ3])^2)/L^5 + A2018gr);
        (*Tables 1-2 of Asgharzadeh et al.(2018)*)

In[∘]:= A2018gλz = 3 * G * ρ * r3 * r3 * ((z - z3) * Sin[λ - λ3])/L^5;
        (*Tables 1-2 of Asgharzadeh et al.(2018)*)

In[∘]:= A2018gzr = 3 * G * ρ * r3 * ((z - z3) * (r - r3 * Cos[λ - λ3]))/L^5;
        (*Tables 1-2 of Asgharzadeh et al.(2018)*)

In[∘]:= A2018gzλ = 1/r * 3 * G * ρ * r3 * r * r3 * ((z - z3) * Sin[λ - λ3])/L^5;
        (*Tables 1-2 of Asgharzadeh et al.(2018)*)

In[∘]:= A2018gzz = -G * ρ * r3 * (L^2 - 3 * (z - z3)^2)/L^5;
        (*Tables 1-2 of Asgharzadeh et al.(2018)*)
```

5. Comparison between the formulas in Table D5 and the formulas in Eqs. (4a), (4b), (4c), (5), (6), and Tables 1-2 of Asgharzadeh et al. (2018) for the GV and GGT

```
In[ ]:= FullSimplify[PhyVr - A2018gr]
Out[ ]=
0

In[ ]:= FullSimplify[PhyVλ - A2018gλ]
Out[ ]=
0

In[ ]:= FullSimplify[PhyVz - A2018gz]
Out[ ]=
0

In[ ]:= FullSimplify[PhyVrr - A2018grr]
Out[ ]=
0

In[ ]:= FullSimplify[PhyVrλ - A2018grλ]
Out[ ]=
0

In[ ]:= FullSimplify[PhyVrλ - A2018gλr]
Out[ ]=
0

In[ ]:= FullSimplify[PhyVrz - A2018grz]
Out[ ]=
0

In[ ]:= FullSimplify[PhyVrz - A2018gzr]
Out[ ]=
0

In[ ]:= FullSimplify[PhyVλλ - A2018gλλ]
Out[ ]=
0

In[ ]:= FullSimplify[PhyVλz - A2018gλz]
Out[ ]=
0

In[ ]:= FullSimplify[PhyVλz - A2018gzλ]
Out[ ]=
0

In[ ]:= FullSimplify[PhyVzz - A2018gzz]
Out[ ]=
0
```

6. Test Laplace’s equation for the GGT in Table D5

In[ ]:=
```
Laplace = FullSimplify[PhyVrr + PhyVλλ + PhyVzz]
```
Out[ ]=
```
0
```

In[ ]:=
```
NotebookSave[EvaluationNotebook[]];
```