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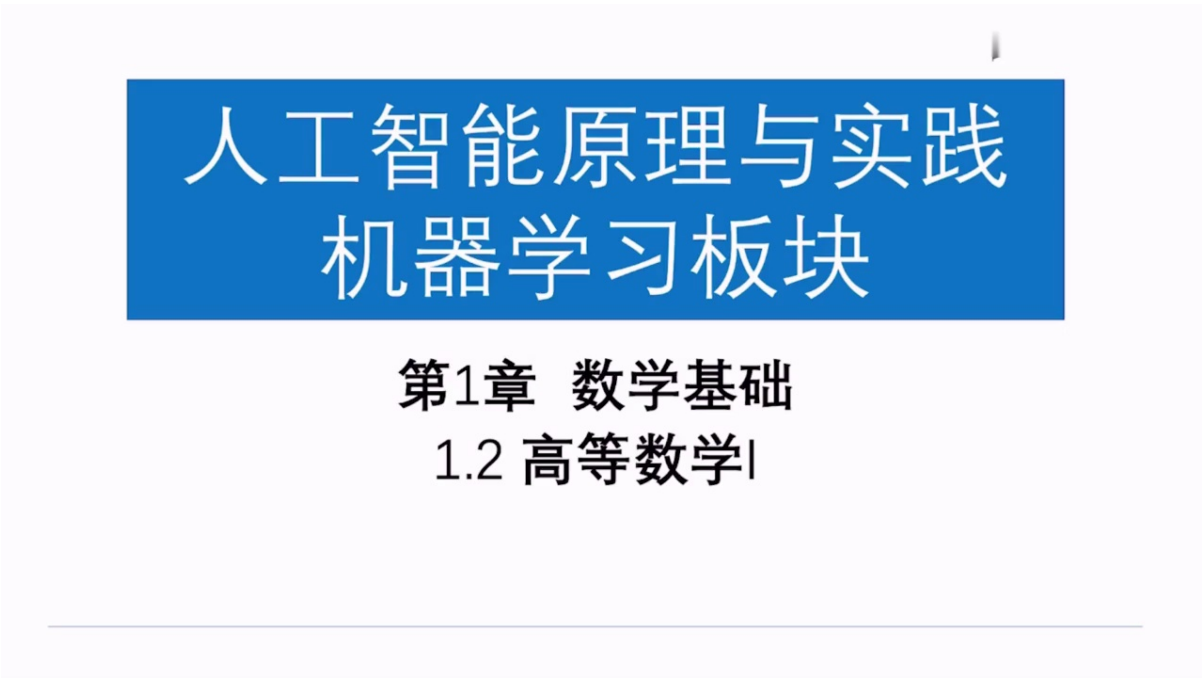

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■■■: 00:00:41.999

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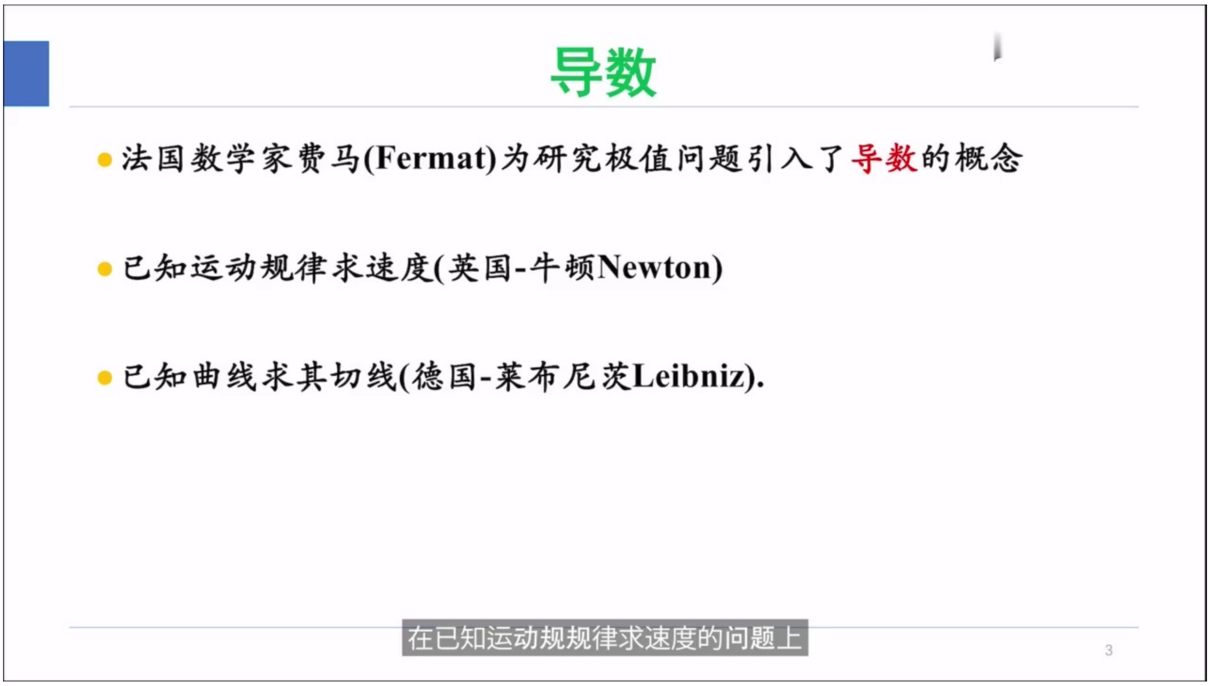

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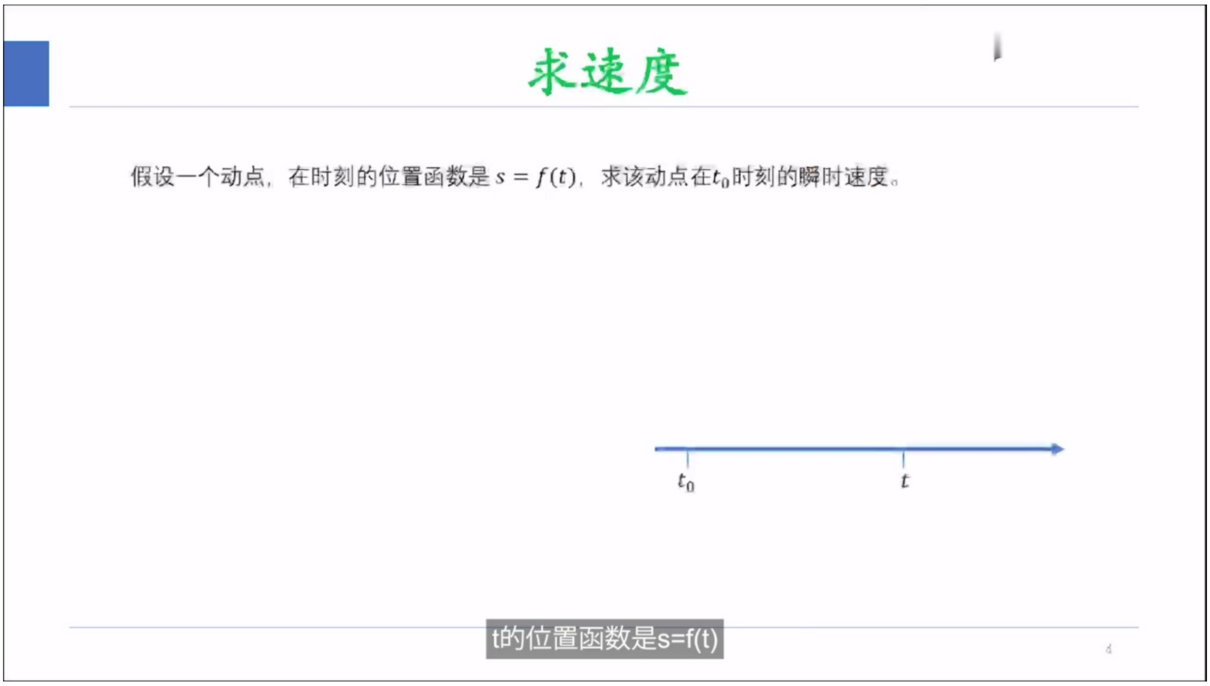
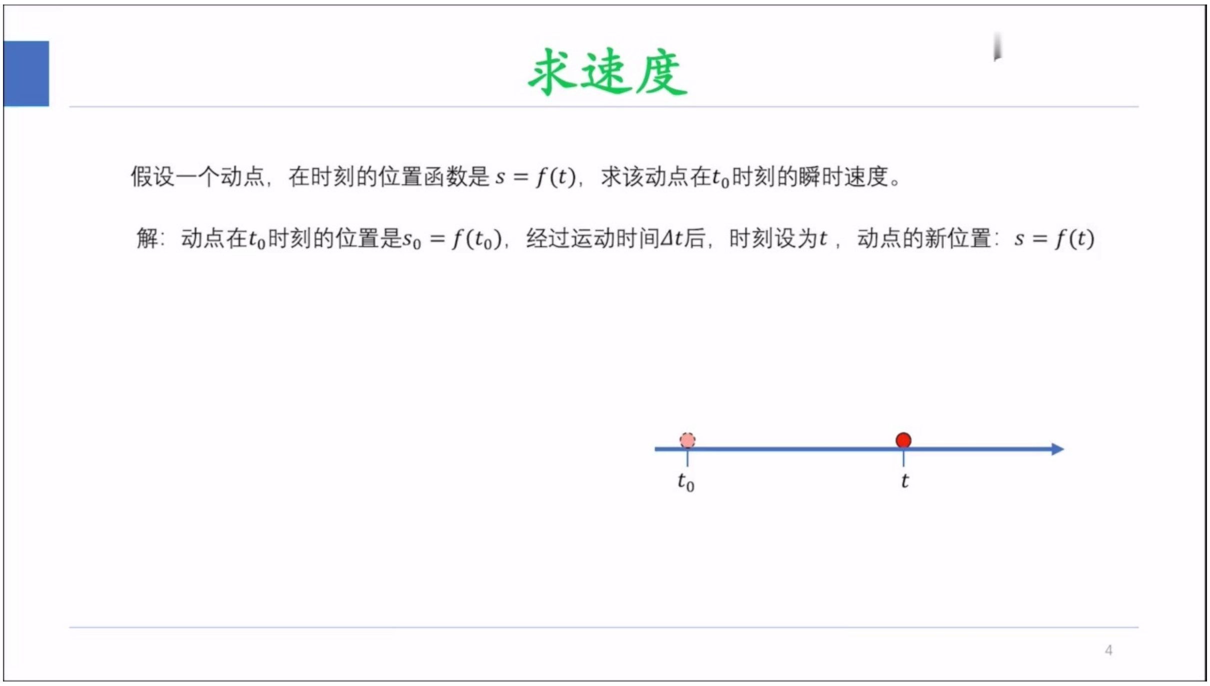
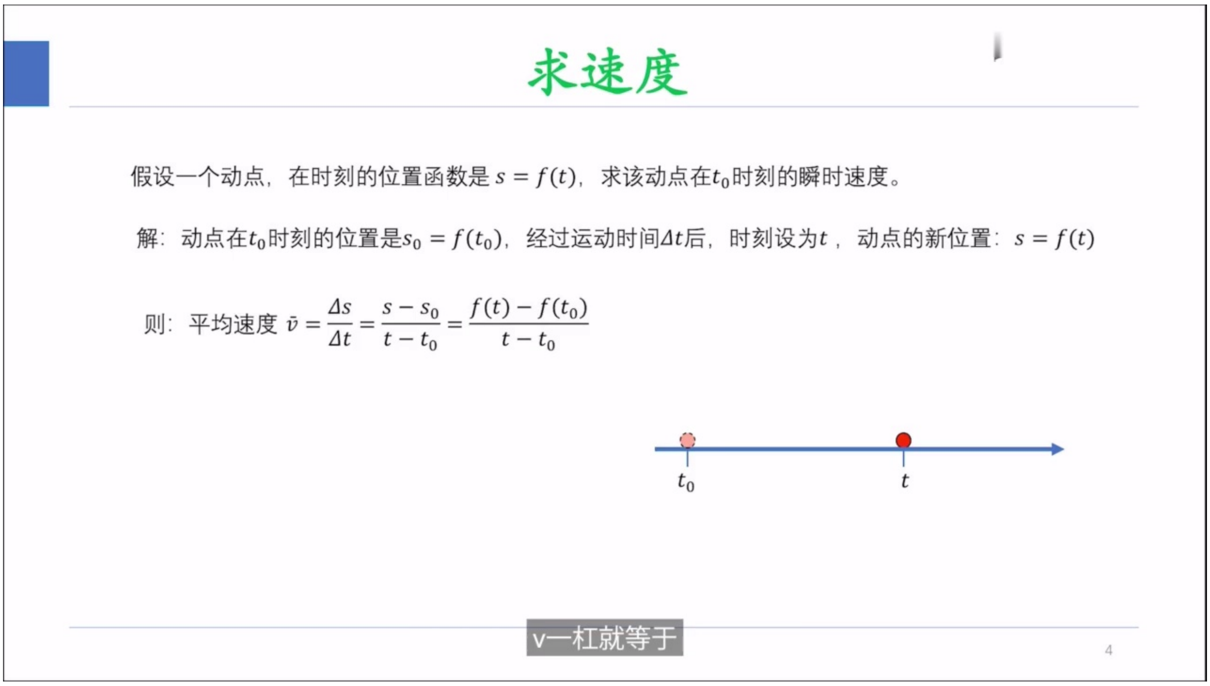


■ 9 ■ / ■ 81 ■
■■: keyframe_00-01-23-999_0008.jpg


求速度
假设一个动点，在时刻的位置函数是 $s=f(t)$，求该动点在t_0时刻的瞬时速度。
解：动点在t_0时刻的位置是$s_0=f(t_0)$，经过运动时间Δt后，时刻设为t，动点的新位置：$s=f(t)$
t_0
t
4

■■■: 00:02:05.999



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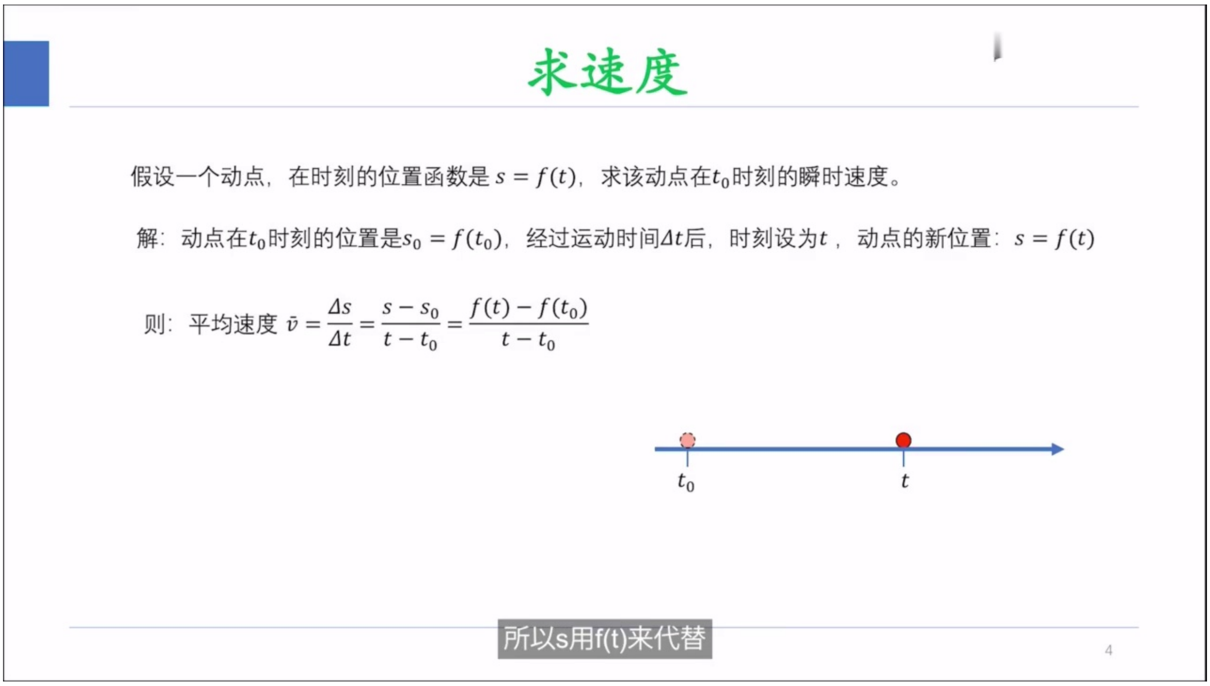

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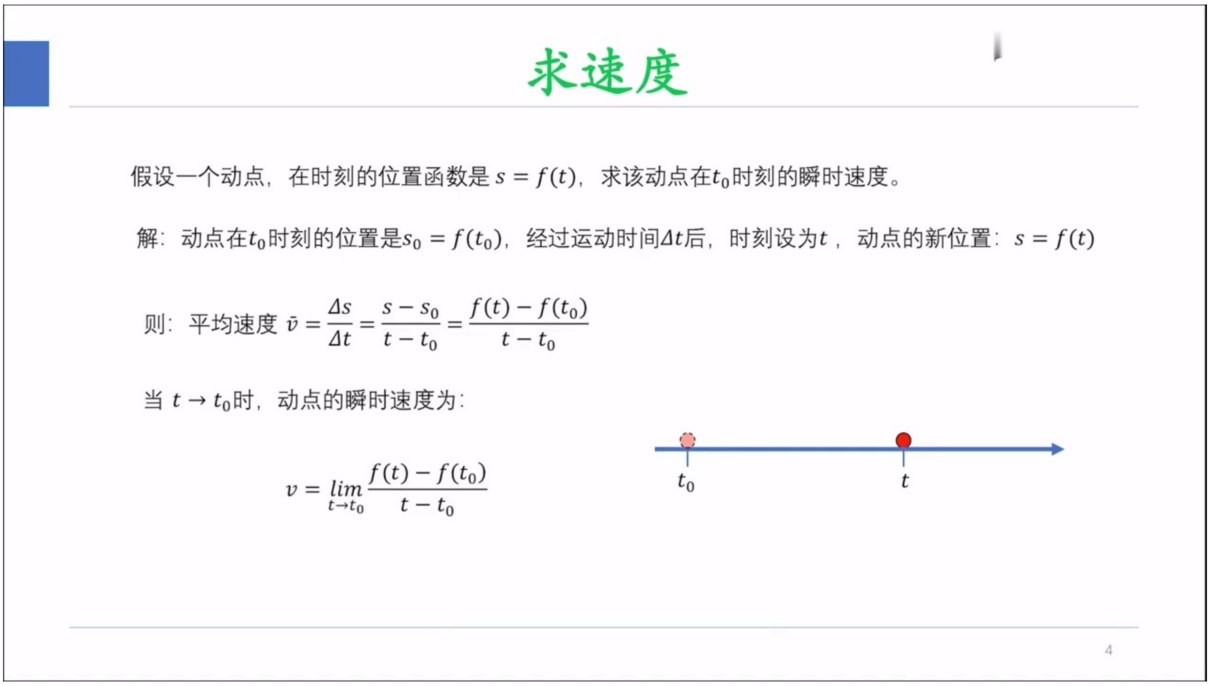

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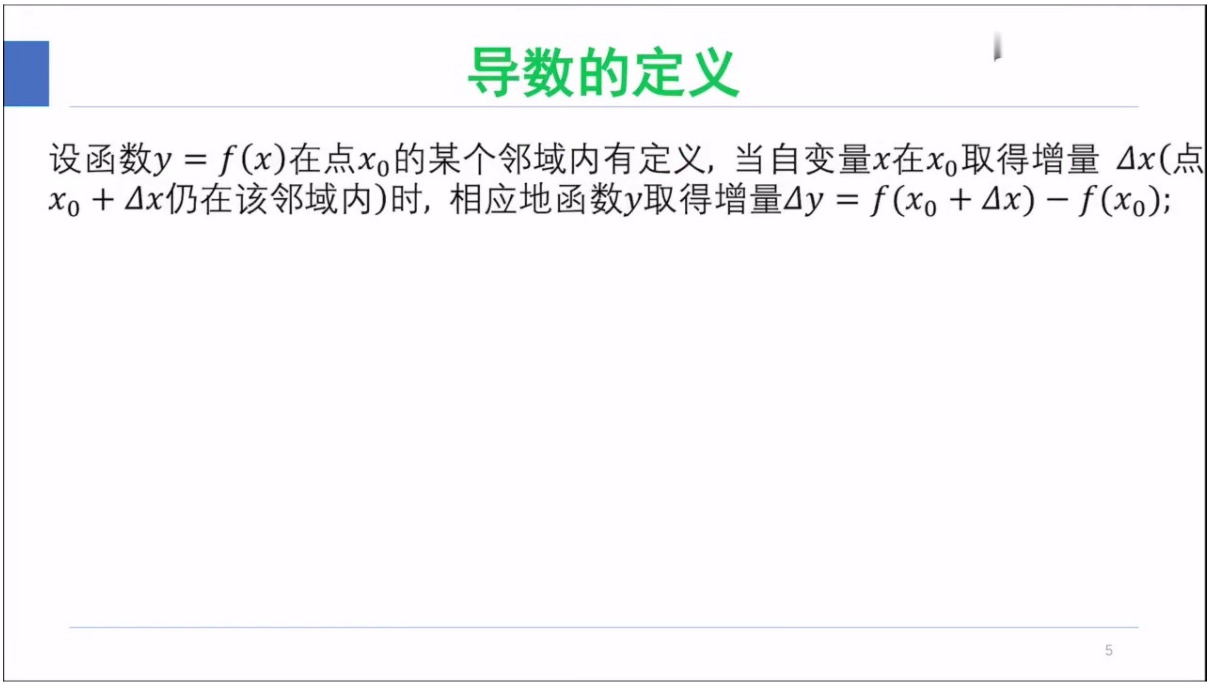

