

homework1

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1 实验要求

使用 C/C++ 实现两种解线性方程组的算法：

1. 列主元 Gauss 消元法
2. Gauss-Seidel 迭代法

2 算法原理

2.1 待解线性方程组

考虑两点边值问题

$$\begin{cases} \varepsilon \frac{d^2 y}{dx^2} + \frac{dy}{dx} = a, & 0 < a < 1 \\ y(0) = 0, & y(1) = 1 \end{cases}$$

容易知道它们的精确解为

$$y = \frac{1-a}{1-e^{-\frac{1}{\varepsilon}}}(1-e^{-\frac{x}{\varepsilon}}) + ax$$

为了把微分方程离散，把 $[0, 1]$ 区间 n 等分，令 $h = \frac{1}{n}$,

$$x_i = ih, \quad i = 1, 2, \dots, n-1$$

得到差分方程

$$\varepsilon \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} + \frac{y_{i+1}}{h} = a$$

简化为

$$(\varepsilon + h)y_{i+1} - (2\varepsilon + h)y_i + \varepsilon y_{i-1} = ah^2$$

从而离散后得到的微分方程组的系数矩阵为

$$A = \begin{bmatrix} -(2\varepsilon + h) & \varepsilon + h & & & \\ \varepsilon & -(2\varepsilon + h) & \varepsilon + h & & \\ & \varepsilon & -(2\varepsilon + h) & \ddots & \\ & & \ddots & \ddots & \varepsilon + h \\ & & & \varepsilon & -(2\varepsilon + h) \end{bmatrix}$$

2.2 列主元 Gauss 消元法

列主元 Gauss 消元法是对顺序 Gauss 消元法的一种改进，顺序 Gauss 消元可行的充分必要条件是矩阵 A 的各阶顺序主子式不为 0，但是只要 $\det A \neq 0$ ，方程组 $Ax = b$ 就有解，故顺序 Gauss 消元本身具有局限性。列主元 Gauss 消元法对每个 $\{k, k = 1, 2, \dots, n-1\}$ 在消元前，选出 $\{|a_{kk}^{(k-1)}|, |a_{k+1,k}^{(k-1)}|, \dots, |a_{n,k}^{(k-1)}|\}$ 中绝对值最大的元素 $a_{mk}^{(k-1)}$ ，对 k 行和 m 行进行交换后，再做消元运算，由于 $\det A \neq 0$ ，可证 $\{a_{kk}^{(k-1)}, a_{k+1,k}^{(k-1)}, \dots, a_{n,k}^{(k-1)}\}$ 中至少有一个元素不为 0，因此，列主行消元总是可行的。

2.3 Gauss-Seidel 迭代法

Gauss-Seidel 迭代法与 jacobi 迭代步骤大致相同，其迭代的核心部分是计算迭代式的过程中，要及时将 $x_i^{(k+1)}$ 放到 $x_i^{(k)}$ 的位置上。

$$\begin{cases} x_1^{k+1} = x_1^k + x_2^k + \dots + x_n^k + b_1 \\ x_2^{k+1} = x_1^{k+1} + x_2^k + \dots + x_n^k + b_2 \\ \vdots \\ x_n^{k+1} = x_1^{k+1} + x_2^{k+1} + \dots + x_n^k + b_n \end{cases}$$

2.4 对实验结果的修正

事实上, 对实验给定的待解方程组, 部分初始值是已知的, 若在计算过程中不对初值条件进行修正, 将会产生较大误差, 故根据已知条件 $y(0) = 0, y(1) = 1$ 对初始条件进行修正 $b[99] = ah^2 - \varepsilon - h$ 。

