



# Testing the Tolerance Principle on Corpus Data

## How well can the Tolerance Principle explain past tense overregularization?

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### INTRODUCTION

Rule learning can lead to overgeneralization errors (e.g. *He falled* [1]). But what leads to rule learning in the first place? To predict when a rule will be productive, Yang [2, 3] proposes the Tolerance Principle (TP). It captures the insight that too many exceptions make rule learning inefficient and impossible. It quantifies the theoretical threshold number of exceptions ( $\theta$ ) that a learner can tolerate, as in (1):

$$(1) \theta = N/\ln(N) \quad (2) \theta = N/H(N)$$

$N$  is the number of types or items in the corpus that the rule is defined over and  $\ln$  is the natural logarithm. If the exceptions  $e$  are no larger than  $\theta$ , rule acquisition can take place.

TP has successfully predicted morphosyntactic performance in artificial language learning [4] but has not accounted for Adam's and Eve's data on past tense acquisition [3], using only verb types in children's own utterances ( $U_c$ ) to represent  $N$ . Yang attributed that failure to sampling effects [3, p. 88]. Here we aim to preserve Yang's insight but develop a different version of TP which better represents the distribution of  $N$  and  $N$  itself. In particular, we replace the natural log of  $N$  with a value that provides the best fit to corpus data.

### METHODOLOGY

This study evaluates and revises Yang's method and proposes a new method to compensate for sampling effects on corpus data. The new method is tested on Adam's and Eve's data.

#### Summary of Adam's and Eve's data

	Adam	Eve
Age of first recording	2;3	1;6
Age of first overregularized verb	2;11	1;10
No. of files in between	18	10

#### 1. Better represent the distribution of $N$

In TP, estimated maximum irregularities  $\theta$  are estimated using  $N/\ln(N)$ , assuming that the distribution of  $N$  is Zipfian. But a Zipfian distribution is not guaranteed for small sample sizes. Therefore, we measure the actual distribution of  $N$  for each child and parent and use the empirically estimated log to calculate the denominator on the right hand side of (1), namely, (2).

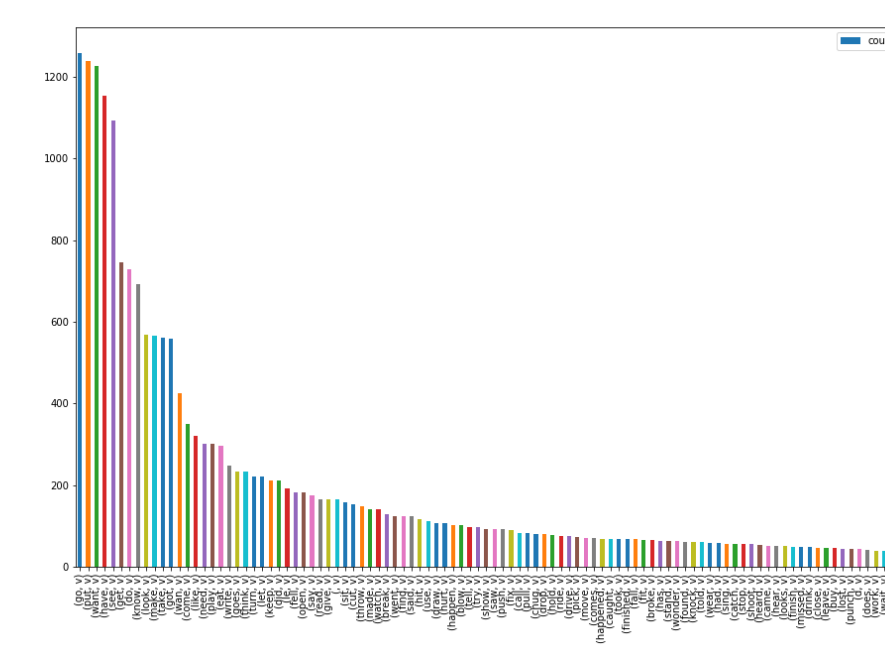


Fig.1. Adam's distribution  
(log=-0.64)

