

# New Kernel with Multi-Scale KV Pooling

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# 1 Forward Pass

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**Algorithm 1** Forward Pass with Multi-Scale KV Pooling and Masking

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**Require:** Matrices  $\mathbf{Q}, \mathbf{K}, \mathbf{V} \in \mathbb{R}^{N \times d}$  in HBM, block sizes  $B_r, B_c$ ,

- 1: sparse-block mask  $mask \in \{0, 1, 0.5, 0.25, 0.125\}^{T_r \times T_c}$
- 2: **Generate multi-scale KV blocks via pooling:**
- 3: pooling(x, zoom\_ratio) applies average pooling to merge each group of zoom\_ratio adjacent tokens into a single token.
- 4:  $\mathbf{K}_1, \mathbf{V}_1 \leftarrow \mathbf{K}, \mathbf{V}$
- 5: Blocks:  $T_c$  blocks of size  $B_c \times d$
- 6:  $\mathbf{K}_2, \mathbf{V}_2 \leftarrow \text{pooling}(\mathbf{K}_1, 2), \text{pooling}(\mathbf{V}_1, 2)$
- 7: Blocks:  $T_c$  blocks of size  $(B_c/2) \times d$
- 8:  $\mathbf{K}_4, \mathbf{V}_4 \leftarrow \text{pooling}(\mathbf{K}_2, 2), \text{pooling}(\mathbf{V}_2, 2)$
- 9: Blocks:  $T_c$  blocks of size  $(B_c/4) \times d$
- 10:  $\mathbf{K}_8, \mathbf{V}_8 \leftarrow \text{pooling}(\mathbf{K}_4, 2), \text{pooling}(\mathbf{V}_4, 2)$
- 11: Blocks:  $T_c$  blocks of size  $(B_c/8) \times d$
- 12:  $T_r \leftarrow \lceil N/B_r \rceil, T_c \leftarrow \lceil N/B_c \rceil$
- 13: Split  $\mathbf{Q}, \mathbf{O}$  into  $\{\mathbf{Q}_i\}, \{\mathbf{O}_i\}$ , each  $B_r \times d$
- 14: **for**  $i = 1$  to  $T_r$  **do**
- 15: Load  $\mathbf{Q}_i$  to SRAM; initialize  $\mathbf{O}_i^{(0)} \leftarrow 0, \ell_i^{(0)} \leftarrow 0, m_i^{(0)} \leftarrow -\infty$
- 16: **for**  $j = 1$  to  $T_c$  **do**
- 17:  $\alpha \leftarrow mask_{i,j}$
- 18: **if**  $\alpha = 0$  **then**
- 19: **continue**
- 20: **else**
- 21: **Select KV:** based on  $\alpha$
- 22: **if**  $\alpha = 1$  **then**
- 23:  $\mathbf{K}_{\text{sel}}, \mathbf{V}_{\text{sel}} \leftarrow \mathbf{K}_1[j], \mathbf{V}_1[j]$
- 24: **else if**  $\alpha = 0.5$  **then**
- 25:  $\mathbf{K}_{\text{sel}}, \mathbf{V}_{\text{sel}} \leftarrow \mathbf{K}_2[j], \mathbf{V}_2[j]$
- 26: **else if**  $\alpha = 0.25$  **then**
- 27:  $\mathbf{K}_{\text{sel}}, \mathbf{V}_{\text{sel}} \leftarrow \mathbf{K}_4[j], \mathbf{V}_4[j]$
- 28: **else if**  $\alpha = 0.125$  **then**
- 29:  $\mathbf{K}_{\text{sel}}, \mathbf{V}_{\text{sel}} \leftarrow \mathbf{K}_8[j], \mathbf{V}_8[j]$
- 30: **end if**
- 31: Compute scores:  $\mathbf{S}_i^{(j)} = \mathbf{Q}_i \mathbf{K}_{\text{sel}}^\top$
- 32: Apply compensation:  $\mathbf{S}_i^{(j)} += \log(1/\alpha)$
- 33: Update max:  $m_i^{(j)} = \max(m_i^{(j-1)}, \text{rowmax}(\mathbf{S}_i^{(j)}))$
- 34: Compute exp:  $\tilde{\mathbf{P}}_i^{(j)} = \exp(\mathbf{S}_i^{(j)} - m_i^{(j)})$
- 35: Accumulate sum:  $\ell_i^{(j)} = e^{m_i^{(j-1)} - m_i^{(j)}} \ell_i^{(j-1)} + \text{rowsum}(\tilde{\mathbf{P}}_i^{(j)})$
- 36: Update output:  $\mathbf{O}_i^{(j)} = e^{m_i^{(j-1)} - m_i^{(j)}} \mathbf{O}_i^{(j-1)} + \tilde{\mathbf{P}}_i^{(j)} \mathbf{V}_{\text{sel}}$
- 37: **end if**
- 38: **end for**
- 39: Finalize:  $\mathbf{O}_i = \mathbf{O}_i^{(T_c)} / \ell_i^{(T_c)}, L_i = m_i^{(T_c)} + \log(\ell_i^{(T_c)})$
- 40: Write  $\mathbf{O}_i, L_i$  to HBM
- 41: **end for**
- 42: **return**  $\mathbf{O}, L$