3 实验结果

未修正前, $\varepsilon = 1, 0.1, 0.01, 0.0001$ 的结果分别为

```
epsilon :1
Result of Gauss Elimination With Pivoting
-0.00283707 -0.00599604 -0.00842777 -0.01103828 -0.01366021
-0.0161664 -0.0195964 -0.02309528 -0.0267384 -0.0304476
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-7.6808556 -7.6940856 -7.7073276 -7.7205816 -7.7338476
-7.7471256 -7.7604156 -7.7737176 -7.7870316 -7.8003576
-7.8136956 -7.8270456 -7.8404076 -7.8537816 -7.8671676
-7.8805656 -7.8939756 -7.9073976 -7.9208316 -7.9342776
-7.9477356 -7.9612056 -7.9746876 -7.9881816 -8.0016876
-8.0152056 -8.0287356 -8.0422776 -8.0558316 -8.0693976
-8.0829756 -8.0965756 -8.1101876 -8.1238116 -8.1374476
-8.1510956 -8.1647556 -8.1784276 -8.1921116 -8.2058076
-8.2195156 -8.2332356 -8.2469676 -8.2607116 -8.2744676
-8.2882356 -8.3020156 -8.3158076 -8.3296116 -8.3434276
-8.3572556 -8.3710956 -8.3849476 -8.3988116 -8.4126876
-8.4265756 -8.4404756 -8.4543876 -8.4683116 -8.4822476
-8.4961956 -8.5101556 -8.5241276 -8.5381116 -8.5521076
-8.5661156 -8.5801356 -8.5941676 -8.6082116 -8.6222676
-8.6363356 -8.6504156 -8.6645076 -8.6786116 -8.6927276
-8.7068556 -8.7209956 -8.7351476 -8.7493116 -8.7634876
-8.7776756 -8.7918756 -8.8060876 -8.8203116 -8.8345476
-8.8487956 -8.8630556 -8.8773276 -8.8916116 -8.9059076
-8.9202156 -8.9345356 -8.9488676 -8.9632116 -8.9775676
-8.9919356 -9.0063156 -9.0207076 -9.0351116 -9.0495276
-9.0639556 -9.0783956 -9.0928476 -9.1073116 -9.1217876
-9.1362756 -9.1507856 -9.165307
```

```

epsilon :0.01
Result of Gauss Elimination With Pivoting
-0.2475      -0.36875      -0.426875      -0.453438      -0.464219
-0.467109    -0.466055    -0.463027    -0.459014    -0.454507
-0.449783    -0.444877    -0.432938    -0.434969    -0.429985
-0.424992    -0.419996    -0.414998    -0.409999    -0.405
-0.4         -0.395         -0.39         -0.385         -0.38
-0.375       -0.37         -0.365       -0.36         -0.355
-0.35        -0.345        -0.34         -0.335        -0.33
-0.325       -0.32         -0.315       -0.31         -0.305
-0.3          -0.295        -0.29         -0.285        -0.28
-0.275       -0.27         -0.265       -0.26         -0.255
-0.25        -0.245        -0.24         -0.235        -0.23
-0.225       -0.22         -0.215       -0.21         -0.205
-0.2          -0.195        -0.19         -0.185        -0.18
-0.175       -0.17         -0.165       -0.16         -0.155
-0.15        -0.145        -0.14         -0.135        -0.13
-0.125       -0.12         -0.115       -0.11         -0.105
-0.1          -0.095        -0.09         -0.085        -0.08
-0.075       -0.07         -0.065       -0.06         -0.055
-0.05        -0.045        -0.04         -0.035        -0.03
-0.025       -0.02         -0.015       -0.01         -0.005
Acc : 1.30666

```

(e)

```

epsilon :0.01
Result of Gauss Seidel Iteration
-0.247493    -0.36874         -0.426865         -0.453428         -0.46421
-0.467102    -0.466049    -0.463023    -0.45901         -0.454504
-0.449751    -0.444875    -0.432937    -0.434968         -0.429984
-0.424992    -0.419996    -0.414998    -0.409999         -0.404999
-0.4          -0.395         -0.39         -0.385         -0.38
-0.375       -0.37         -0.365       -0.36         -0.355
-0.35        -0.345        -0.34         -0.335        -0.33
-0.325       -0.32         -0.315       -0.31         -0.305
-0.3          -0.295        -0.29         -0.285        -0.28
-0.275       -0.27         -0.265       -0.26         -0.255
-0.25        -0.245        -0.24         -0.235        -0.23
-0.225       -0.22         -0.215       -0.21         -0.205
-0.2          -0.195        -0.19         -0.185        -0.18
-0.175       -0.17         -0.165       -0.16         -0.155
-0.15        -0.145        -0.14         -0.135        -0.13
-0.125       -0.12         -0.115       -0.11         -0.105
-0.1          -0.095        -0.09         -0.085        -0.08
-0.075       -0.07         -0.065       -0.06         -0.055
-0.05        -0.045        -0.04         -0.035        -0.03
-0.025       -0.02         -0.015       -0.01         -0.005
Acc : 1.30666