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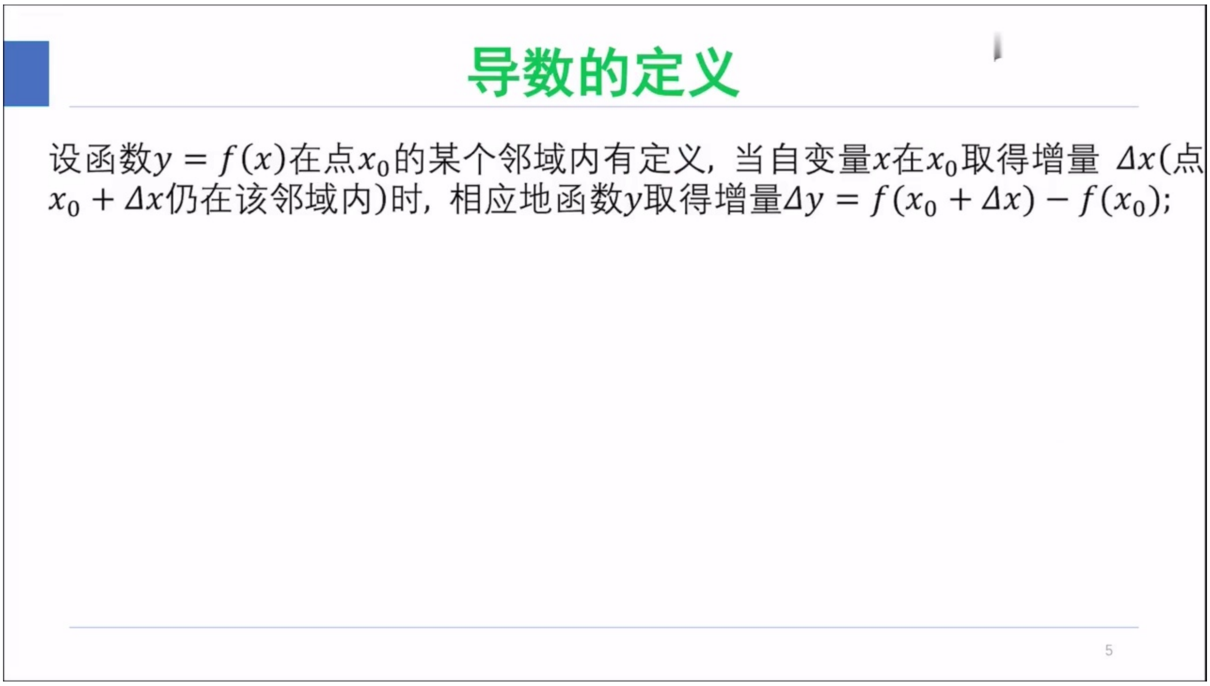
■■■: 00:03:29.999



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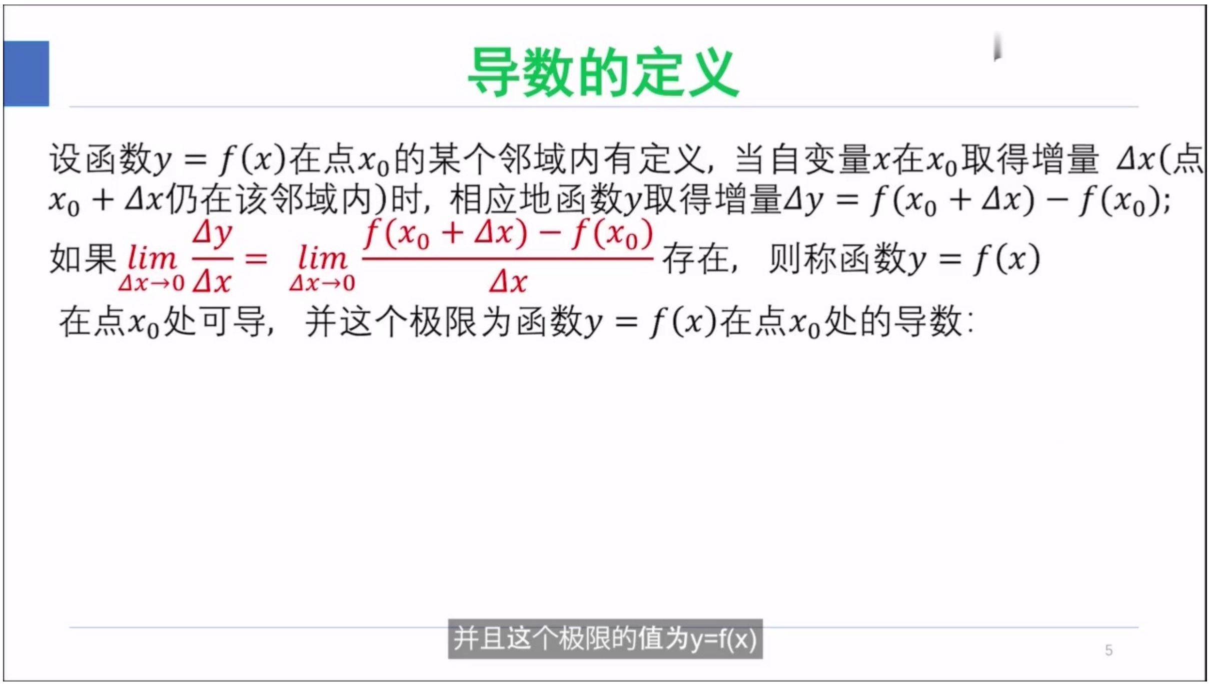

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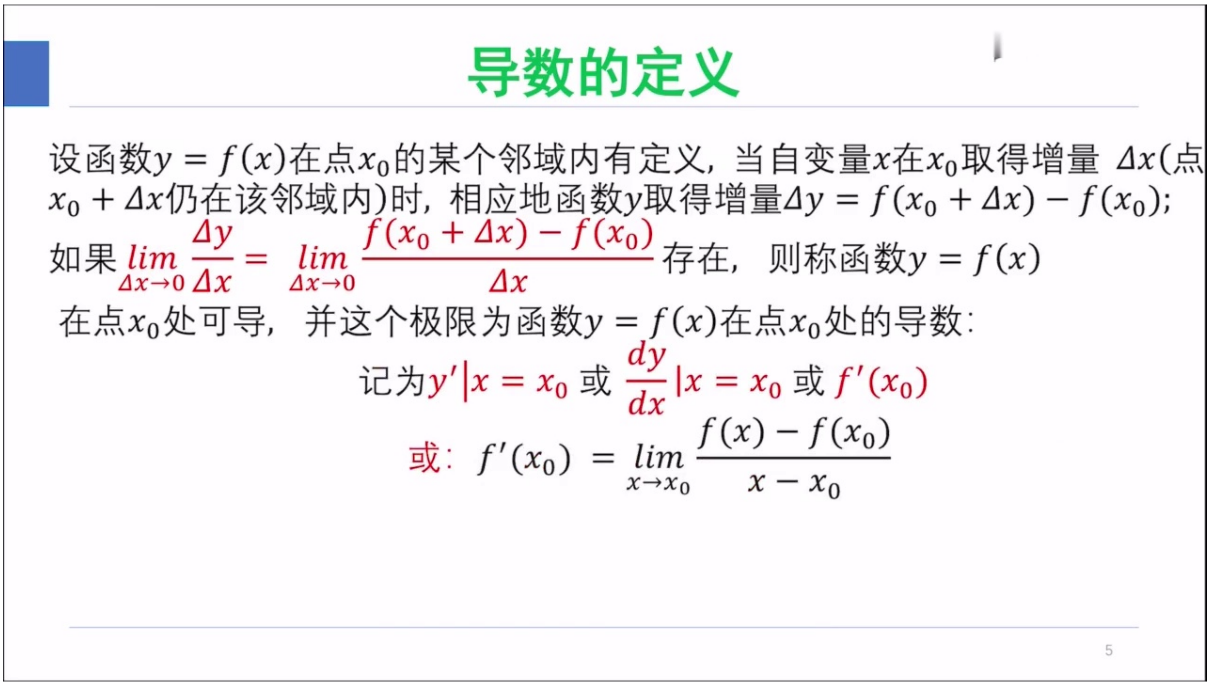
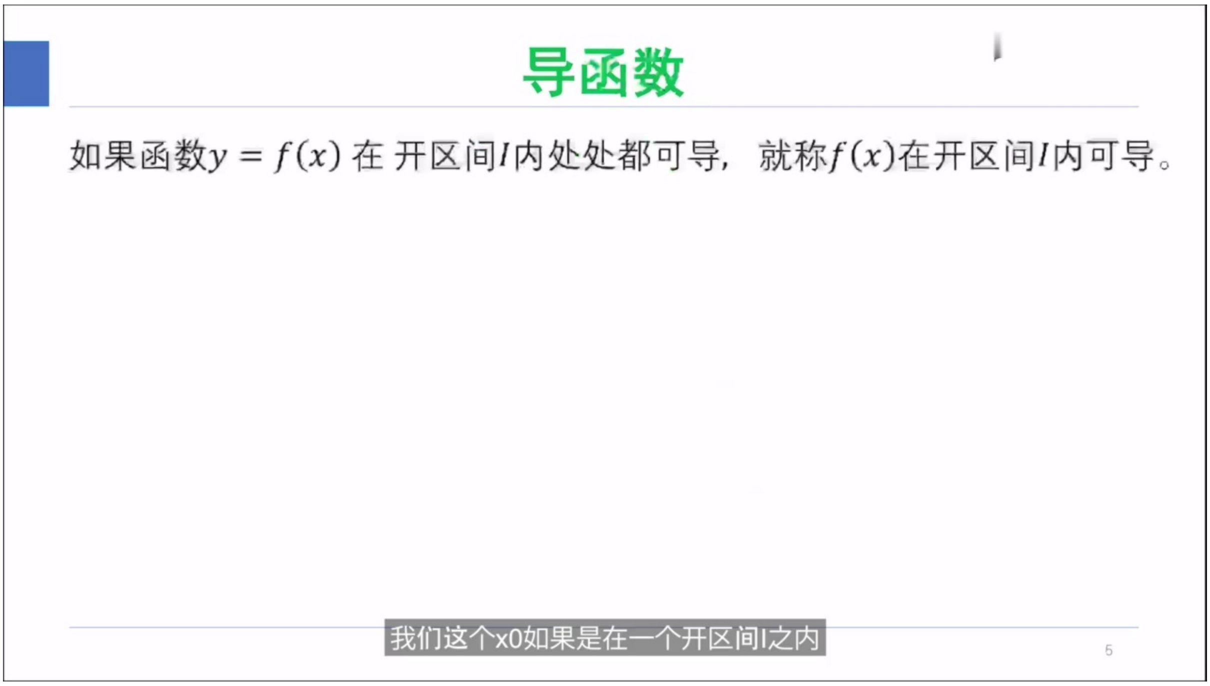


导数的定义
设函数$y=f(x)$在点x_0的某个邻域内有定义，当自变量x在x_0取得增量 Δx(点$x_0+\Delta x$仍在该邻域内)时，相应地函数y取得增量$\Delta y=f(x_0+\Delta x)-f(x_0)$;
如果$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$存在，则称函数$y=f(x)$
在点x_0处可导，并这个极限为函数$y=f(x)$在点x_0处的导数:
记为$y'|x=x_0$ 或 $\frac{dy}{dx}|x=x_0$ 或 $f'(x_0)$
或：$f'(x_0)=\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}$
5

■■■: 00:04:32.999



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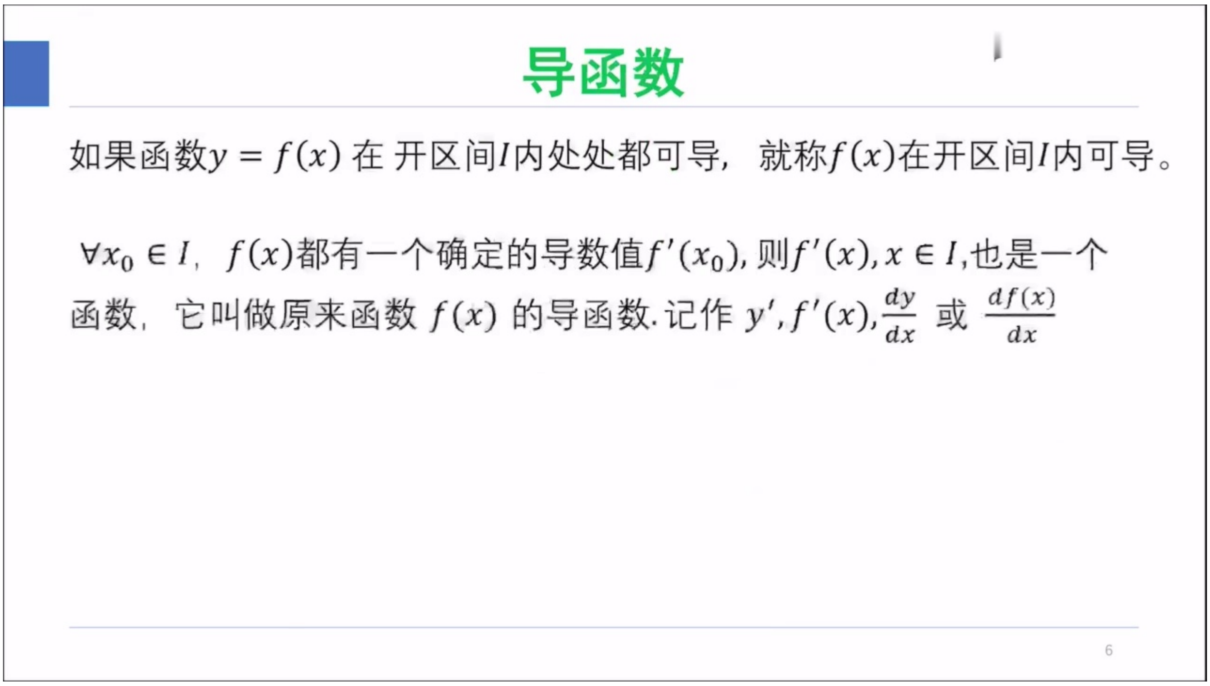

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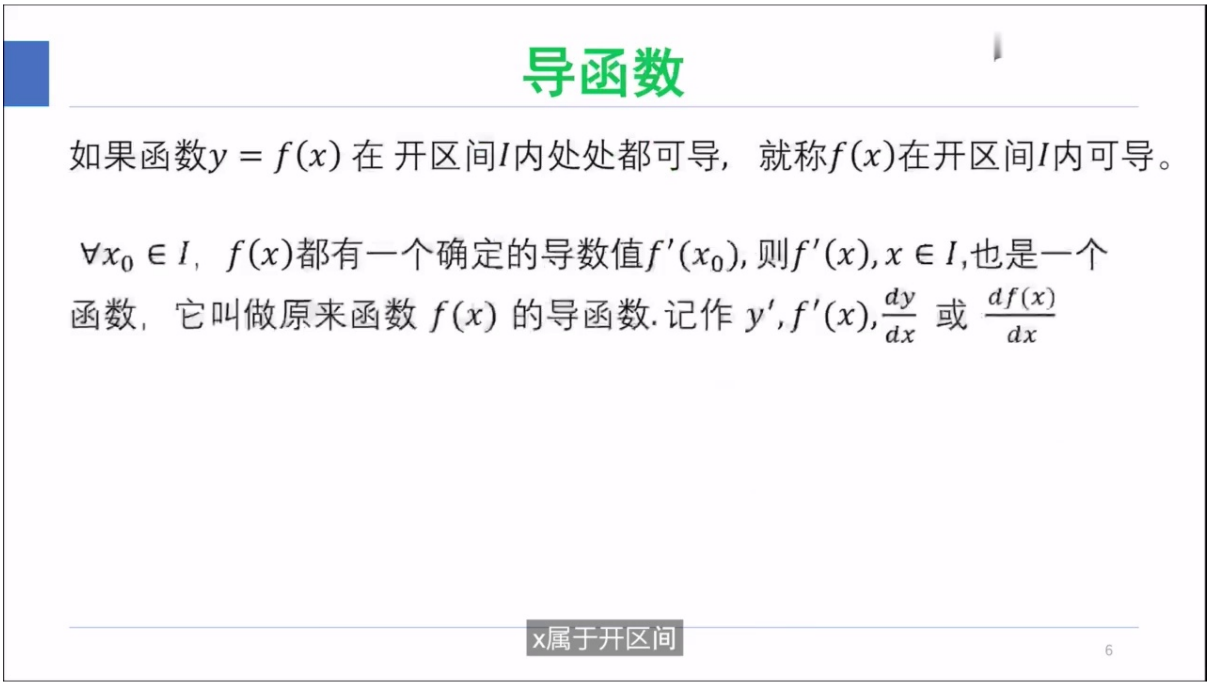

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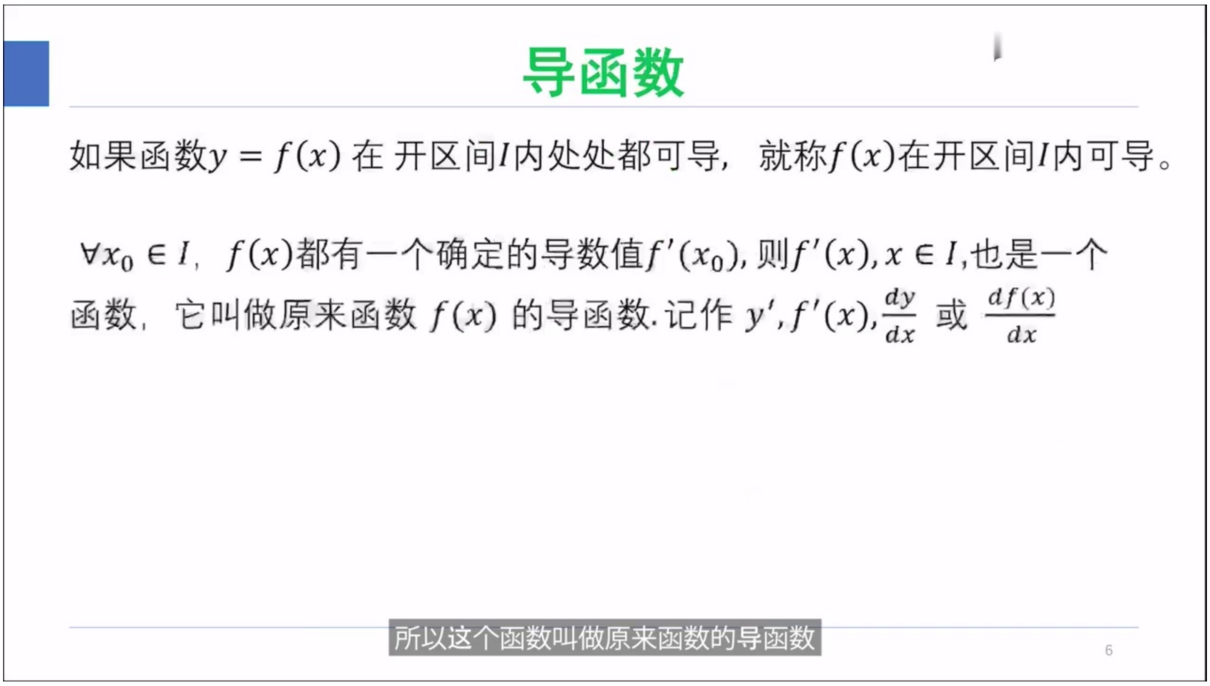
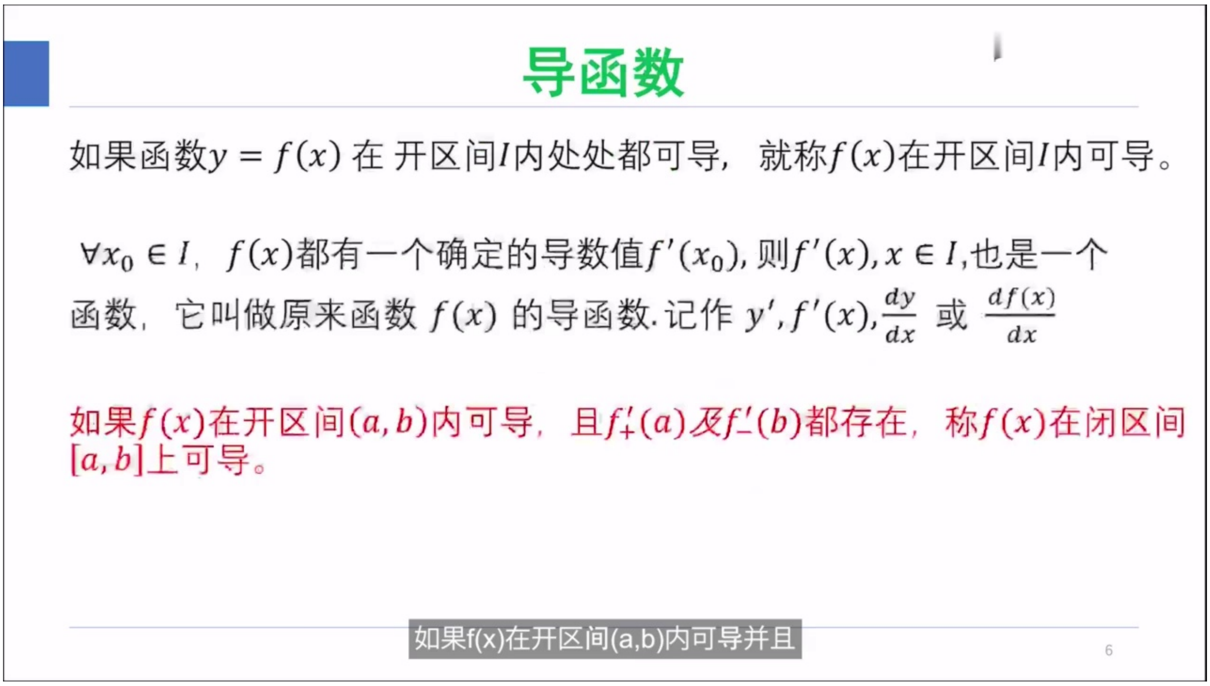