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## 2 New Backward Pass

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### Algorithm 2 FlashAttention-MSNew Backward Pass with Multi-Scale KV Pooling and Masking

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**Require:** Matrices  $\mathbf{Q}, \mathbf{K}, \mathbf{V}, \mathbf{O}, \mathbf{dO} \in \mathbb{R}^{N \times d}$  in HBM, vector  $\mathbf{L} \in \mathbb{R}^N$  in HBM, block sizes  $B_r, B_c$ , sparse-block mask  $mask \in \{0, 1, 0.5, 0.25, 0.125\}^{T_r \times T_c}$ .

- 1: INITIALIZATION FOR BACKWARD PASS
- 2:  $T_r \leftarrow \lceil N/B_r \rceil, T_c \leftarrow \lceil N/B_c \rceil$
- 3: Divide  $\mathbf{Q}, \mathbf{O}, \mathbf{dO}$  into  $\{\mathbf{Q}_i\}, \{\mathbf{O}_i\}, \{\mathbf{dO}_i\}$ , each  $B_r \times d$ ; divide  $\mathbf{L}$  into  $\{\mathbf{L}_i\}$ , each  $B_r$
- 4: Initialize  $\mathbf{dQ} = (0)_{N \times d}$  in HBM, divided into  $\{\mathbf{dQ}_i\}$ ; initialize  $\mathbf{dK}_1 = (0)_{N \times d}, \mathbf{dV}_1 = (0)_{N \times d}$  in HBM
- 5: Compute  $D = \text{rowsum}(\mathbf{dO} \circ \mathbf{O}) \in \mathbb{R}^N$ , write to HBM, divide into  $\{D_i\}$ , each  $B_r$
- 6: **for**  $\alpha \in \{1, 0.5, 0.25, 0.125\}$  **do**
- 7:     PROCESSMASK( $\alpha$ )
- 8: **end for**
- 9: BACKWARD FOR POOLING( $\mathbf{dK}_1, \mathbf{dV}_1, \mathbf{dK}_2, \mathbf{dV}_2, \mathbf{dK}_4, \mathbf{dV}_4, \mathbf{dK}_8, \mathbf{dV}_8$ )
- 10: **return**  $\mathbf{dQ}, \mathbf{dK}_1, \mathbf{dV}_1$

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### Algorithm 3 INITIALIZATION FOR NEW BACKWARD PASS WITH MULTI-SCALE KV POOLING

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**Require:** Matrices  $\mathbf{Q}, \mathbf{K}, \mathbf{V}, \mathbf{O}, \mathbf{dO} \in \mathbb{R}^{N \times d}$  in HBM, vector  $\mathbf{L} \in \mathbb{R}^N$  in HBM, block sizes  $B_r, B_c$