```

(f)

```

epsilon :0.0001
Result of Gauss Elimination With Pivoting
-0.495      -0.49495      -0.49         -0.485         -0.48
-0.475      -0.47         -0.465       -0.46         -0.455
-0.45        -0.445       -0.44         -0.435        -0.43
-0.425      -0.42         -0.415       -0.41         -0.405
-0.4          -0.395       -0.39         -0.385        -0.38
-0.375      -0.37         -0.365       -0.36         -0.355
-0.35        -0.345       -0.34         -0.335        -0.33
-0.325      -0.32         -0.315       -0.31         -0.305
-0.3          -0.295       -0.29         -0.285        -0.28
-0.275      -0.27         -0.265       -0.26         -0.255
-0.25        -0.245       -0.24         -0.235        -0.23
-0.225      -0.22         -0.215       -0.21         -0.205
-0.2          -0.195       -0.19         -0.185        -0.18
-0.175      -0.17         -0.165       -0.16         -0.155
-0.15        -0.145       -0.14         -0.135        -0.13
-0.125      -0.12         -0.115       -0.11         -0.105
-0.1          -0.095       -0.09         -0.085        -0.08
-0.075      -0.07         -0.065       -0.06         -0.055
-0.05        -0.045       -0.04         -0.035        -0.03
-0.025      -0.02         -0.015       -0.01         -0.005
Acc : 1.31139

```

(g)

```

epsilon :0.0001
Result of Gauss Seidel Iteration
-0.495      -0.49495      -0.49         -0.485         -0.48
-0.475      -0.47         -0.465       -0.46         -0.455
-0.45        -0.445       -0.44         -0.435        -0.43
-0.425      -0.42         -0.415       -0.41         -0.405
-0.4          -0.395       -0.39         -0.385        -0.38
-0.375      -0.37         -0.365       -0.36         -0.355
-0.35        -0.345       -0.34         -0.335        -0.33
-0.325      -0.32         -0.315       -0.31         -0.305
-0.3          -0.295       -0.29         -0.285        -0.28
-0.275      -0.27         -0.265       -0.26         -0.255
-0.25        -0.245       -0.24         -0.235        -0.23
-0.225      -0.22         -0.215       -0.21         -0.205
-0.2          -0.195       -0.19         -0.185        -0.18
-0.175      -0.17         -0.165       -0.16         -0.155
-0.15        -0.145       -0.14         -0.135        -0.13
-0.125      -0.12         -0.115       -0.11         -0.105
-0.1          -0.095       -0.09         -0.085        -0.08
-0.075      -0.07         -0.065       -0.06         -0.055
-0.05        -0.045       -0.04         -0.035        -0.03
-0.025      -0.02         -0.015       -0.01         -0.005
Acc : 1.31139

```

(h)

修正后, $\varepsilon = 1, 0.1, 0.01, 0.0001$ 的结果分别为

```
epsilon :1
Result of Gauss Elimination With Pivoting
0.0127309 0.0253852 0.0379038 0.0504073 0.0628965
0.0752522 0.0875351 0.0997458 0.111885 0.123954
0.136952 0.147882 0.159743 0.171535 0.183201
0.194492 0.206313 0.218041 0.229594 0.240903
0.252239 0.263512 0.274723 0.285873 0.296961
0.307989 0.318958 0.329868 0.340719 0.351512
0.362247 0.372926 0.383549 0.394116 0.404628
0.415085 0.425495 0.435838 0.446135 0.45638
0.466512 0.476713 0.486803 0.496843 0.506833
0.516773 0.526665 0.536508 0.546303 0.556051
0.565752 0.575406 0.585014 0.594576 0.604094
0.613566 0.622994 0.632379 0.64172 0.651018
0.660273 0.669487 0.678658 0.687788 0.696878
0.705927 0.714939 0.723905 0.732835 0.741728
0.750578 0.759392 0.768169 0.776908 0.78561
0.794275 0.802905 0.811498 0.820055 0.828578
0.837065 0.845518 0.853937 0.862322 0.870674
0.878992 0.887277 0.89553 0.903751 0.91194
0.920097 0.928223 0.936318 0.944382 0.952416
0.96042 0.968395 0.976339 0.984255 0.992142
Acc : 0.010442
```