导函数
如果函数$y = f(x)$ 在 开区间I内处处都可导， 就称$f(x)$在开区间I内可导。
$\forall x_0 \in I$， $f(x)$都有一个确定的导数值$f'(x_0)$, 则$f'(x), x \in I$,也是一个
函数， 它叫做原来函数 $f(x)$ 的导函数. 记作 $y', f'(x), \frac{dy}{dx}$ 或 $\frac{df(x)}{dx}$
所以这个函数叫做原来函数的导函数
6

■■■: 00:05:56.999



■ 22 ■ / ■ 81 ■
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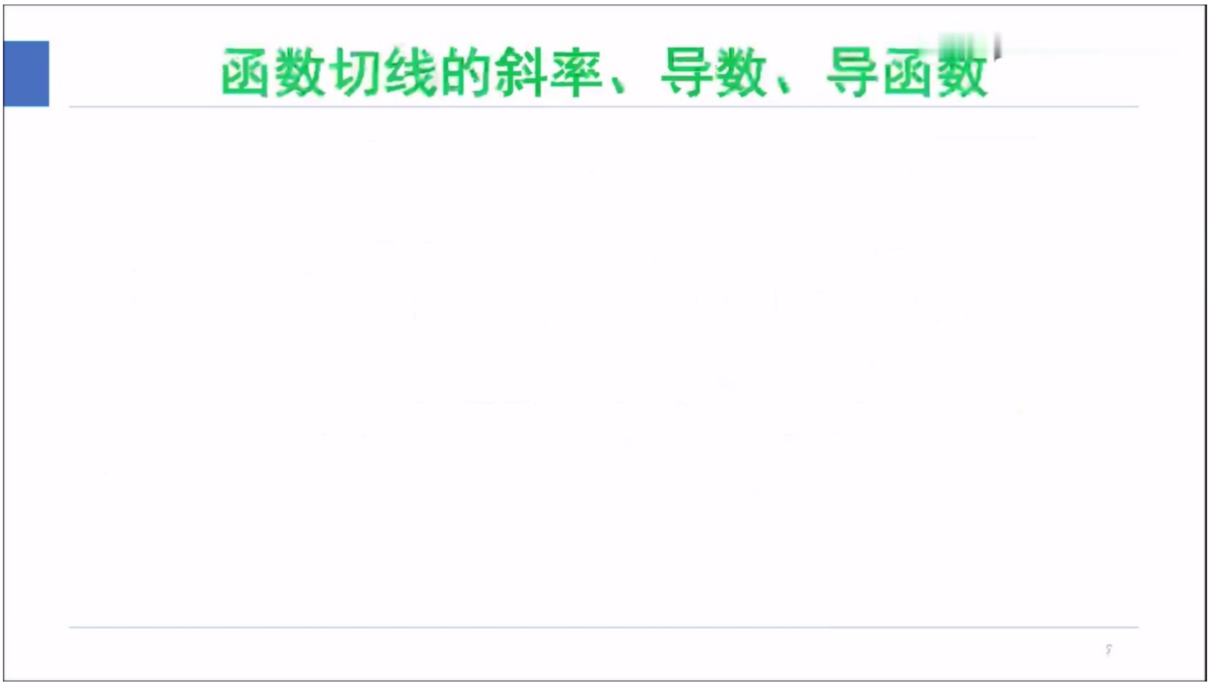

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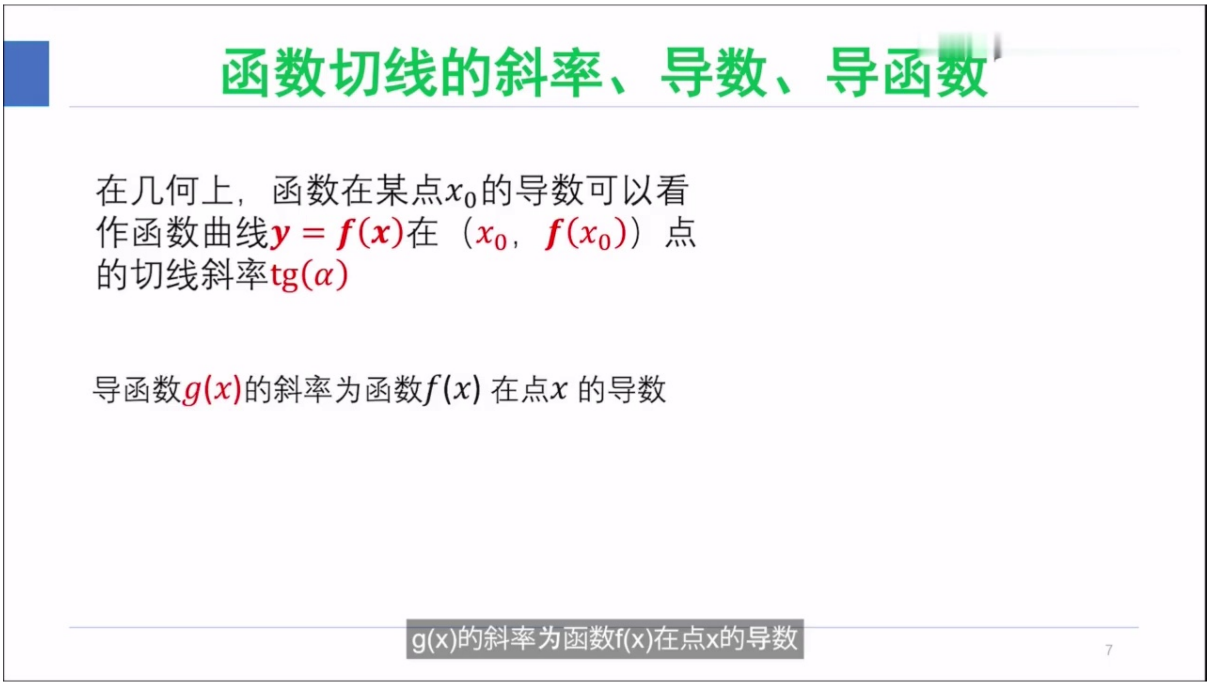

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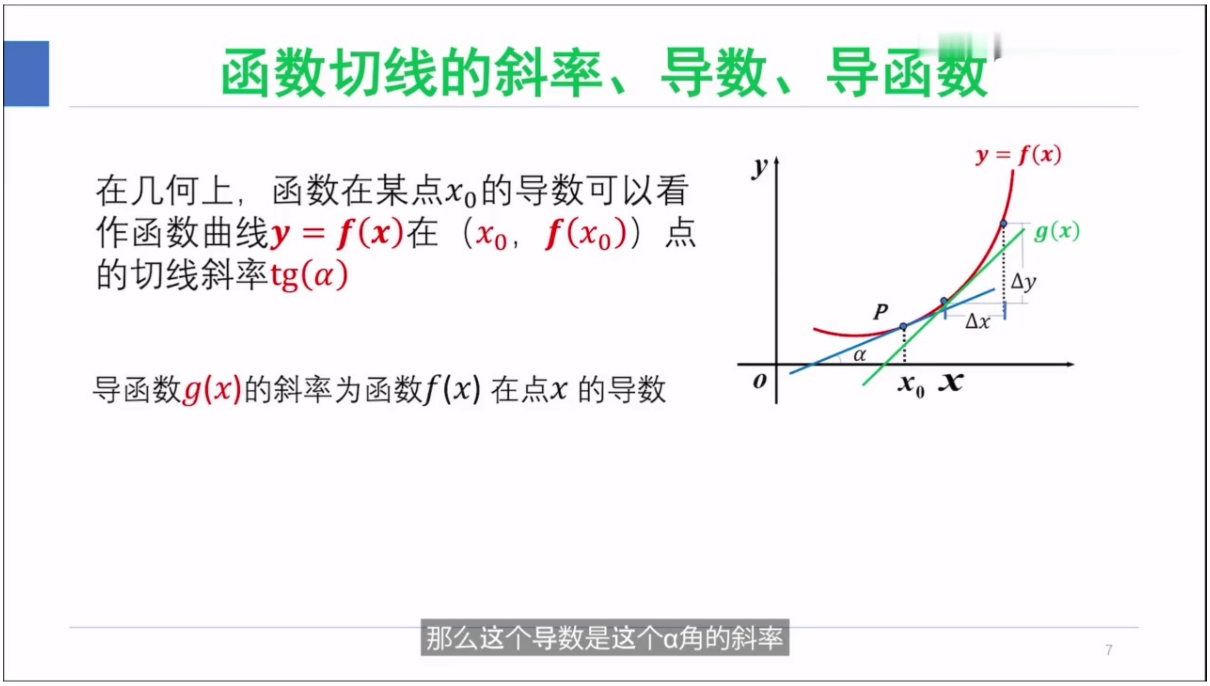

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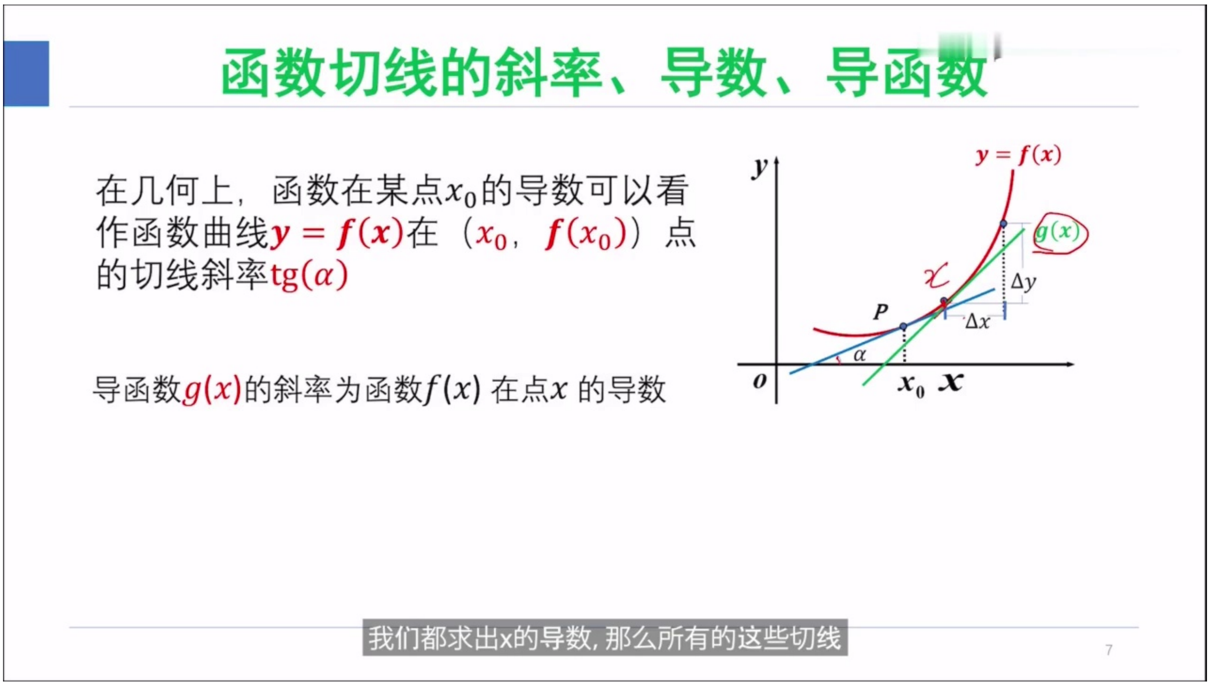

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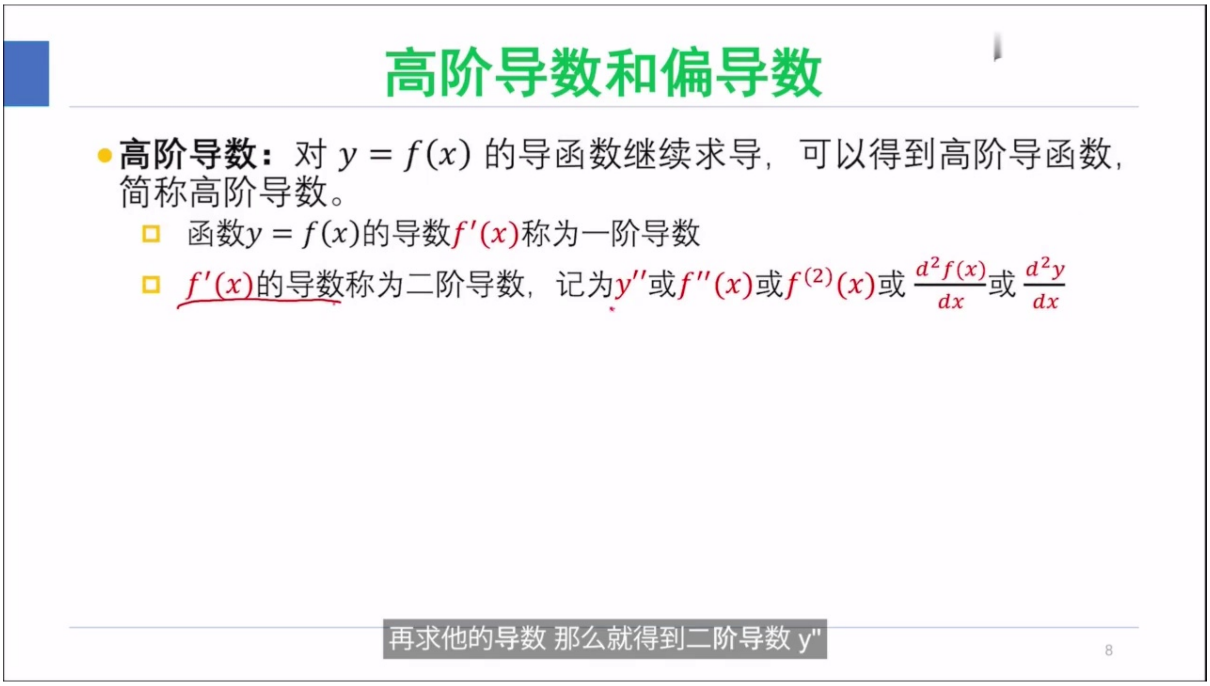

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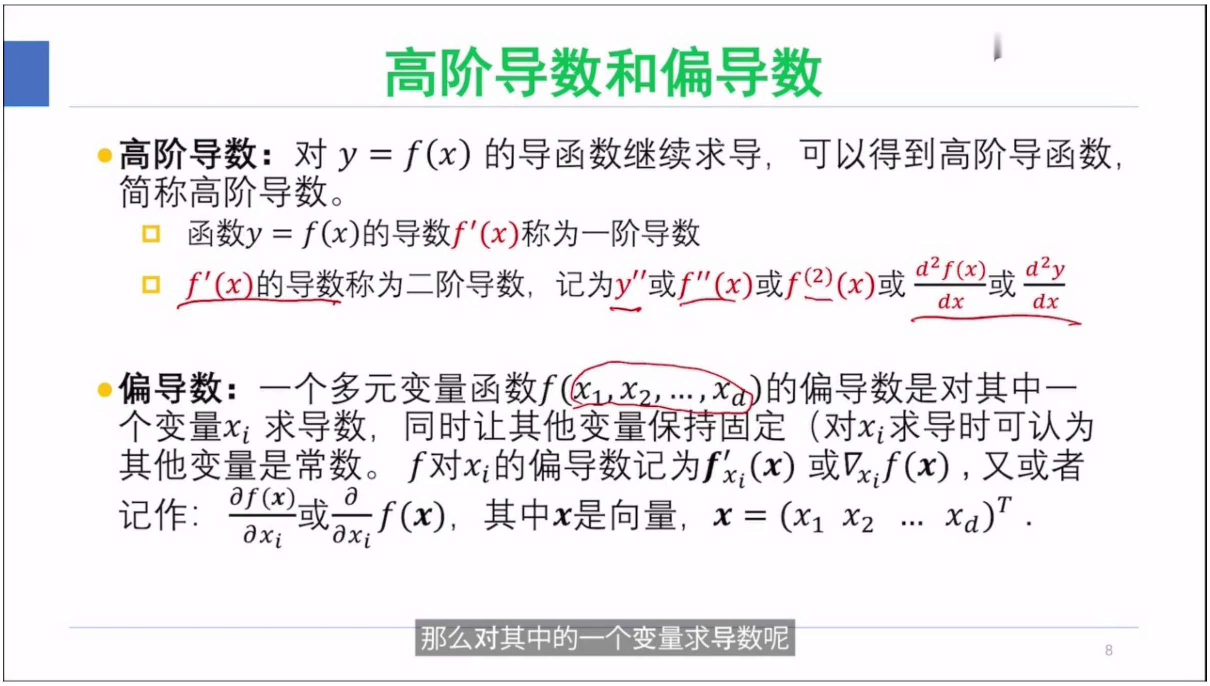

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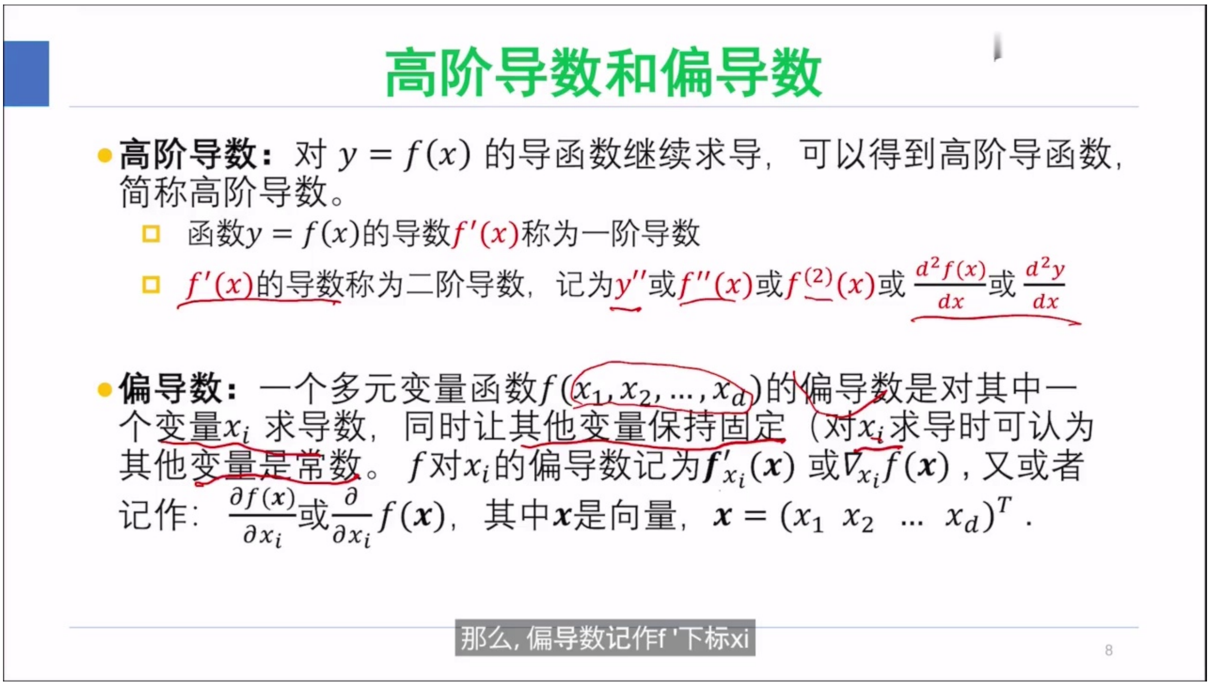

■ 29 ■ / ■ 81 ■

■■: keyframe_00-08-23-999_0028.jpg

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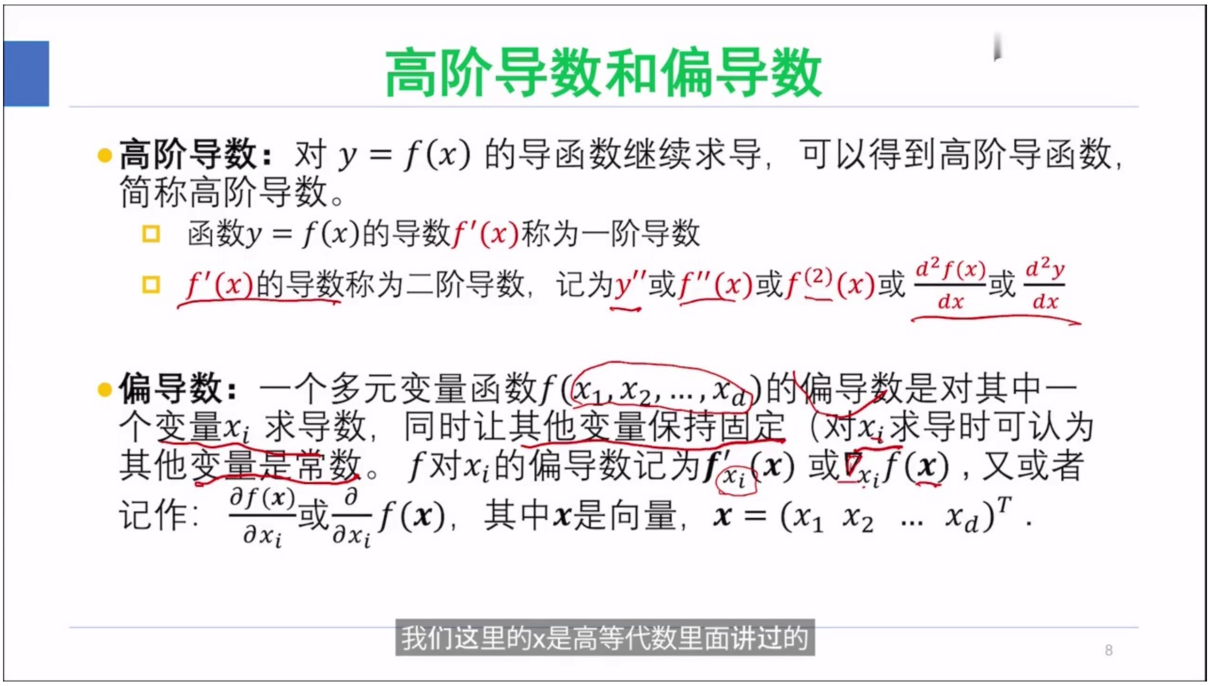