- 1: **procedure** INITIALIZE( $\mathbf{Q}, \mathbf{K}, \mathbf{V}, \mathbf{O}, \mathbf{dO}, \mathbf{L}, B_r, B_c$ )
- 2:     **Generate multi-scale KV blocks via pooling:**
- 3:     *pooling*( $x, \text{zoom\_ratio}$ ) applies average pooling to merge each group of *zoom\_ratio* adjacent tokens into a single token.
- 4:      $\mathbf{K}_1, \mathbf{V}_1 \leftarrow \mathbf{K}, \mathbf{V}$   $\triangleright \text{zoom\_ratio}=1$
- 5:     Divide  $\mathbf{K}_1, \mathbf{V}_1$  into  $T_c$  blocks  $\mathbf{K}_1[1], \dots, \mathbf{K}_1[T_c], \mathbf{V}_1[1], \dots, \mathbf{V}_1[T_c]$  of size  $B_c \times d$  each in HBM.
- 6:      $\mathbf{K}_2, \mathbf{V}_2 \leftarrow \text{pooling}(\mathbf{K}_1, 2), \text{pooling}(\mathbf{V}_1, 2)$   $\triangleright \text{zoom\_ratio}=2$
- 7:     Divide  $\mathbf{K}_2, \mathbf{V}_2$  into  $T_c$  blocks  $\mathbf{K}_2[1], \dots, \mathbf{K}_2[T_c], \mathbf{V}_2[1], \dots, \mathbf{V}_2[T_c]$  of size  $\lceil B_c/2 \rceil \times d$  each in HBM.
- 8:      $\mathbf{K}_4, \mathbf{V}_4 \leftarrow \text{pooling}(\mathbf{K}_2, 2), \text{pooling}(\mathbf{V}_2, 2)$   $\triangleright \text{zoom\_ratio}=4$
- 9:     Divide  $\mathbf{K}_4, \mathbf{V}_4$  into  $T_c$  blocks  $\mathbf{K}_4[1], \dots, \mathbf{K}_4[T_c], \mathbf{V}_4[1], \dots, \mathbf{V}_4[T_c]$  of size  $\lceil B_c/4 \rceil \times d$  each in HBM.
- 10:      $\mathbf{K}_8, \mathbf{V}_8 \leftarrow \text{pooling}(\mathbf{K}_4, 2), \text{pooling}(\mathbf{V}_4, 2)$   $\triangleright \text{zoom\_ratio}=8$
- 11:     Divide  $\mathbf{K}_8, \mathbf{V}_8$  into  $T_c$  blocks  $\mathbf{K}_8[1], \dots, \mathbf{K}_8[T_c], \mathbf{V}_8[1], \dots, \mathbf{V}_8[T_c]$  of size  $\lceil B_c/8 \rceil \times d$  each in HBM.
- 12:      $T_r \leftarrow \lceil N/B_r \rceil, T_c \leftarrow \lceil N/B_c \rceil$
- 13:     Divide  $\mathbf{Q}$  into  $\{\mathbf{Q}_i\}_{i=1}^{T_r}$ ,  $\mathbf{O}$  into  $\{\mathbf{O}_i\}_{i=1}^{T_r}$ ,  $\mathbf{dO}$  into  $\{\mathbf{dO}_i\}_{i=1}^{T_r}$ , each  $B_r \times d$ , and  $\mathbf{L}$  into  $\{\mathbf{L}_i\}_{i=1}^{T_r}$ , each  $B_r$ , all in HBM.
- 14:     Initialize  $\mathbf{dQ} = (0)_{N \times d}$  in HBM, divide into  $T_r$  blocks  $\mathbf{dQ}_1, \dots, \mathbf{dQ}_{T_r}$ , each  $B_r \times d$ .
- 15:     Initialize and Divide  $\mathbf{dK}_1, \mathbf{dV}_1 \in (0)_{N \times d}$  into  $T_c$  blocks  $\mathbf{dK}_1[j], \mathbf{dV}_1[j]$ , each  $B_c \times d$  in HBM.
- 16:     Initialize and Divide  $\mathbf{dK}_2, \mathbf{dV}_2 \in (0)_{\lceil N/2 \rceil \times d}$  into  $T_c$  blocks  $\mathbf{dK}_2[j], \mathbf{dV}_2[j]$ , each  $\lceil B_c/2 \rceil \times d$  in HBM.
- 17:     Initialize and Divide  $\mathbf{dK}_4, \mathbf{dV}_4 \in (0)_{\lceil N/4 \rceil \times d}$  into  $T_c$  blocks  $\mathbf{dK}_4[j], \mathbf{dV}_4[j]$ , each  $\lceil B_c/4 \rceil \times d$  in HBM.
- 18:     Initialize and Divide  $\mathbf{dK}_8, \mathbf{dV}_8 \in (0)_{\lceil N/8 \rceil \times d}$  into  $T_c$  blocks  $\mathbf{dK}_8[j], \mathbf{dV}_8[j]$ , each  $\lceil B_c/8 \rceil \times d$  in HBM.
- 19:     Compute  $D = \text{rowsum}(\mathbf{dO} \circ \mathbf{O}) \in \mathbb{R}^N$ , write to HBM, divide into  $\{D_i\}_{i=1}^{T_r}$ , each  $B_r$ .
- 20: **end procedure**