(i)

```
epsilon :1
Result of Gauss Seidel Iteration
0.0126923 0.0253086 0.0378495 0.0503016 0.0627087
0.0750284 0.0872759 0.0994452 0.111557 0.123593
0.135559 0.147456 0.159289 0.171048 0.182745
0.194375 0.20594 0.217441 0.228878 0.240252
0.251564 0.262814 0.274002 0.28513 0.296196
0.307207 0.318157 0.32905 0.339884 0.350662
0.361383 0.372049 0.382559 0.393214 0.403716
0.414164 0.424559 0.434901 0.445192 0.455431
0.465619 0.475757 0.485845 0.495883 0.505873
0.515814 0.525707 0.535552 0.545351 0.555103
0.564809 0.574409 0.584084 0.593654 0.60318
0.612662 0.6221 0.631496 0.640848 0.650159
0.659428 0.668555 0.677541 0.686387 0.695092
0.705157 0.714193 0.72317 0.732119 0.741027
0.749899 0.758733 0.767529 0.776289 0.785012
0.793698 0.802349 0.810964 0.819544 0.828089
0.836599 0.845075 0.853517 0.861925 0.8703
0.878642 0.886951 0.895227 0.903472 0.911684
0.919865 0.928015 0.936133 0.944221 0.952279
0.960306 0.968304 0.976271 0.98421 0.992119
Acc : 0.00963603
```

(j)

```
epsilon :0.1
Result of Gauss Elimination With Pivoting
0.050003 0.0959148 0.138107 0.176919 0.212656
0.2456 0.276003 0.304996 0.33009 0.354176
0.376827 0.3973 0.416639 0.434675 0.451526
0.467299 0.482093 0.495997 0.509051 0.521449
0.533139 0.54422 0.554748 0.564774 0.574343
0.583497 0.592273 0.600705 0.608826 0.616663
0.624242 0.631587 0.638718 0.645656 0.652417
0.659013 0.665475 0.671798 0.678001 0.684095
0.690089 0.695993 0.701815 0.707562 0.713241
0.718859 0.72442 0.72993 0.735394 0.740816
0.746199 0.751548 0.756804 0.762152 0.767414
0.772652 0.777869 0.783065 0.788244 0.793407
0.798555 0.803689 0.808811 0.813922 0.819023
0.824115 0.829158 0.834274 0.839343 0.844406
0.849463 0.854515 0.859562 0.864605 0.869643
0.874679 0.879711 0.88474 0.889767 0.894791
0.899813 0.904833 0.909851 0.914868 0.919883
0.924896 0.929909 0.93492 0.93993 0.944939
0.949948 0.954956 0.959963 0.964969 0.969975
0.97498 0.979985 0.984989 0.989993 0.994997
Acc : 0.0133884
```

(k)

```
epsilon :0.1
Result of Gauss Seidel Iteration
0.0499691 0.0958504 0.138015 0.176802 0.212518
0.245442 0.275828 0.303906 0.329888 0.353962
0.376304 0.397099 0.416402 0.434433 0.45118
0.467051 0.481844 0.495747 0.508841 0.521201
0.532892 0.543975 0.554507 0.564536 0.574109
0.583267 0.592048 0.600485 0.608611 0.616454
0.624038 0.631388 0.638526 0.645469 0.652237
0.658844 0.665306 0.671635 0.677844 0.683944
0.689944 0.695854 0.701681 0.707434 0.713113
0.718741 0.724308 0.729823 0.735292 0.740718
0.746106 0.751459 0.756781 0.762073 0.767339
0.772581 0.777801 0.783002 0.788184 0.79335
0.798501 0.803639 0.808764 0.813878 0.818981
0.824076 0.829162 0.83424 0.839311 0.844376
0.849435 0.854469 0.859538 0.864622 0.869623
0.87466 0.879694 0.884724 0.889752 0.894777
0.899801 0.904822 0.909841 0.914858 0.919874
0.924889 0.929902 0.934914 0.939925 0.944935
0.949944 0.954952 0.95996 0.964967 0.969973
0.974978 0.979984 0.984988 0.989993 0.994996
Acc : 0.0133334
```

