Homework I: DATA130048

Biostatistics

Due Thursday, March 26th, 2020

1 Problem 1: MLE & MME

Suppose that X is a discrete random variable with the following probability mass function, where $0 \leq \theta \leq 1$ is a parameter. The following 10 independent observations were taken from such a distribution: $(3, 0, 2, 1, 3, 2, 1, 0, 2, 1)$

X	0	1	2	3
$P(X)$	$\frac{2\theta}{3}$	$\frac{\theta}{3}$	$\frac{2(1-\theta)}{3}$	$\frac{1-\theta}{3}$

Derive the MLE (maximum likelihood estimator) and MME (method of moments estimator) of θ.

2 Problem 2: MLE & MME

Suppose $X_1, X_2, ..., X_n$ are iid random variables with density function

$$f(x \mid \sigma) = \frac{1}{2\sigma} \exp(-\frac{|x|}{\sigma})$$

Find the MLE and MME of σ.

3 Problem 3: MLE & MME

Suppose that $X_1, ..., X_n$ form a random sample from a uniform distribution on the interval $(0, \theta)$, where parameter $\theta > 0$ is unknown. Find the MME and MLE of θ.

4 Problem 4: Central Limit Theorem & Delta Method

Suppose $X_n \sim Binomial(n, p)$, with $p \neq \frac{1}{2}$. Because $\frac{X_n}{n}$ is the maximum likelihood estimator for p, the maximum likelihood estimator for $p(1-p)$ is $\delta_n = \frac{X_n(n-X_n)}{n^2}$.
Use Central Limit Theorem to show the limiting distribution for $\frac{X_n}{n}$, and use Delta Method to derive the limiting distribution for δ_n.

5 Problem 5: Central Limit Theorem & Delta Method

Suppose $X_1, X_2, ...$ are iid with mean μ and finite variance σ^2, and $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. What is the limiting distribution of $\bar{X}_n^2$?

6 Problem 6: Confidence intervals

Suppose that a clinical trial was conducted to estimate the response rate p of an experimental drug. We observed 13 responses among 65 subjects treated by this drug. Based on the binomial distribution, the estimated response rate is $\hat{p} = 0.2$. Construct the 95% confidence interval for p using the asymptotic normal approximation.