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procedure PROCESSMASK( $\alpha$ )
   $zoom\_ratio \leftarrow 1/\alpha$ 
  for  $j = 1$  to  $T_c$  do
    Select KV and gradient blocks based on  $\alpha$ :
    if  $\alpha = 1$  then
       $\mathbf{K}_{sel} \leftarrow \mathbf{K}_1[j], \mathbf{V}_{sel} \leftarrow \mathbf{V}_1[j], \mathbf{dK}_{sel} \leftarrow \mathbf{dK}_1[j], \mathbf{dV}_{sel} \leftarrow \mathbf{dV}_1[j]$ 
    else if  $\alpha = 0.5$  then
       $\mathbf{K}_{sel} \leftarrow \mathbf{K}_2[j], \mathbf{V}_{sel} \leftarrow \mathbf{V}_2[j], \mathbf{dK}_{sel} \leftarrow \mathbf{dK}_2[j], \mathbf{dV}_{sel} \leftarrow \mathbf{dV}_2[j]$ 
    else if  $\alpha = 0.25$  then
       $\mathbf{K}_{sel} \leftarrow \mathbf{K}_4[j], \mathbf{V}_{sel} \leftarrow \mathbf{V}_4[j], \mathbf{dK}_{sel} \leftarrow \mathbf{dK}_4[j], \mathbf{dV}_{sel} \leftarrow \mathbf{dV}_4[j]$ 
    else if  $\alpha = 0.125$  then
       $\mathbf{K}_{sel} \leftarrow \mathbf{K}_8[j], \mathbf{V}_{sel} \leftarrow \mathbf{V}_8[j], \mathbf{dK}_{sel} \leftarrow \mathbf{dK}_8[j], \mathbf{dV}_{sel} \leftarrow \mathbf{dV}_8[j]$ 
    end if
    Load  $\mathbf{K}_{sel}, \mathbf{V}_{sel}, \mathbf{dK}_{sel}, \mathbf{dV}_{sel}$  from HBM to on-chip SRAM
    for  $i = 1$  to  $T_r$  do
      if  $mask[i, j] \neq \alpha$  then
        continue
      end if
      Load  $\mathbf{Q}_i, \mathbf{dO}_i, \mathbf{L}_i, D_i$  from HBM to on-chip SRAM
      On chip, compute  $\mathbf{S}_i^{(j)} = \mathbf{Q}_i \mathbf{K}_{sel}^\top$ 
      On chip, apply compensation:  $\mathbf{S}_i^{(j)} += \ln(zoom\_ratio)$ 
      On chip, compute  $\mathbf{P}_i^{(j)} = \exp(\mathbf{S}_i^{(j)} - \mathbf{L}_i)$ 
      On chip, compute  $\mathbf{dP}_i^{(j)} = \mathbf{dO}_i \mathbf{V}_{sel}^\top$ 
      On chip, compute  $\mathbf{dS}_i^{(j)} = \mathbf{P}_i^{(j)} \circ (\mathbf{dP}_i^{(j)} - D_i)$ 
      Load  $\mathbf{dQ}_i$  from HBM to on-chip SRAM
      On chip, update  $\mathbf{dQ}_i \leftarrow \mathbf{dQ}_i + \mathbf{dS}_i^{(j)} \mathbf{K}_{sel}$ 
      Write  $\mathbf{dQ}_i$  back to HBM
      On chip, accumulate  $\mathbf{dK}_{sel} \leftarrow \mathbf{dK}_{sel} + (\mathbf{dS}_i^{(j)})^\top \mathbf{Q}_i$ 
      On chip, accumulate  $\mathbf{dV}_{sel} \leftarrow \mathbf{dV}_{sel} + (\mathbf{P}_i^{(j)})^\top \mathbf{dO}_i$ 
    end for
    Write  $\mathbf{dK}_{sel}, \mathbf{dV}_{sel}$  back to HBM
  end for
end procedure

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