■■: keyframe_00-08-44-999_0029.jpg

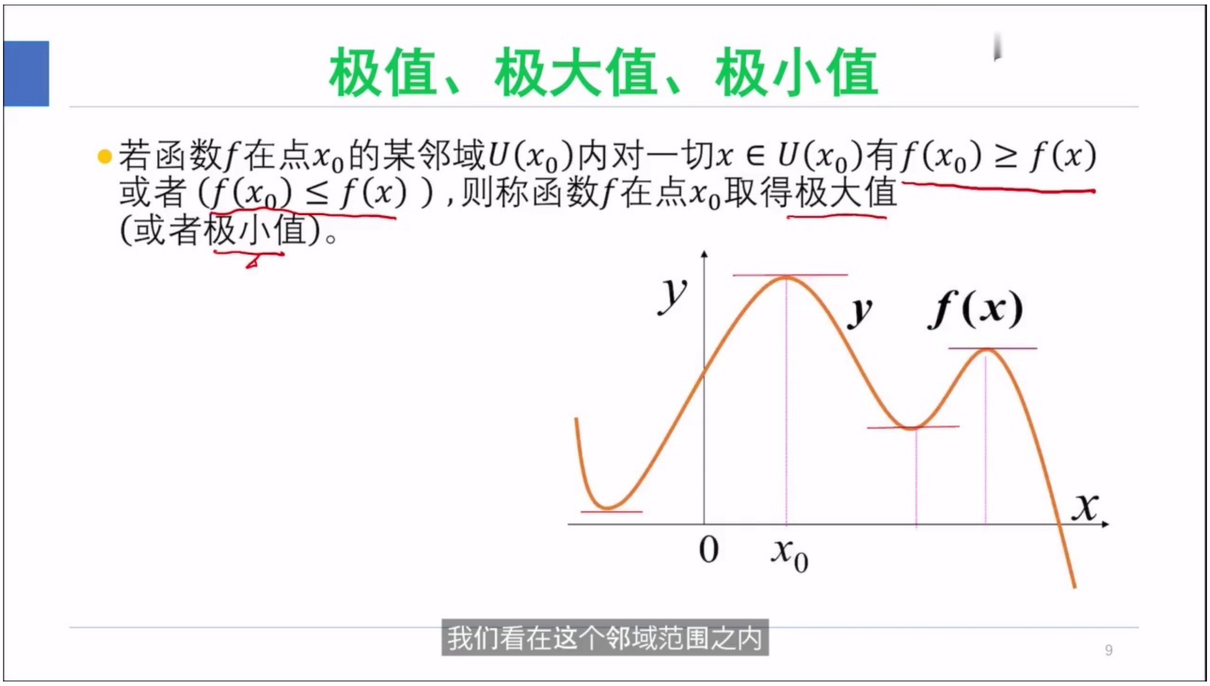
■■■: 00:09:05.999

极值、极大值、极小值

- 若函数f在点x_0的某邻域$U(x_0)$内对一切$x \in U(x_0)$有$f(x_0) \geq f(x)$或者（$f(x_0) \leq f(x)$），则称函数f在点x_0取得极大值(或者极小值)。

■■: keyframe_00-09-05-999_0030.jpg

■■■: 00:09:26.999



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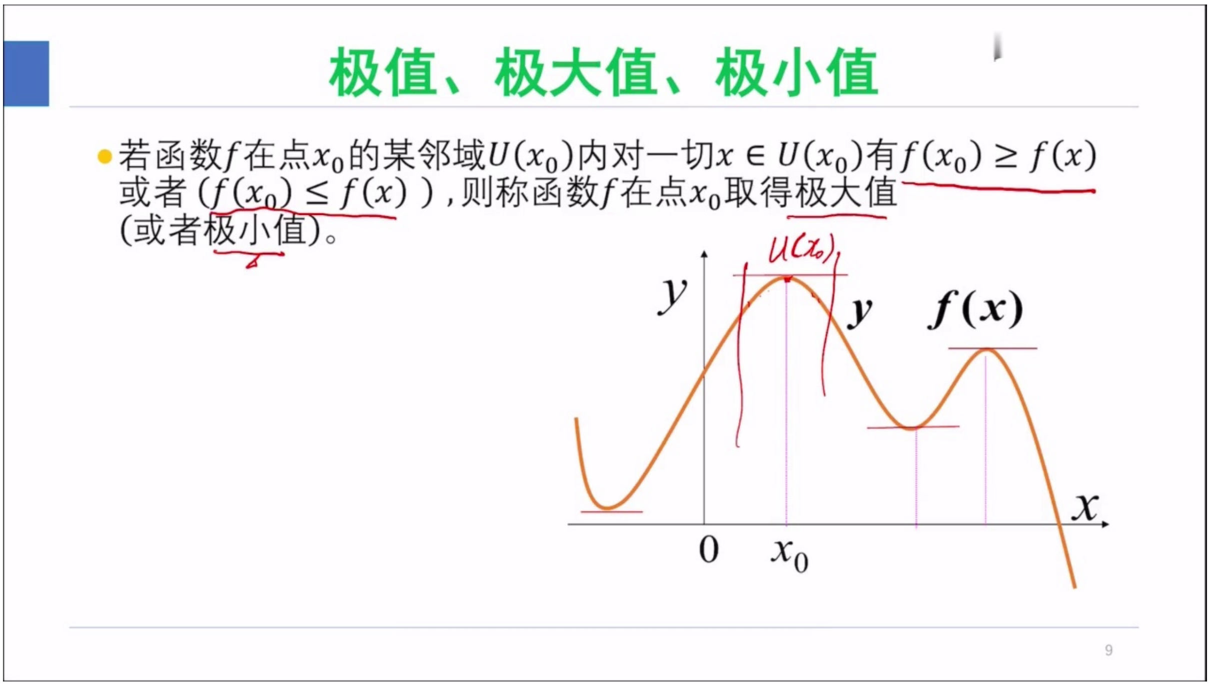

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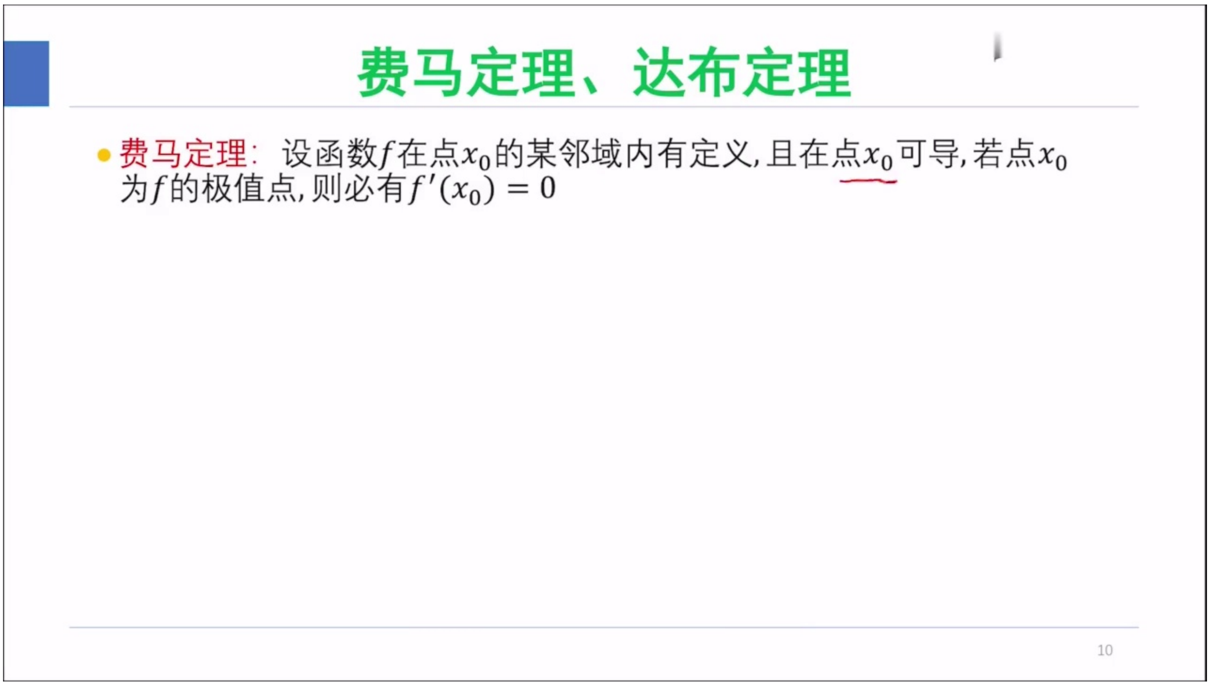

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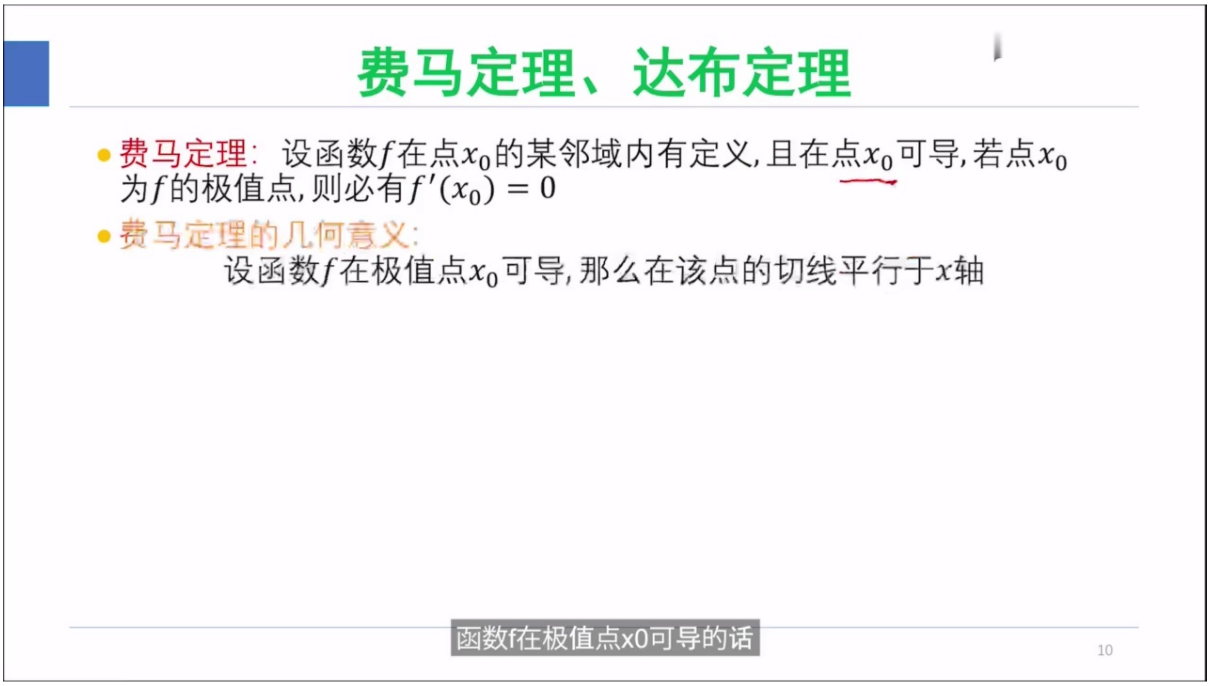

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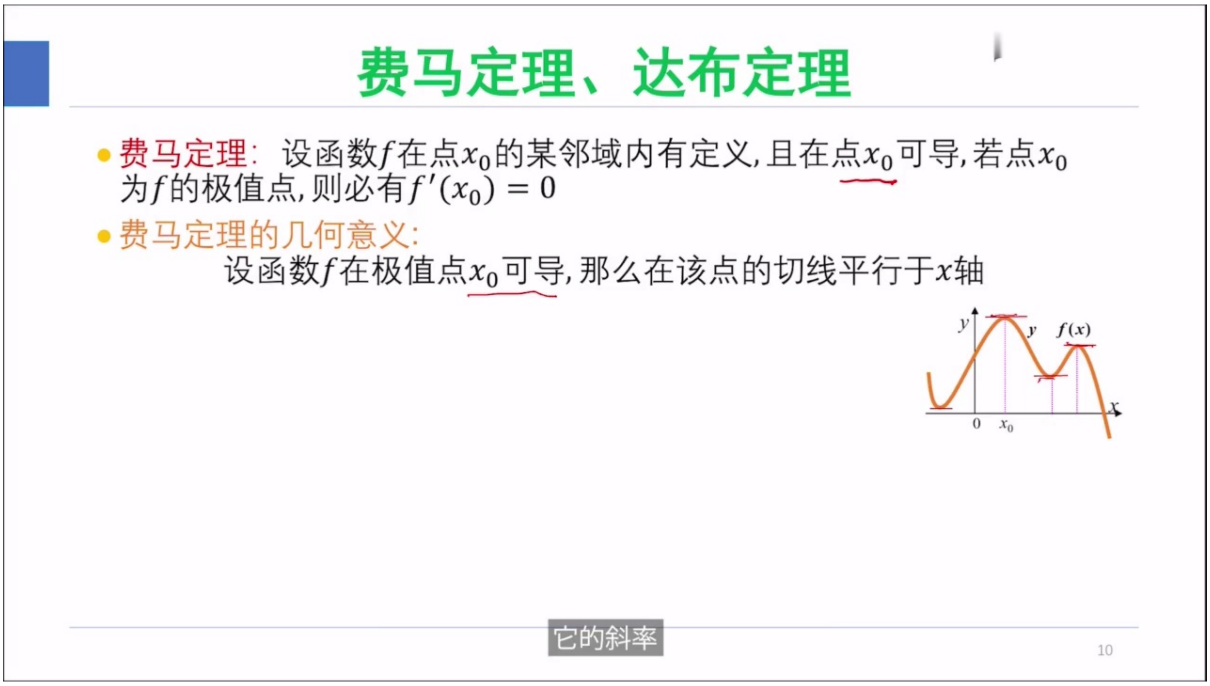


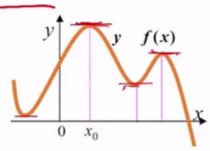
■■: keyframe_00-10-50-999_0035.jpg

■■■: 00:11:11.999

费马定理、达布定理

- 费马定理：设函数f在点x_0的某邻域内有定义，且在点x_0可导，若点x_0为f的极值点，则必有$f'(x_0)=0$
- 费马定理的几何意义：

  设函数f在极值点x_0可导，那么在该点的切线平行于x轴



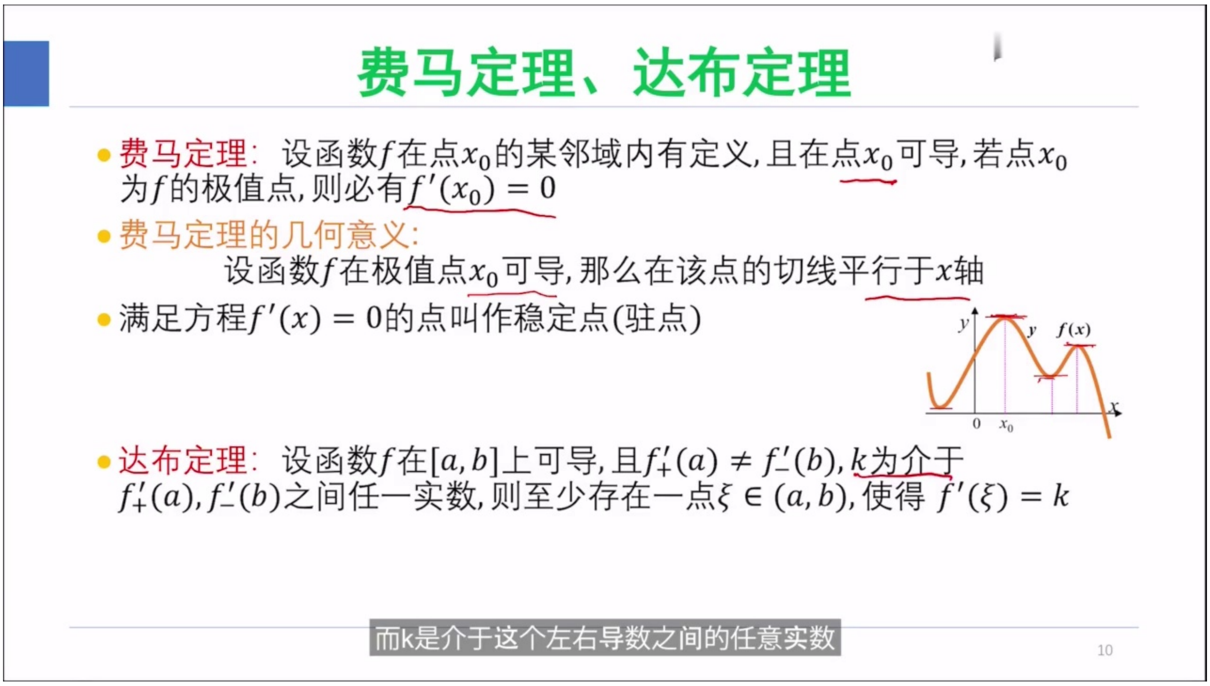
满足这个方程 f'(x0)=0的这个点

10

■ 37 ■ / ■ 81 ■

■■: keyframe_00-11-11-999_0036.jpg

■■■: 00:11:32.999



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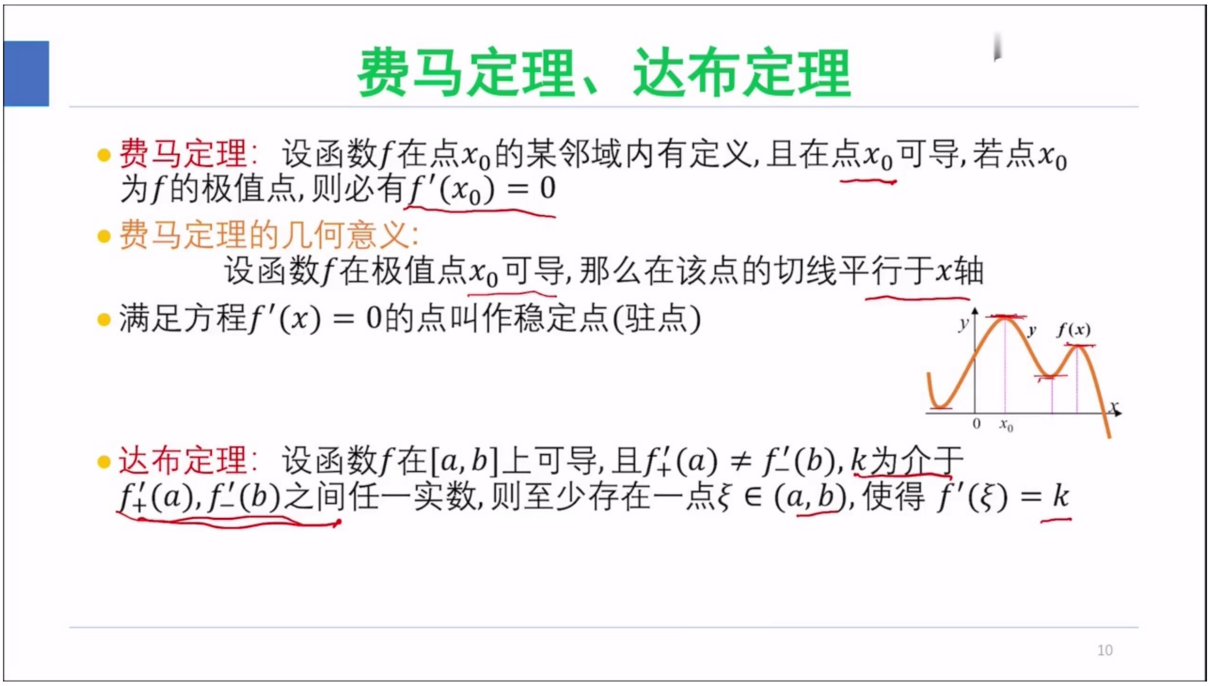

■ 39 ■ / ■ 81 ■
■■: keyframe_00-11-53-999_0038.jpg

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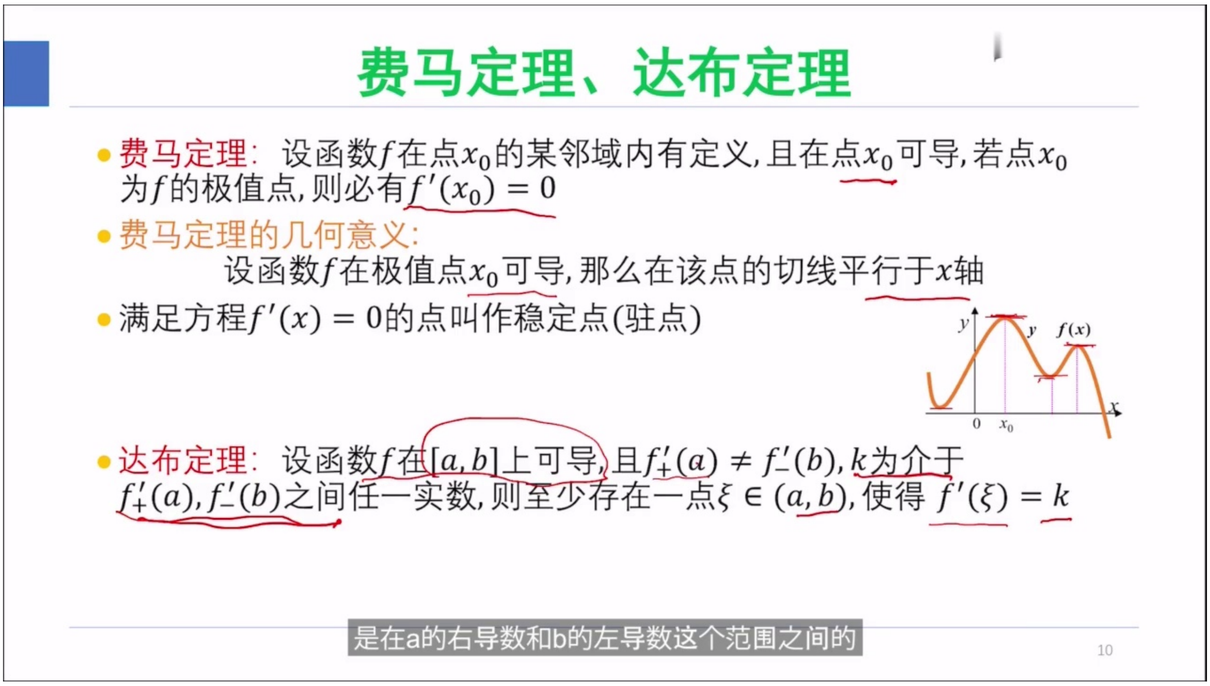