(l)

```

epsilon :0.01
Result of Gauss Elimination With Pivoting
0.2525      0.38125      0.448125      0.484063      0.504531
0.517266    0.526133    0.533066    0.539033    0.544517
0.549788    0.554879    0.55994    0.56497    0.569985
0.574992    0.579996    0.584998    0.589999    0.595
0.6          0.605      0.61        0.615      0.62
0.625      0.63        0.635      0.64        0.645
0.65        0.655      0.66        0.665      0.67
0.675      0.68        0.685      0.69        0.695
0.7          0.705      0.71        0.715      0.72
0.725      0.73        0.735      0.74        0.745
0.75        0.755      0.76        0.765      0.77
0.775      0.78        0.785      0.79        0.795
0.8          0.805      0.81        0.815      0.82
0.825      0.83        0.835      0.84        0.845
0.85        0.855      0.86        0.865      0.87
0.875      0.88        0.885      0.89        0.895
0.9          0.905      0.91        0.915      0.92
0.925      0.93        0.935      0.94        0.945
0.95        0.955      0.96        0.965      0.97
0.975      0.98        0.985      0.99        0.995
Acc : 0.0342812

```

(m)

```

epsilon :0.01
Result of Gauss Seidel Iteration
0.2525      0.38125      0.448125      0.484062      0.504531
0.517266    0.526133    0.533066    0.539033    0.544517
0.549788    0.554879    0.55994    0.56497    0.569985
0.574992    0.579996    0.584998    0.589999    0.595
0.6          0.605      0.61        0.615      0.62
0.625      0.63        0.635      0.64        0.645
0.65        0.655      0.66        0.665      0.67
0.675      0.68        0.685      0.69        0.695
0.7          0.705      0.71        0.715      0.72
0.725      0.73        0.735      0.74        0.745
0.75        0.755      0.76        0.765      0.77
0.775      0.78        0.785      0.79        0.795
0.8          0.805      0.81        0.815      0.82
0.825      0.83        0.835      0.84        0.845
0.85        0.855      0.86        0.865      0.87
0.875      0.88        0.885      0.89        0.895
0.9          0.905      0.91        0.915      0.92
0.925      0.93        0.935      0.94        0.945
0.95        0.955      0.96        0.965      0.97
0.975      0.98        0.985      0.99        0.995
Acc : 0.0342812

```

(n)

```

epsilon :0.0001
Result of Gauss Elimination With Pivoting
0.495099      0.504951      0.51        0.515      0.52
0.525      0.53        0.535      0.54        0.545
0.55        0.555      0.56        0.565      0.57
0.575      0.58        0.585      0.59        0.595
0.6          0.605      0.61        0.615      0.62
0.625      0.63        0.635      0.64        0.645
0.65        0.655      0.66        0.665      0.67
0.675      0.68        0.685      0.69        0.695
0.7          0.705      0.71        0.715      0.72
0.725      0.73        0.735      0.74        0.745
0.75        0.755      0.76        0.765      0.77
0.775      0.78        0.785      0.79        0.795
0.8          0.805      0.81        0.815      0.82
0.825      0.83        0.835      0.84        0.845
0.85        0.855      0.86        0.865      0.87
0.875      0.88        0.885      0.89        0.895
0.9          0.905      0.91        0.915      0.92
0.925      0.93        0.935      0.94        0.945
0.95        0.955      0.96        0.965      0.97
0.975      0.98        0.985      0.99        0.995
Acc : 0.0651735

```

(o)

```

epsilon :0.0001
Result of Gauss Seidel Iteration
0.495099      0.504951      0.51        0.515      0.52
0.525      0.53        0.535      0.54        0.545
0.55        0.555      0.56        0.565      0.57
0.575      0.58        0.585      0.59        0.595
0.6          0.605      0.61        0.615      0.62
0.625      0.63        0.635      0.64        0.645
0.65        0.655      0.66        0.665      0.67
0.675      0.68        0.685      0.69        0.695
0.7          0.705      0.71        0.715      0.72
0.725      0.73        0.735      0.74        0.745
0.75        0.755      0.76        0.765      0.77
0.775      0.78        0.785      0.79        0.795
0.8          0.805      0.81        0.815      0.82
0.825      0.83        0.835      0.84        0.845
0.85        0.855      0.86        0.865      0.87
0.875      0.88        0.885      0.89        0.895
0.9          0.905      0.91        0.915      0.92
0.925      0.93        0.935      0.94        0.945
0.95        0.955      0.96        0.965      0.97
0.975      0.98        0.985      0.99        0.995
Acc : 0.0651735

```

(p)

4 实验分析

在执行解线性方程组的算法之前，首先考虑该方程组是否有已知的初始值，若有则需要做相应的修正，否则将产生较大的误差。