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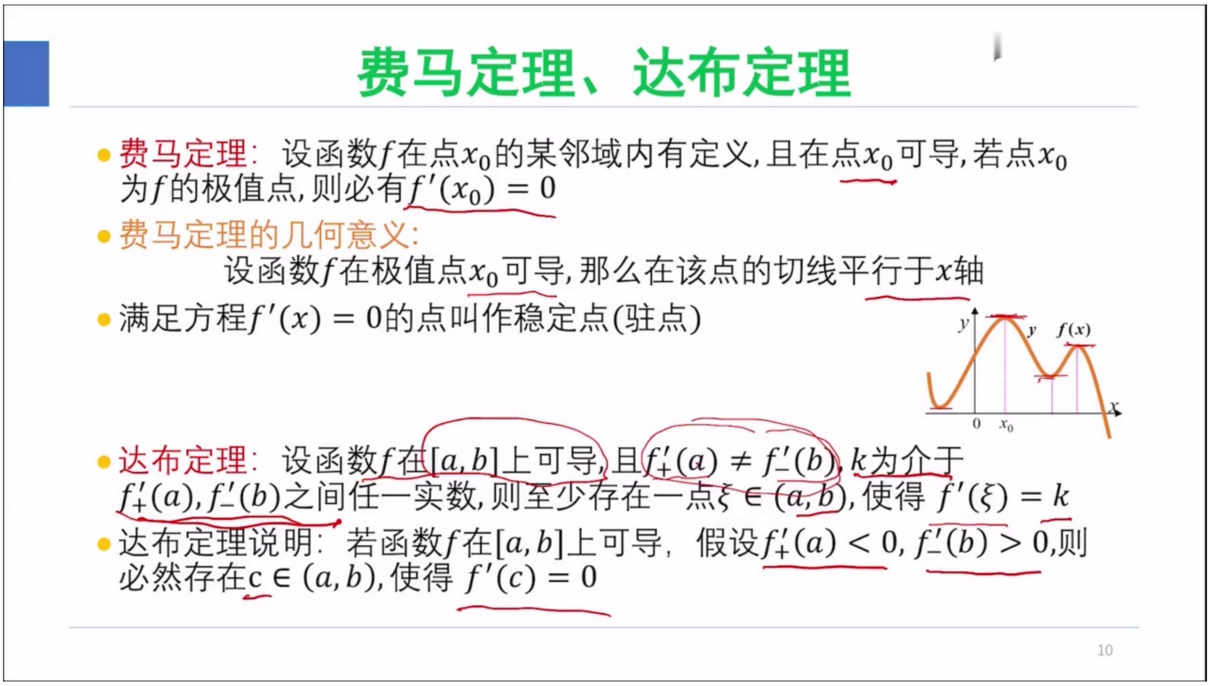

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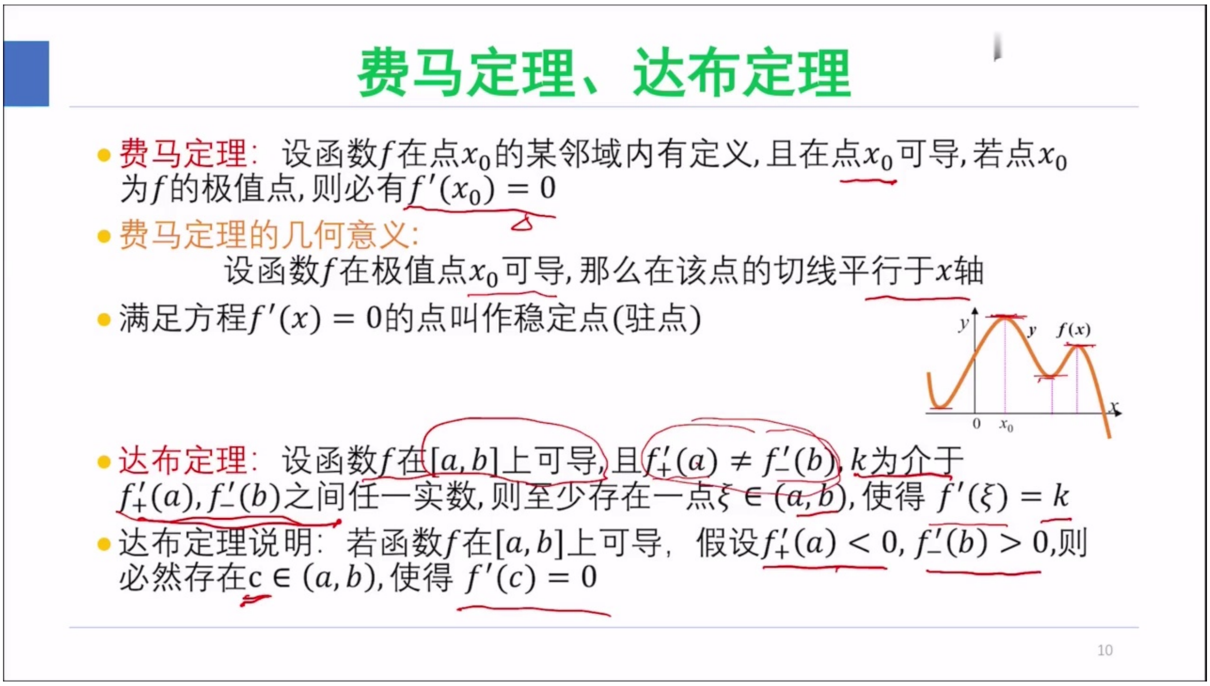
费马定理、达布定理
费马定理：设函数f在点x_0的某邻域内有定义，且在点x_0可导，若点x_0为f的极值点，则必有$f'(x_0)=0$
费马定理的几何意义：
设函数f在极值点x_0可导，那么在该点的切线平行于x轴
满足方程$f'(x)=0$的点叫作稳定点（驻点）
y
y
f(x)
0
x_0
x
达布定理：设函数f在$[a,b]$上可导，且$f'_+(a)\neq f'_-(b)$，k为介于$f'_+(a), f'_-(b)$之间任一实数，则至少存在一点$\xi\in(a,b)$，使得 $f'(\xi)=k$
达布定理说明：若函数f在$[a,b]$上可导，假设$f'_+(a)<0$，$f'_-(b)>0$，则必然存在$c\in(a,b)$，使得 $f'(c)=0$
10

■■■: 00:13:17.999

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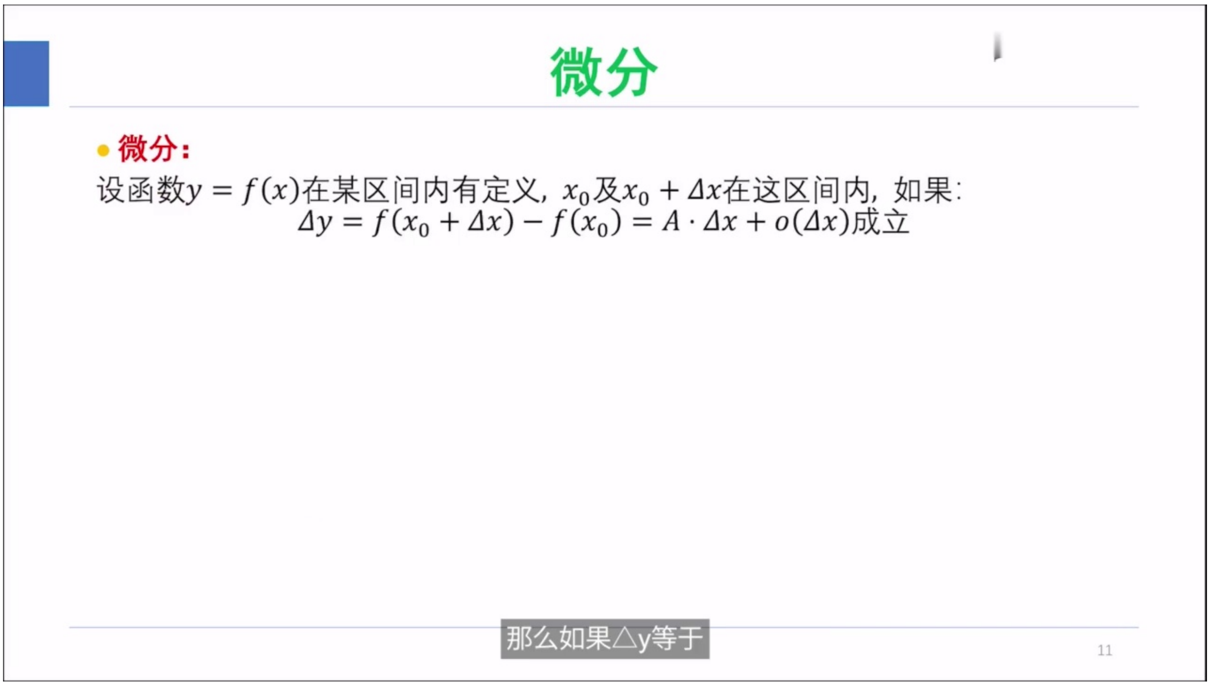
[redacted]: 00:13:38.999



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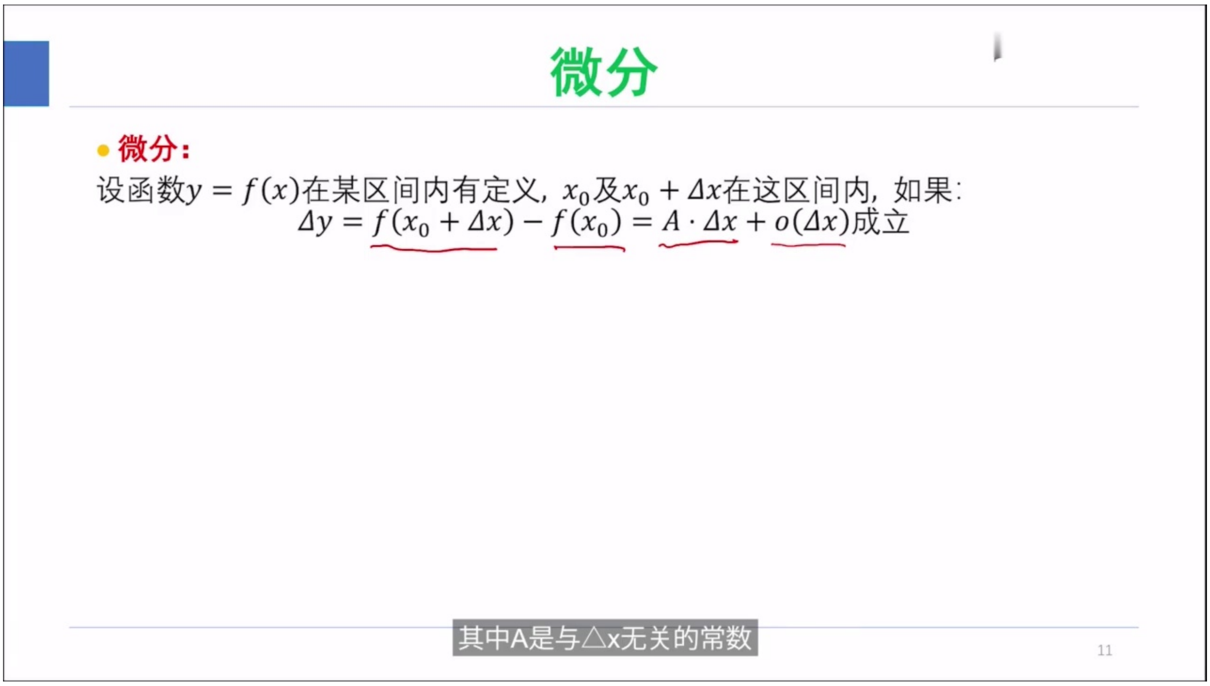

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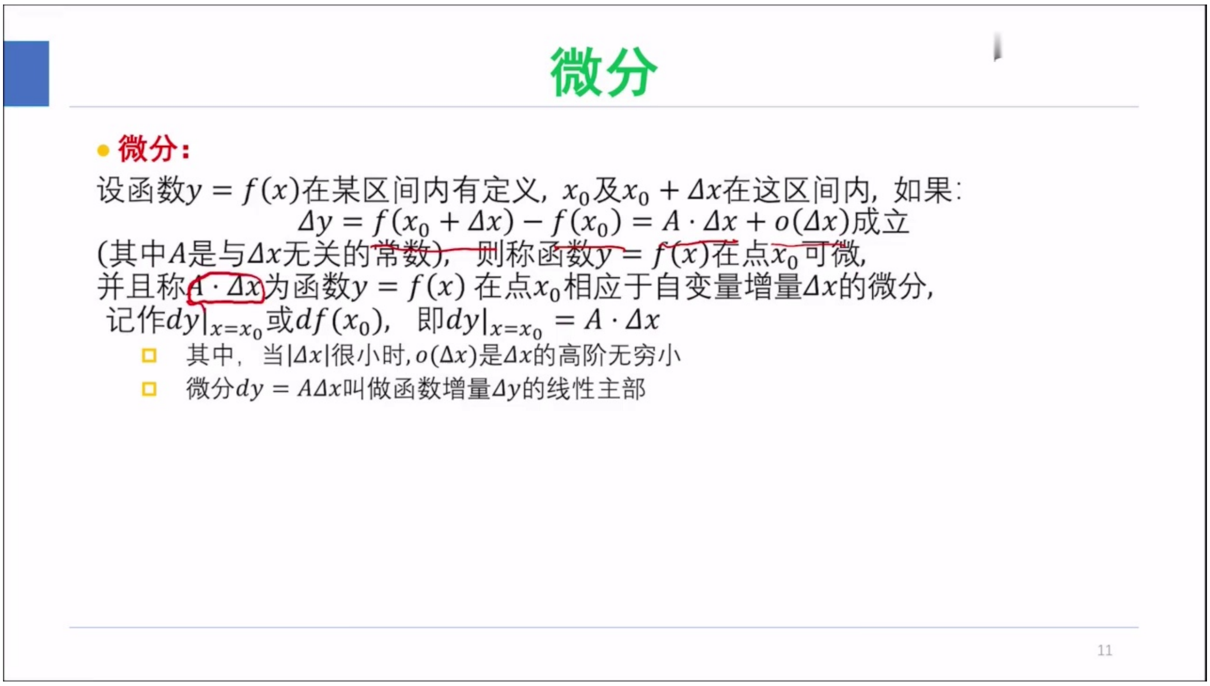

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微分

- **微分：**

设函数$y = f(x)$在某区间内有定义，x_0及$x_0 + \Delta x$在这区间内，如果：

$$\Delta y = f(x_0 + \Delta x) - f(x_0) = A \cdot \Delta x + o(\Delta x)\text{成立}$$

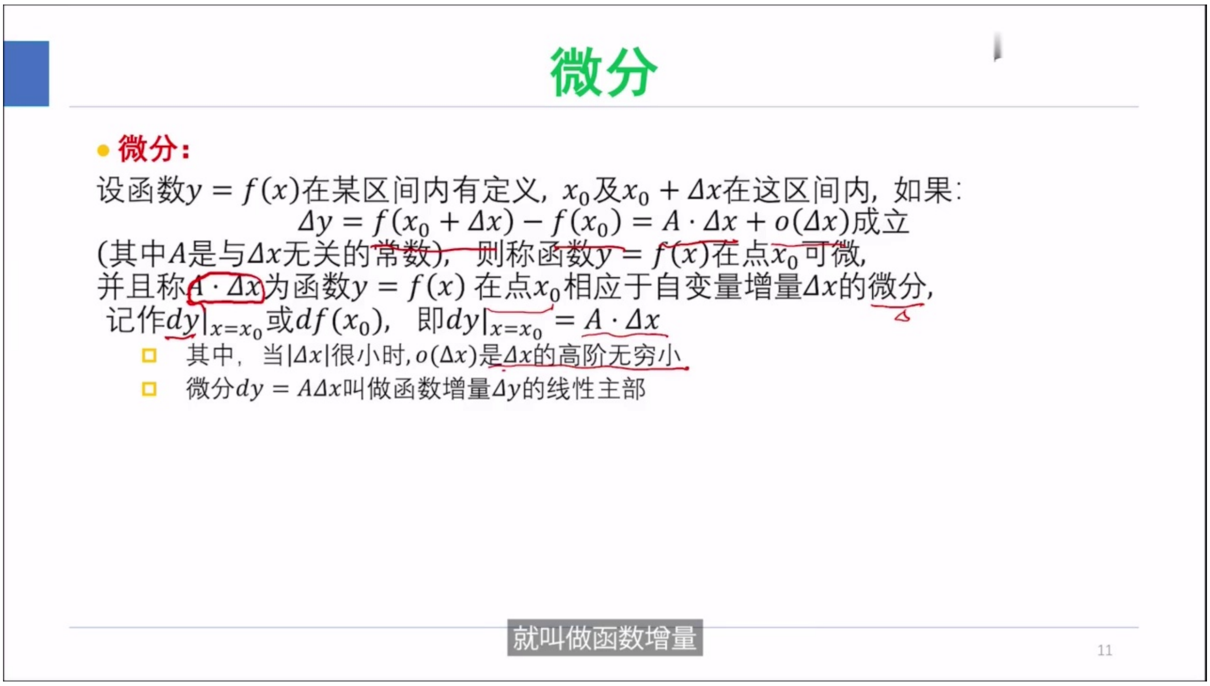
(其中A是与Δx无关的常数)，则称函数$y = f(x)$在点x_0可微，
并且称$A \cdot \Delta x$为函数$y = f(x)$ 在点x_0相应于自变量增量Δx的微分，
记作$dy|_{x=x_0}$或$df(x_0)$，即$dy|_{x=x_0} = A \cdot \Delta x$

- 其中，当$|\Delta x|$很小时，$o(\Delta x)$是Δx的高阶无穷小
- 微分$dy = A\Delta x$叫做函数增量Δy的线性主部

x=x0处的微分或者df(x0)

11

[redacted]: 00:15:02.999



[redacted] 48 [redacted] / [redacted] 81 [redacted]

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■■■: 00:15:23.999

微分

- **微分：**

设函数$y=f(x)$在某区间内有定义，x_0及$x_0+\Delta x$在这区间内，如果：

$$\Delta y=f(x_0+\Delta x)-f(x_0)=A\cdot\Delta x+o(\Delta x)\text{成立}$$

(其中A是与Δx无关的常数)，则称函数$y=f(x)$在点x_0可微，

并且称$A\cdot\Delta x$为函数$y=f(x)$ 在点x_0相应于自变量增量Δx的微分，

记作$dy|_{x=x_0}$或$df(x_0)$， 即$dy|_{x=x_0}=A\cdot\Delta x$

- 其中，当$|\Delta x|$很小时，$o(\Delta x)$是Δx的高阶无穷小
- 微分$dy=A\Delta x$叫做函数增量Δy的线性主部

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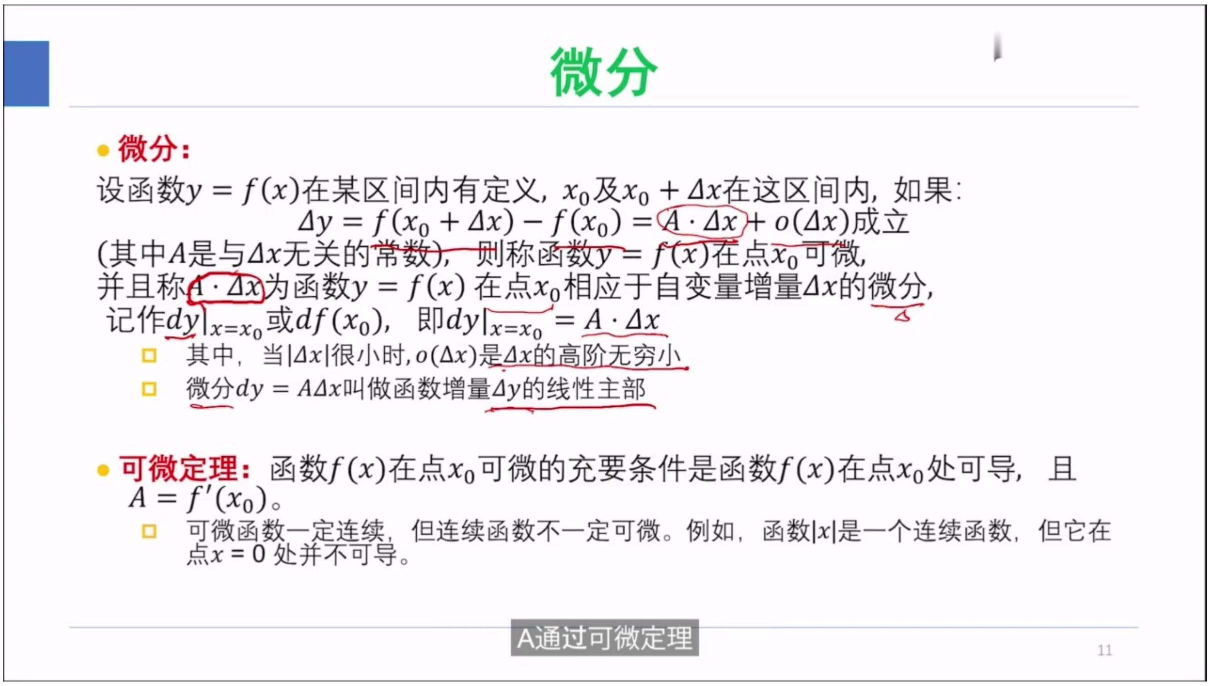

■ 50 ■ / ■ 81 ■
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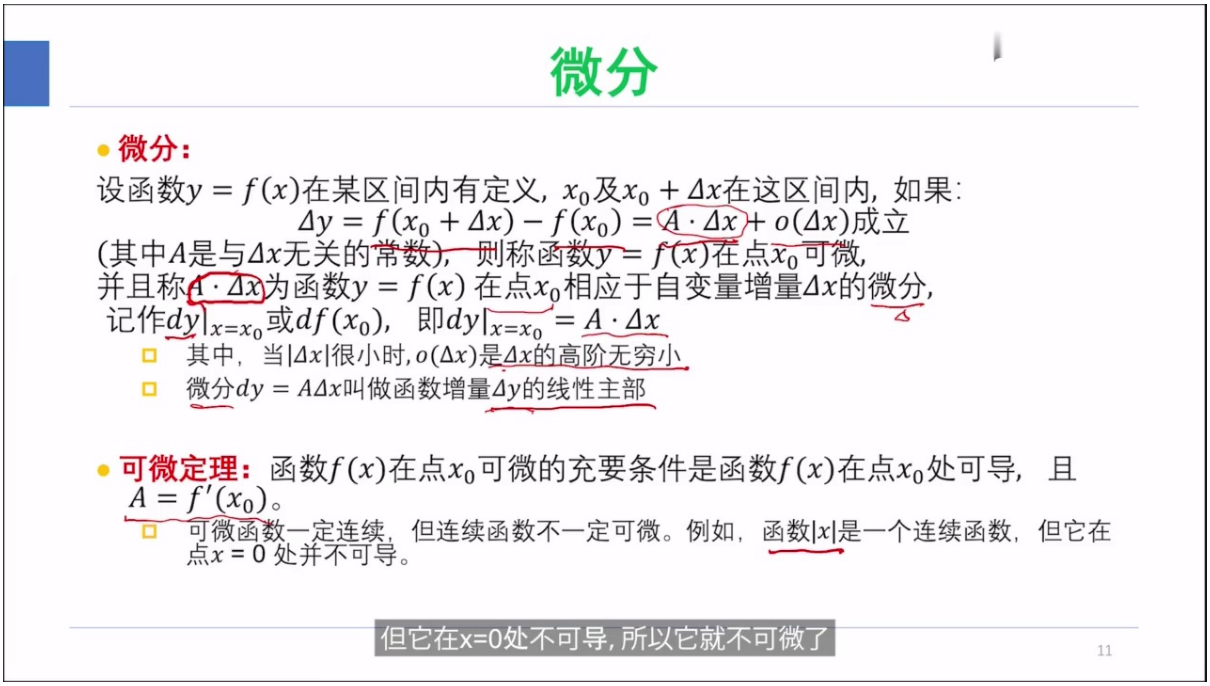
■■■: 00:16:05.999



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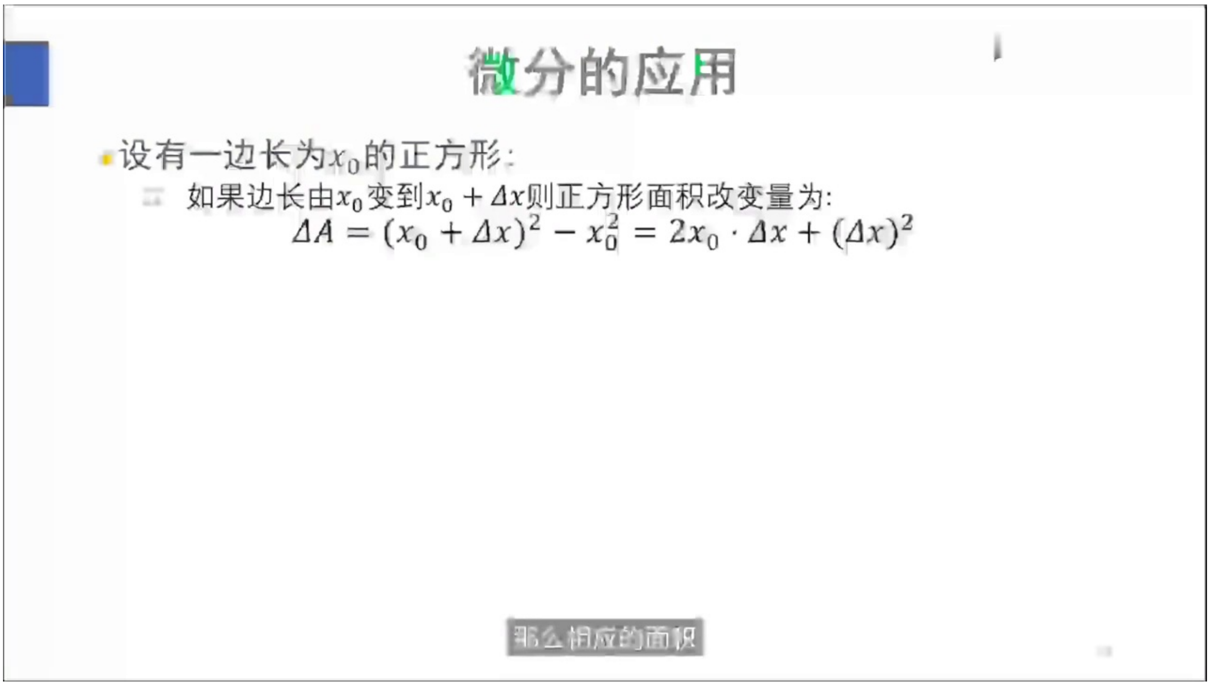

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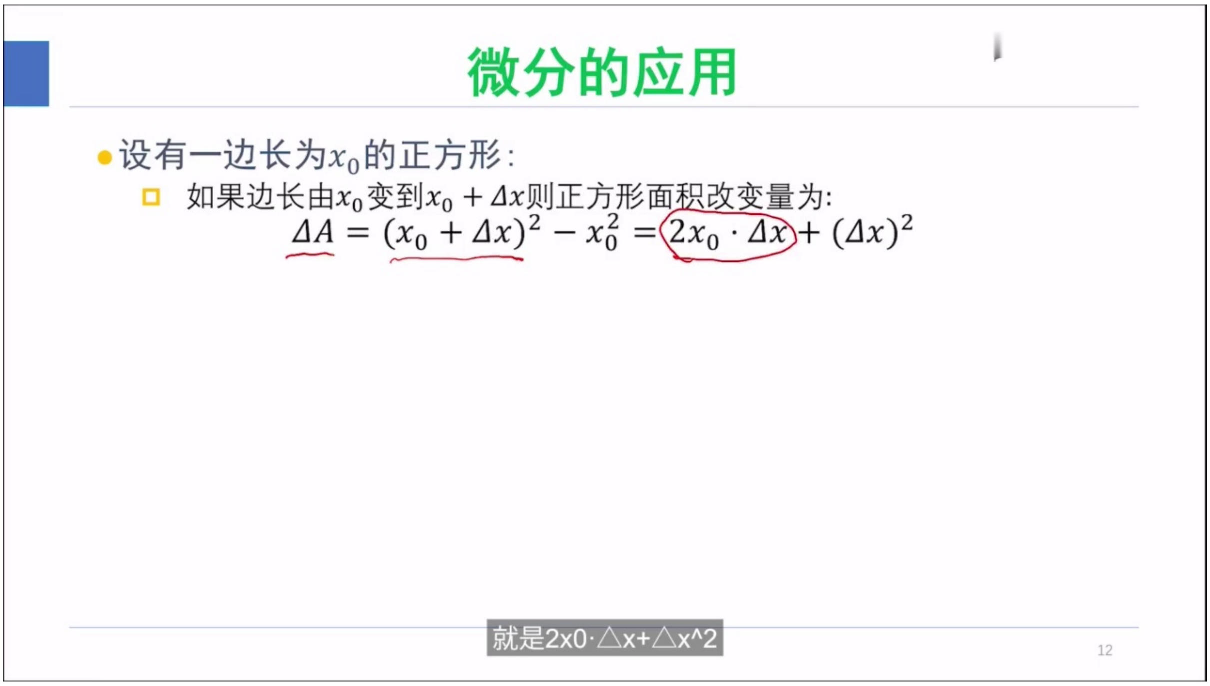

■ 53 ■ / ■ 81 ■

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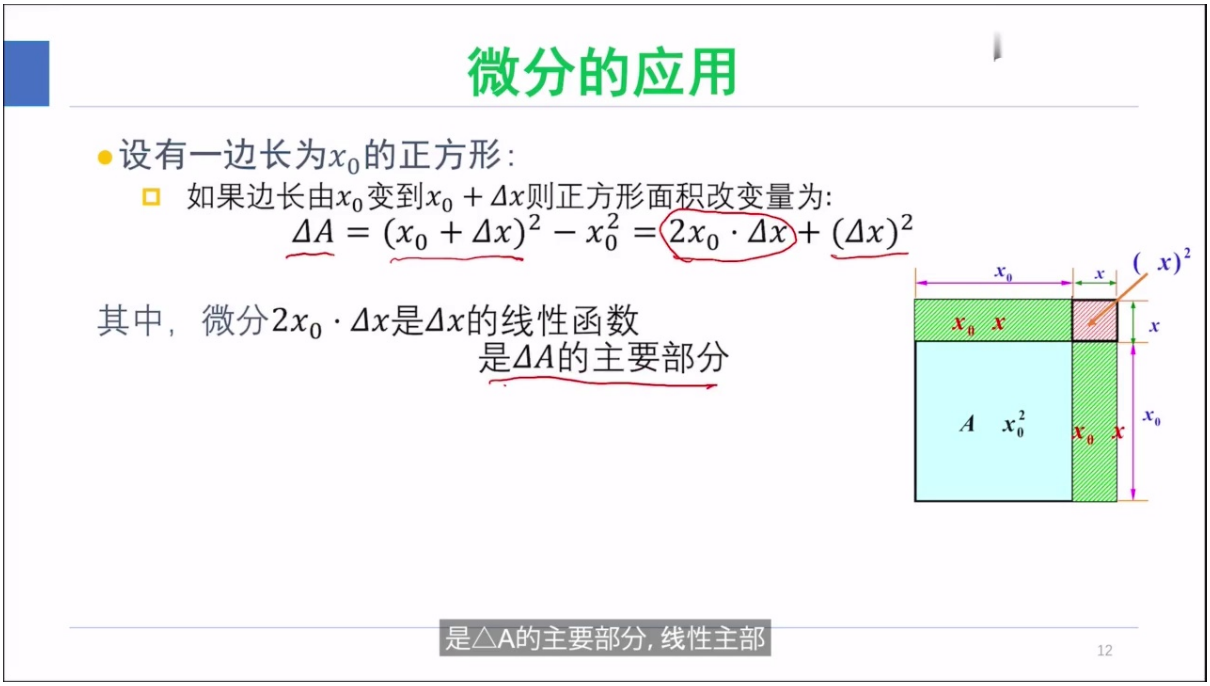

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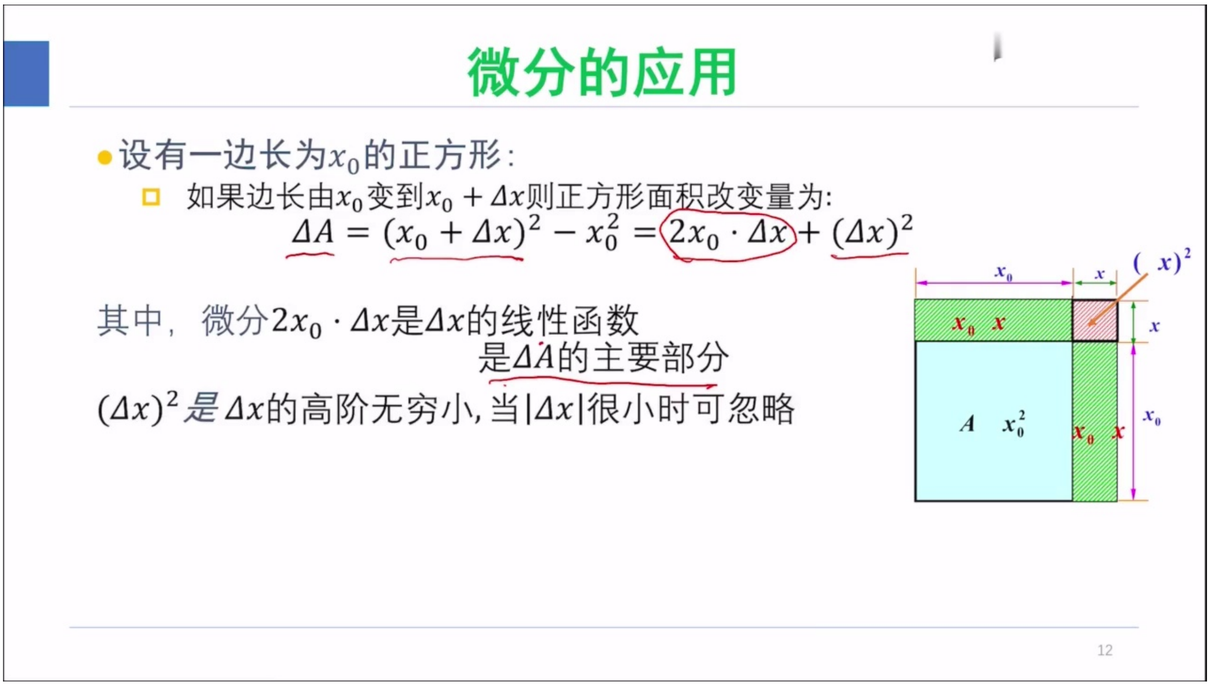

■ 55 ■ / ■ 81 ■
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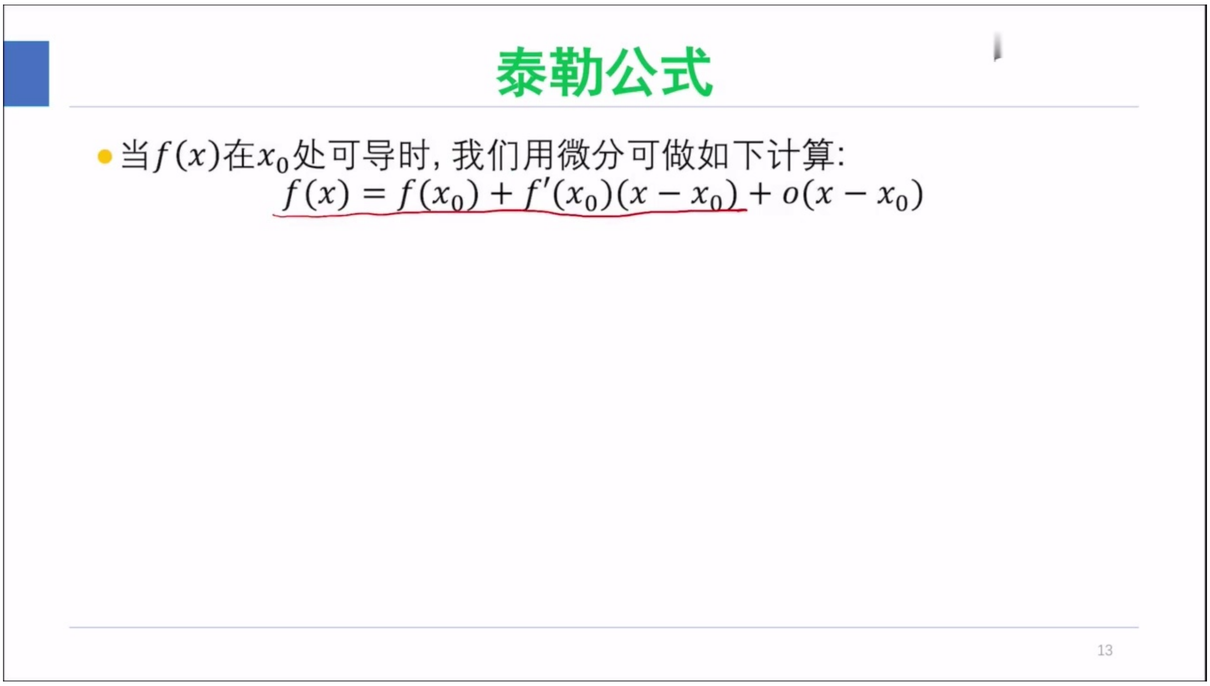

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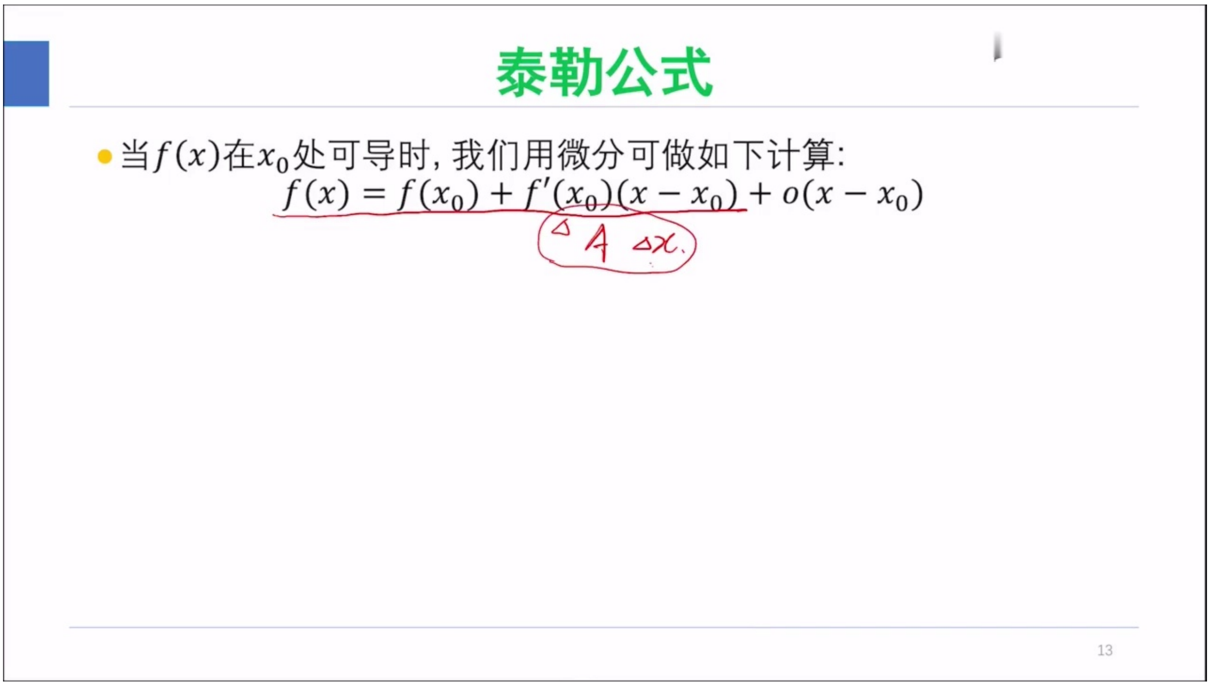

■ 57 ■ / ■ 81 ■

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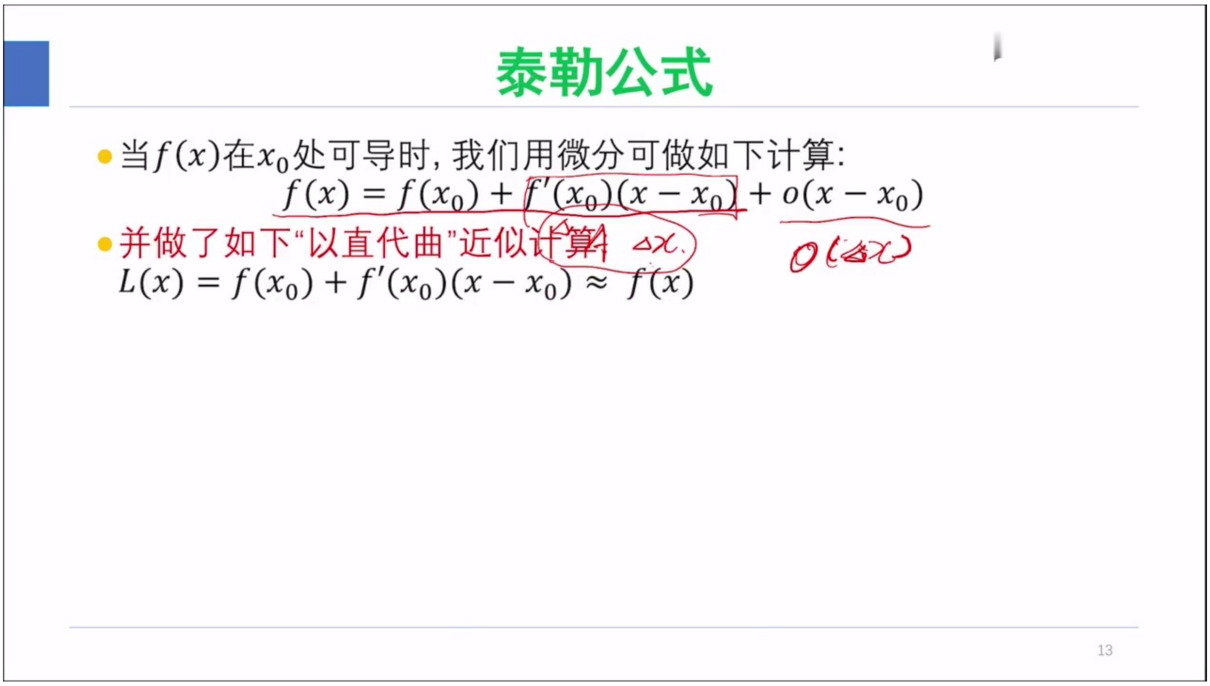

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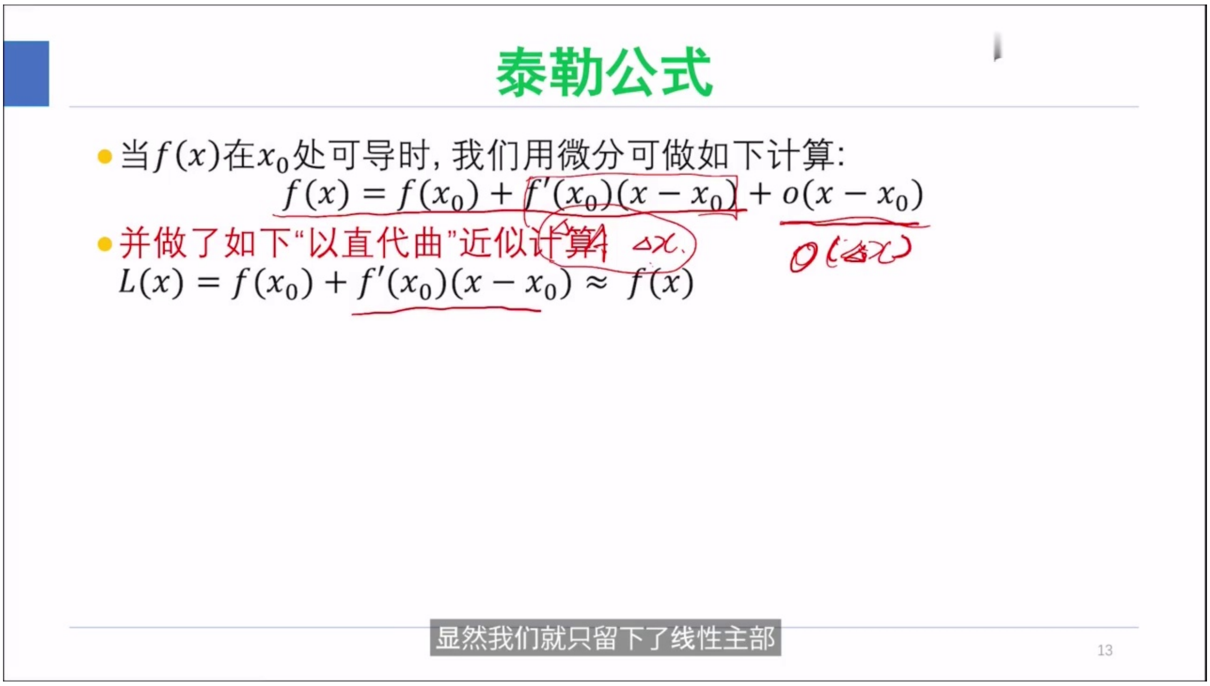

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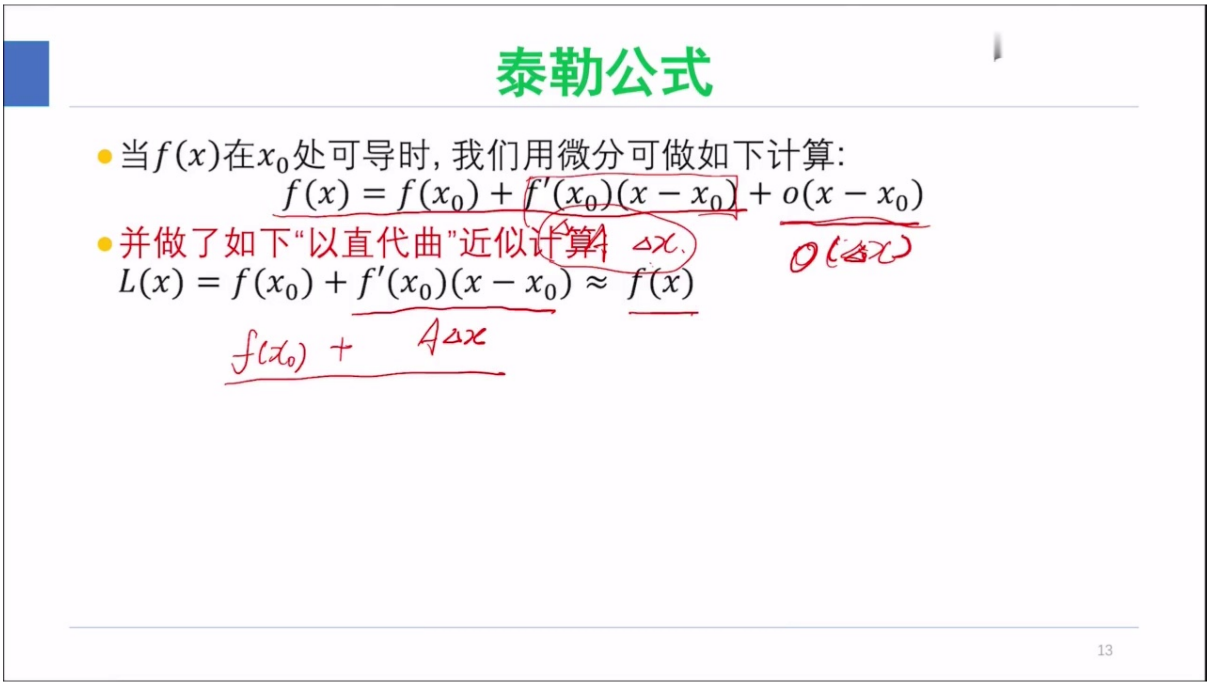

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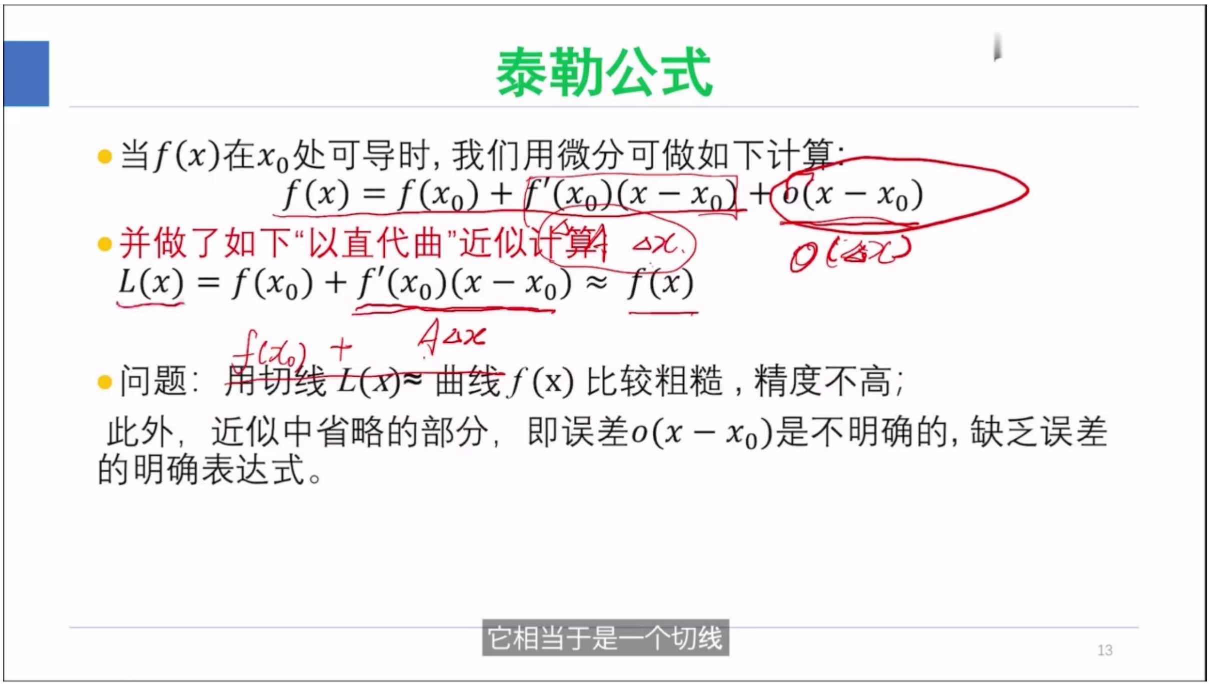

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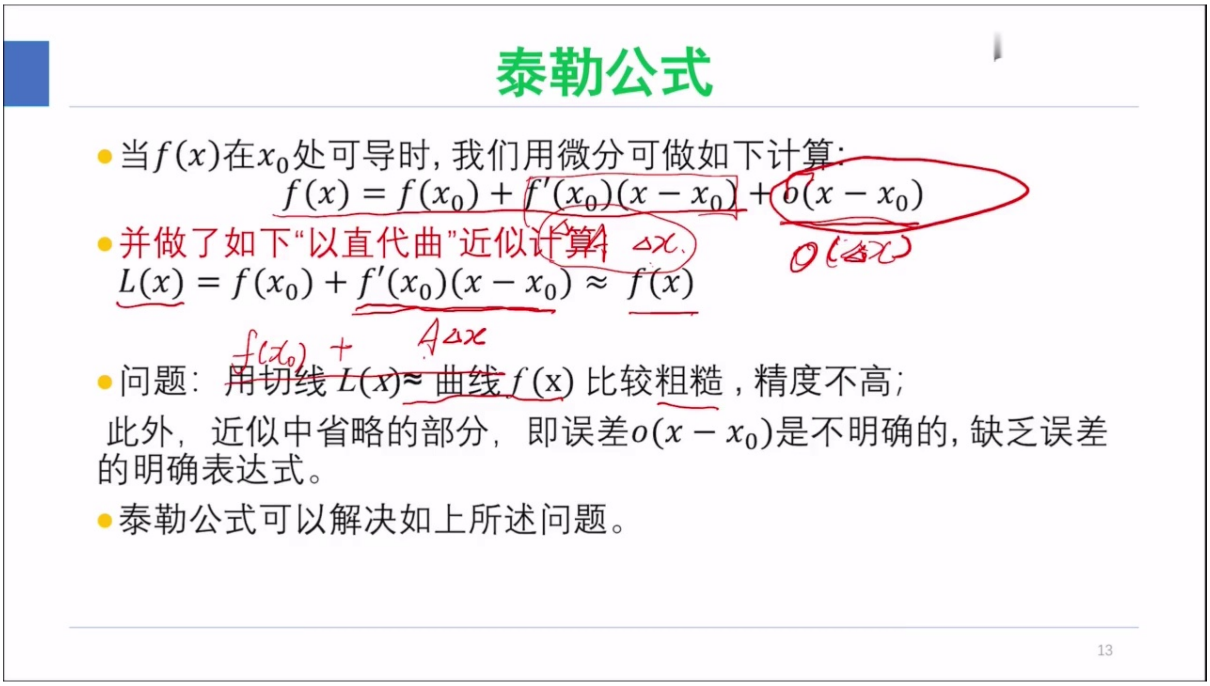

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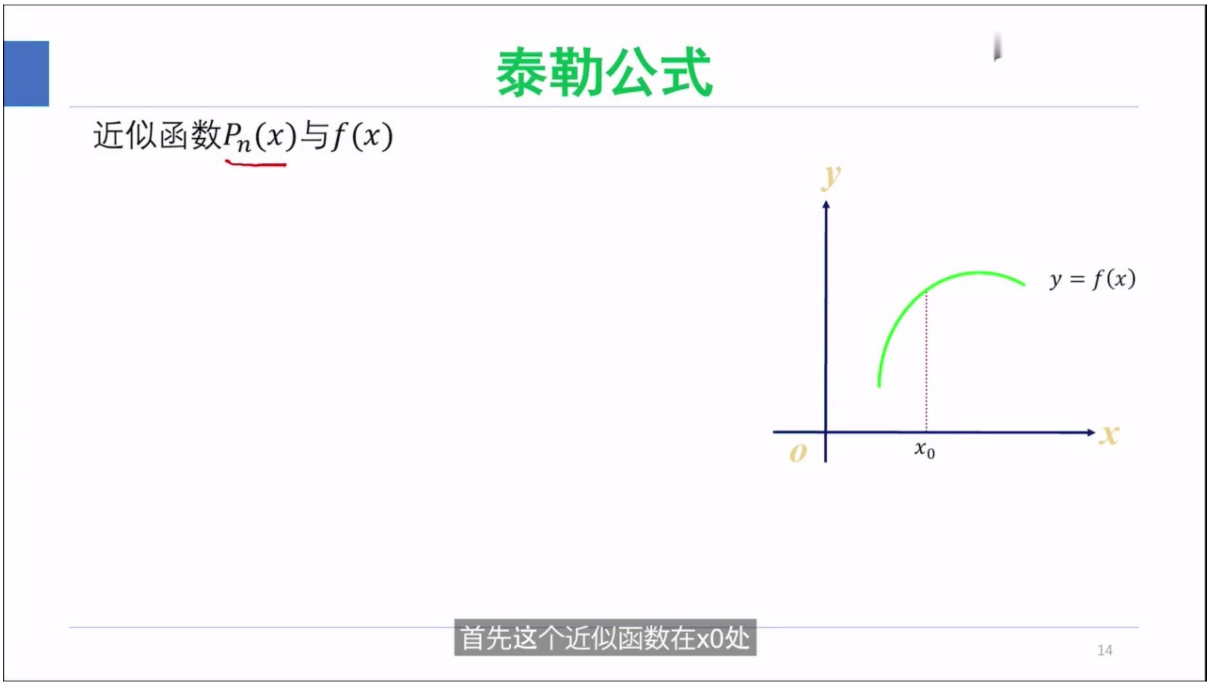

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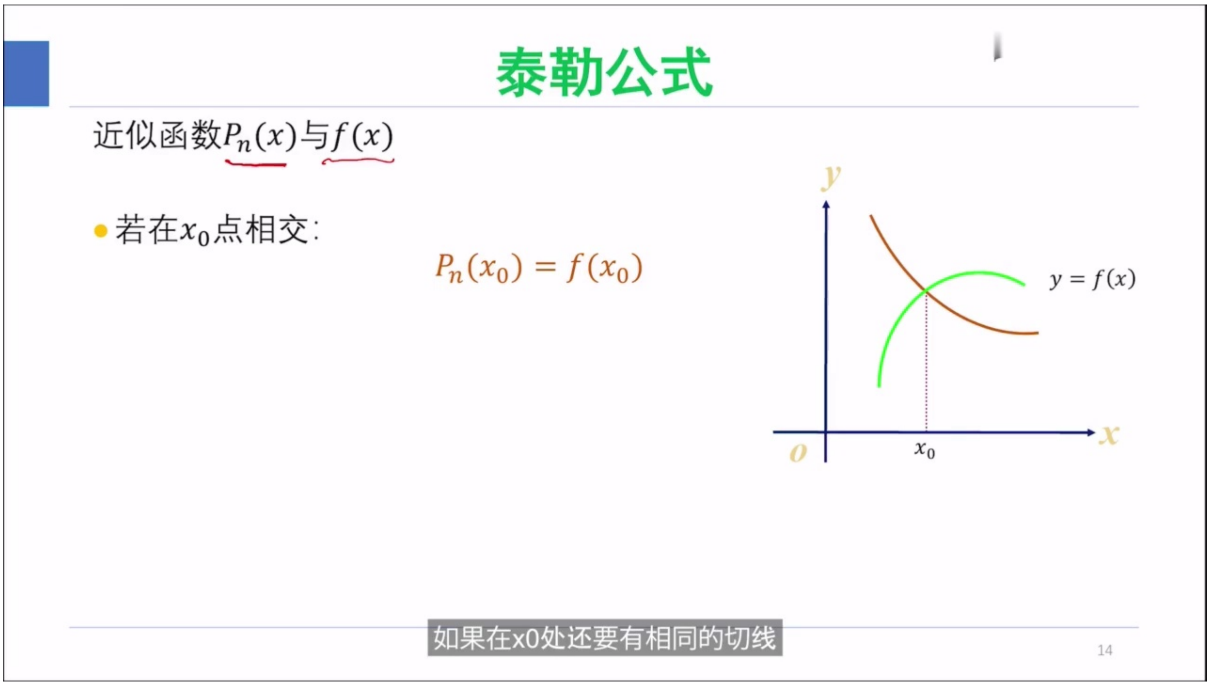

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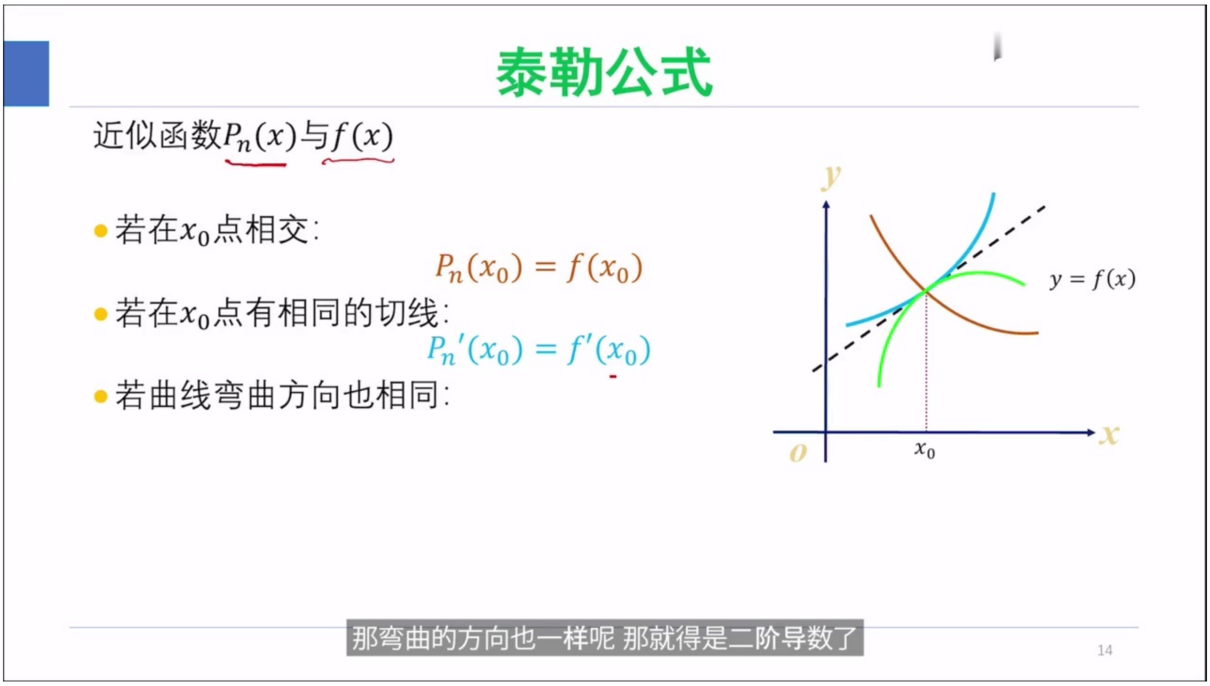

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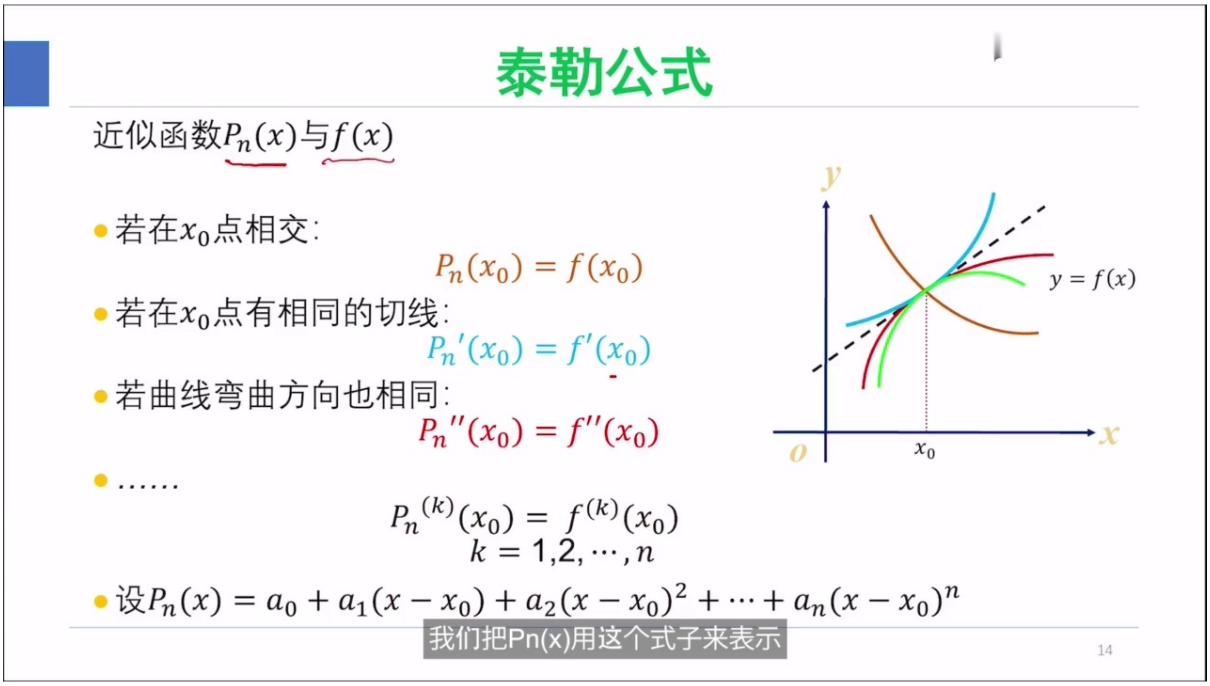

■ 66 ■ / ■ 81 ■
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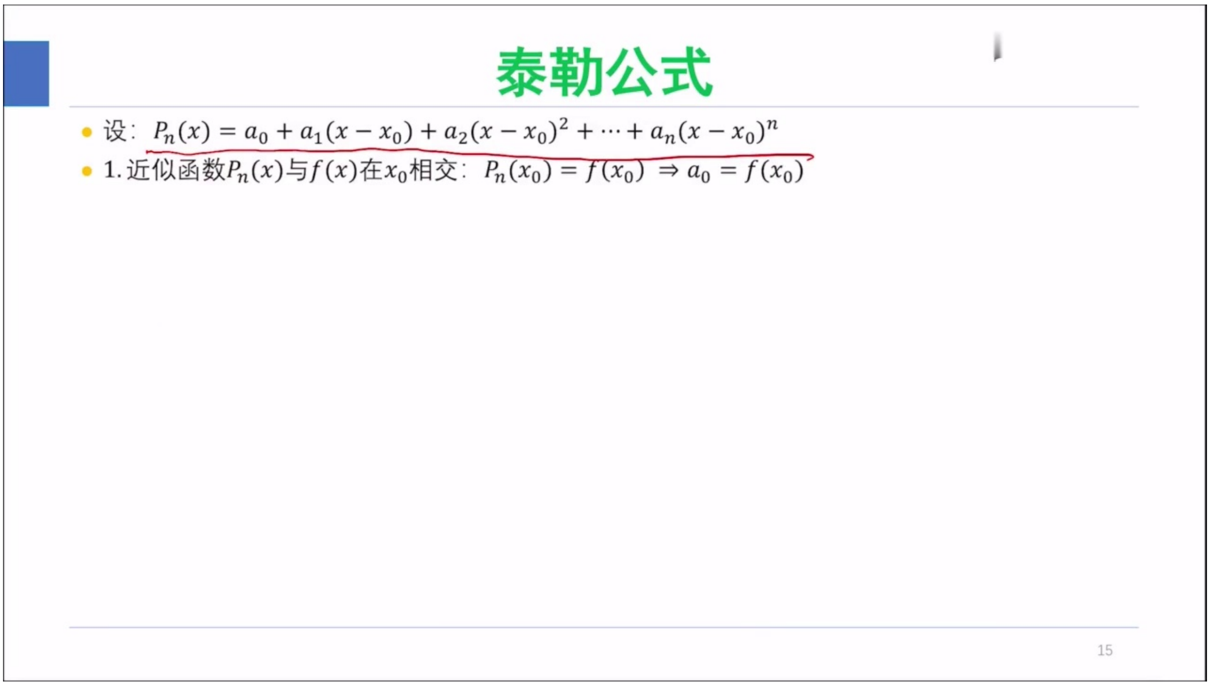

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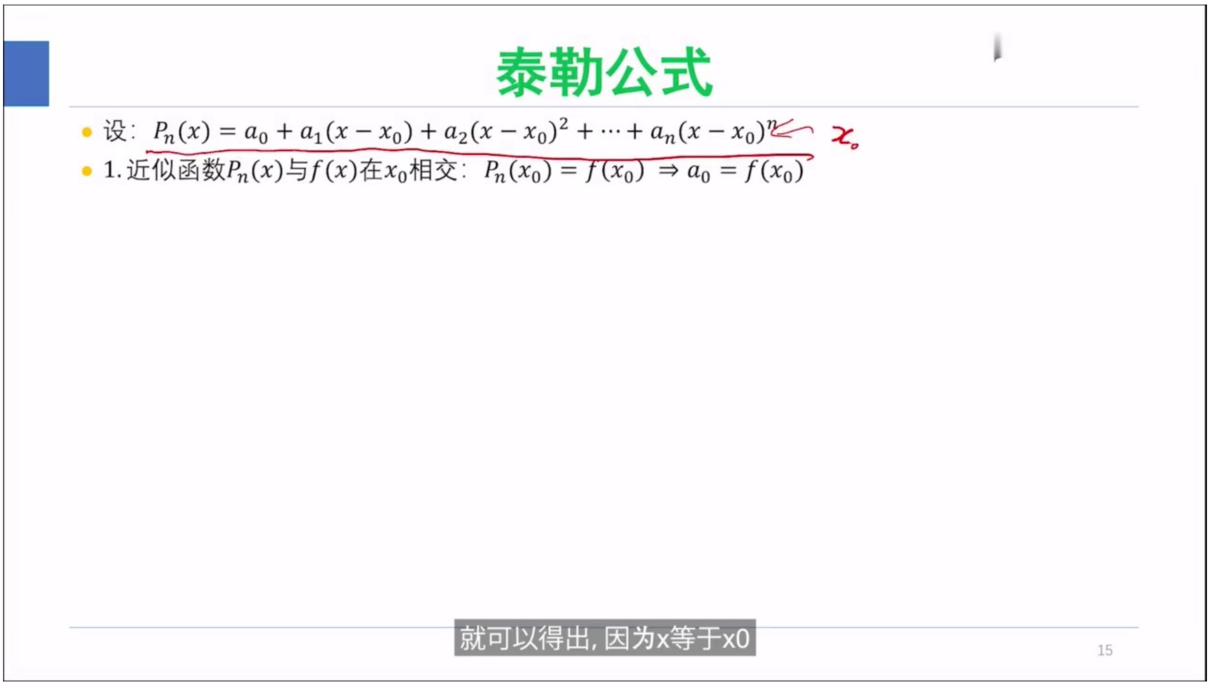

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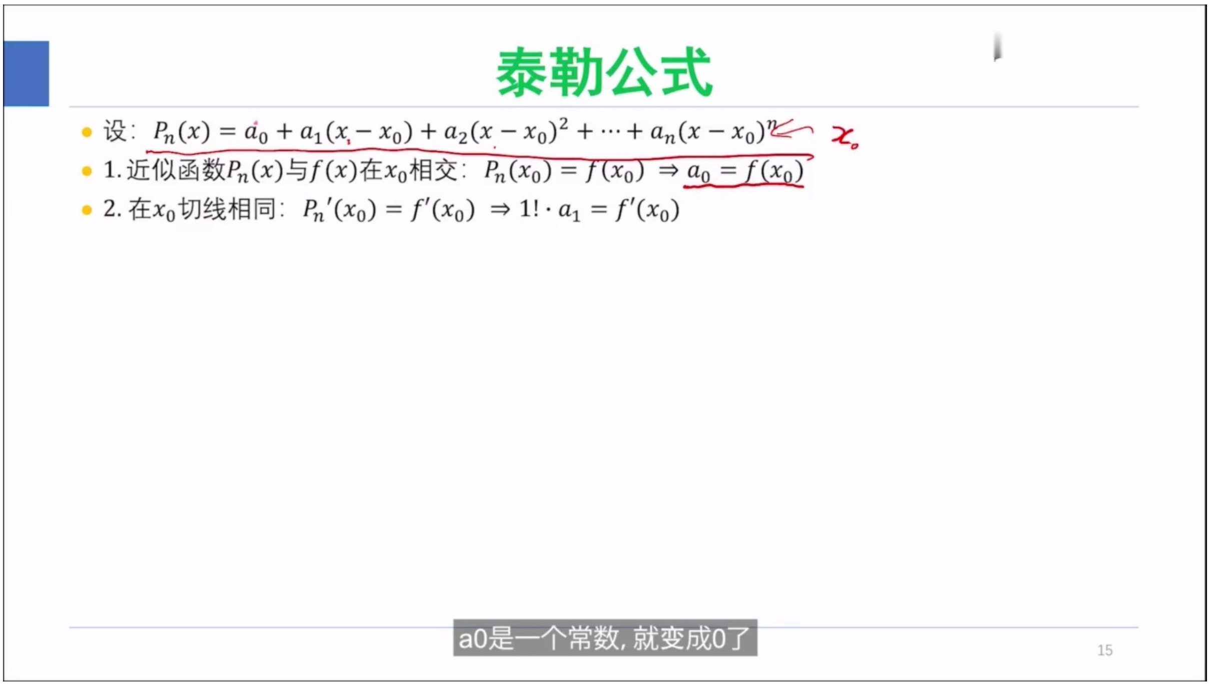

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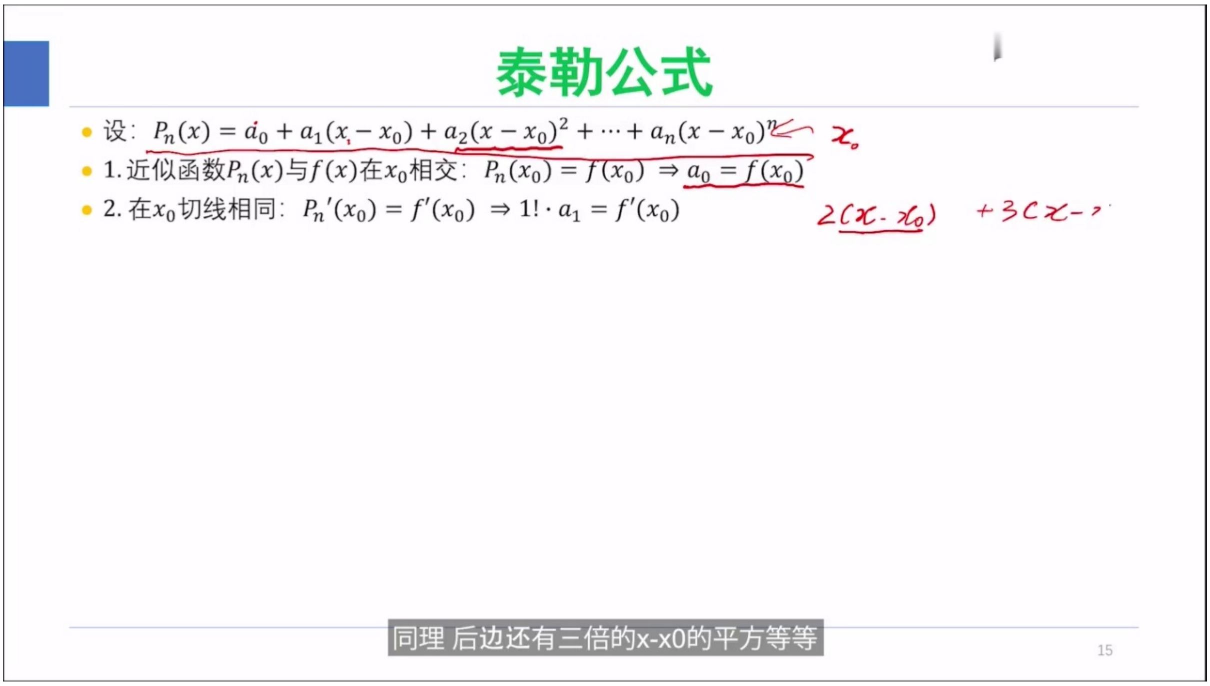

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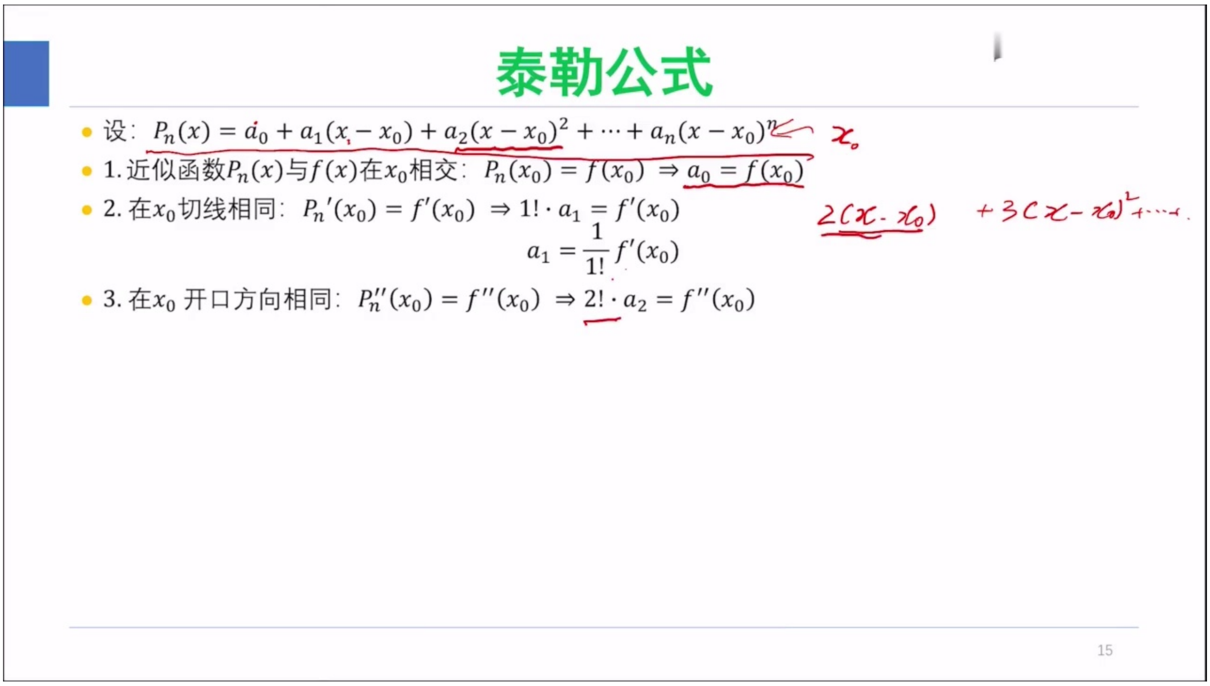

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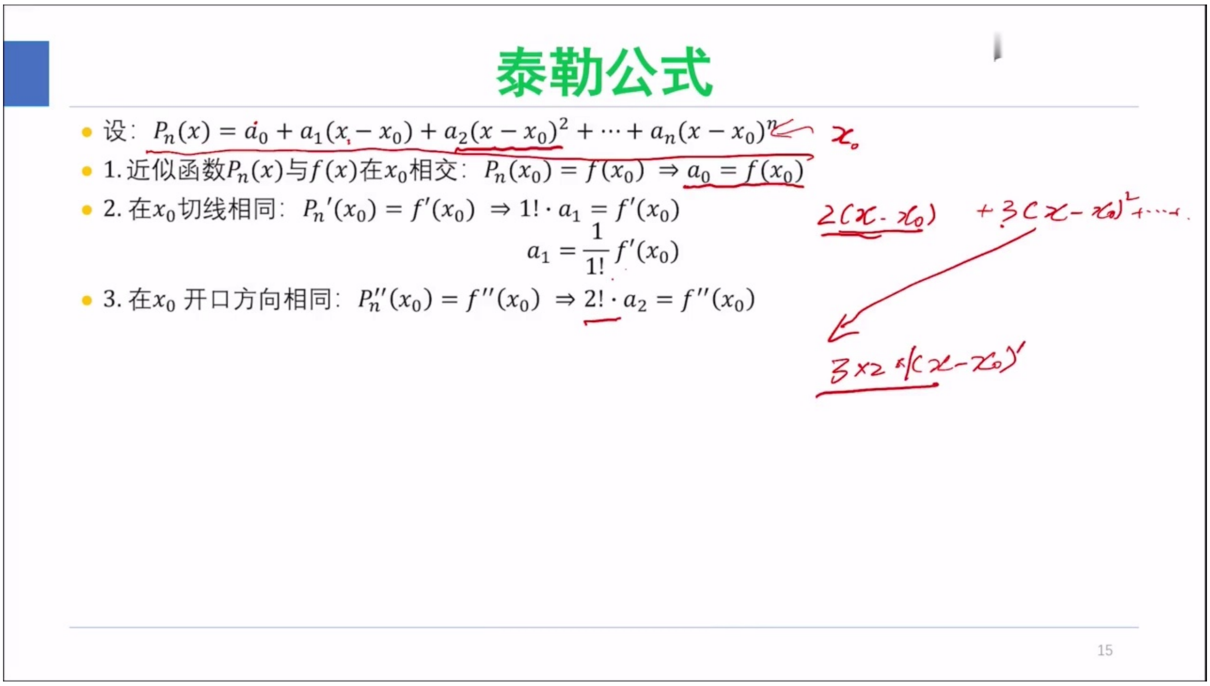

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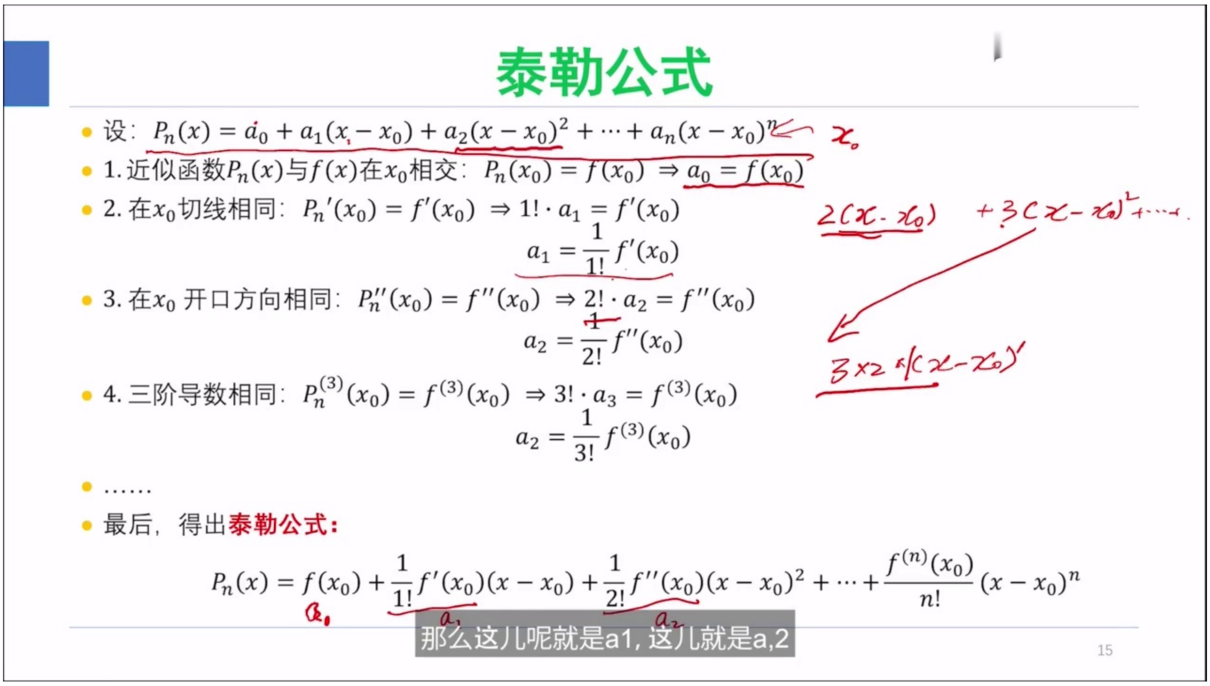

■ 73 ■ / ■ 81 ■
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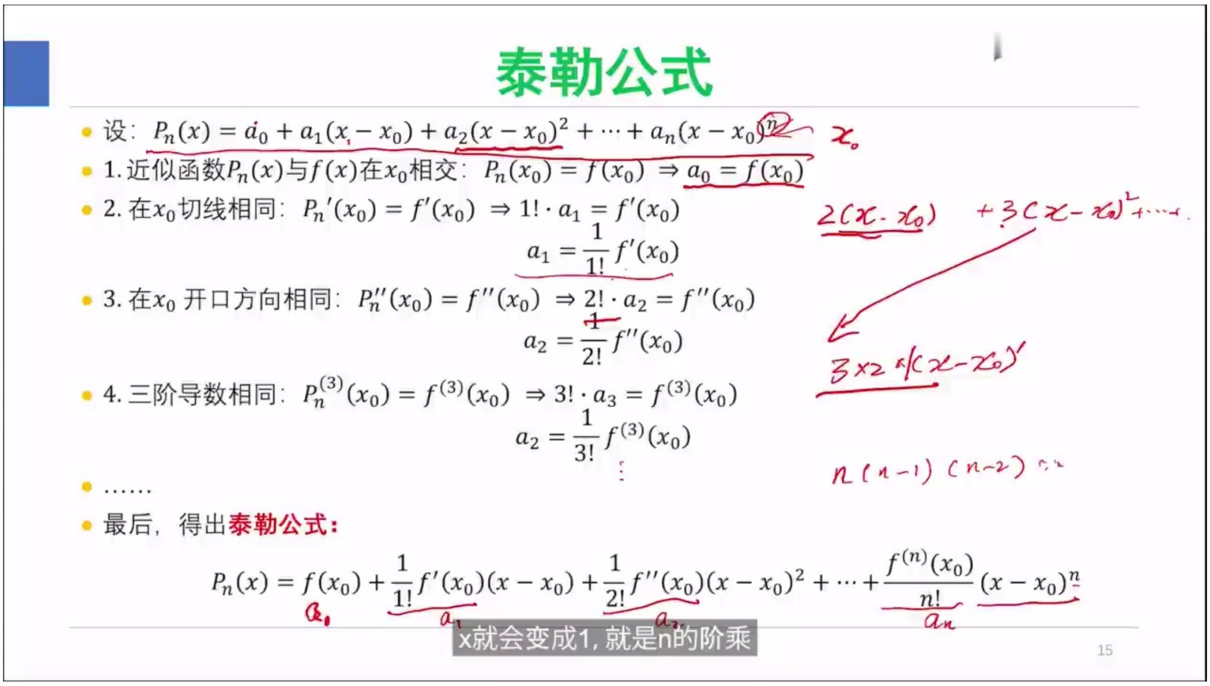

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■■: keyframe_00-25-18-999_0079.jpg

■■■: 00:25:19.999

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