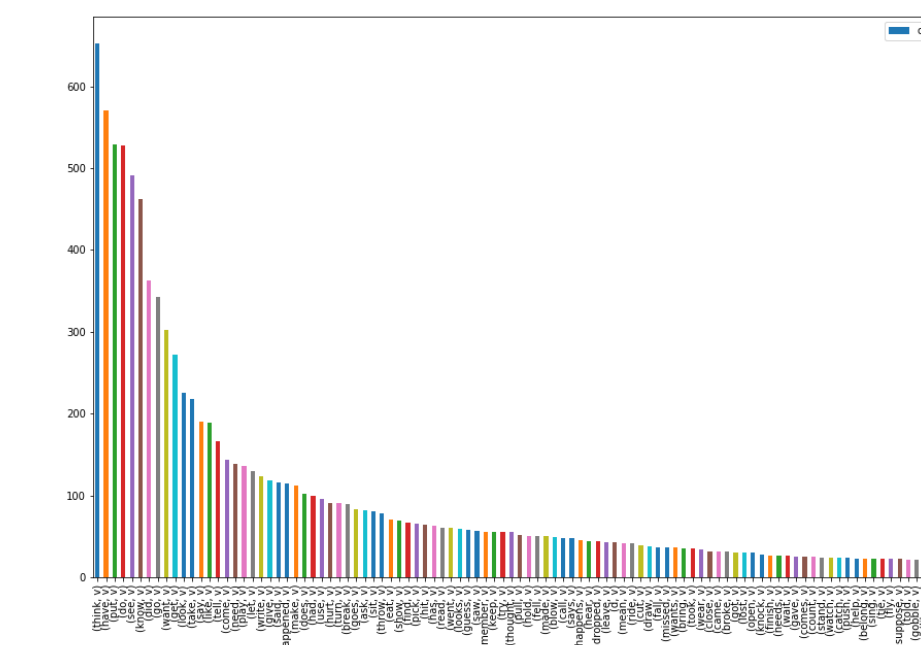


Fig. 2. Adam's mother's distribution  
(log = -0.72)

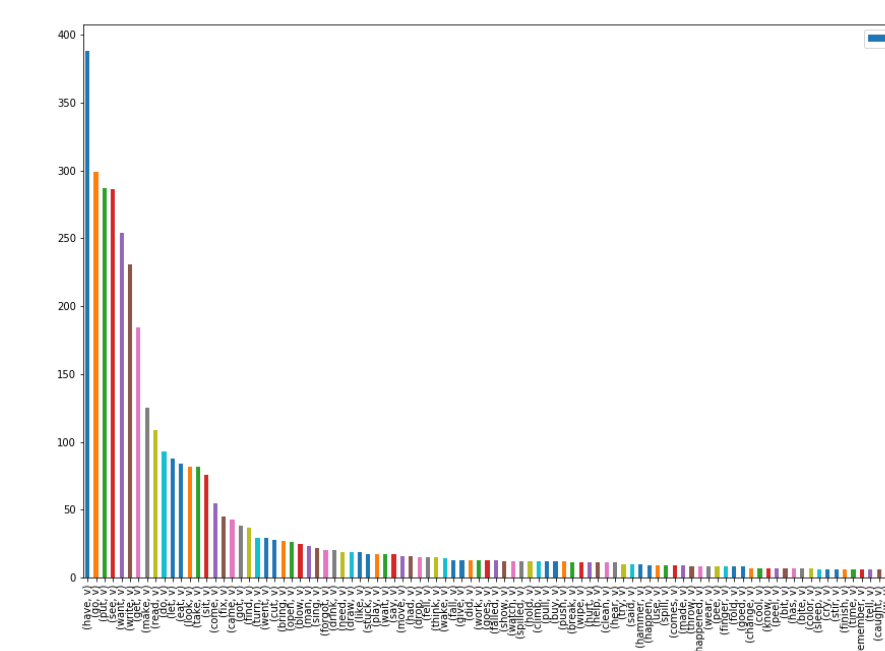


Fig. 3. Eve's distribution  
(log = -0.69)

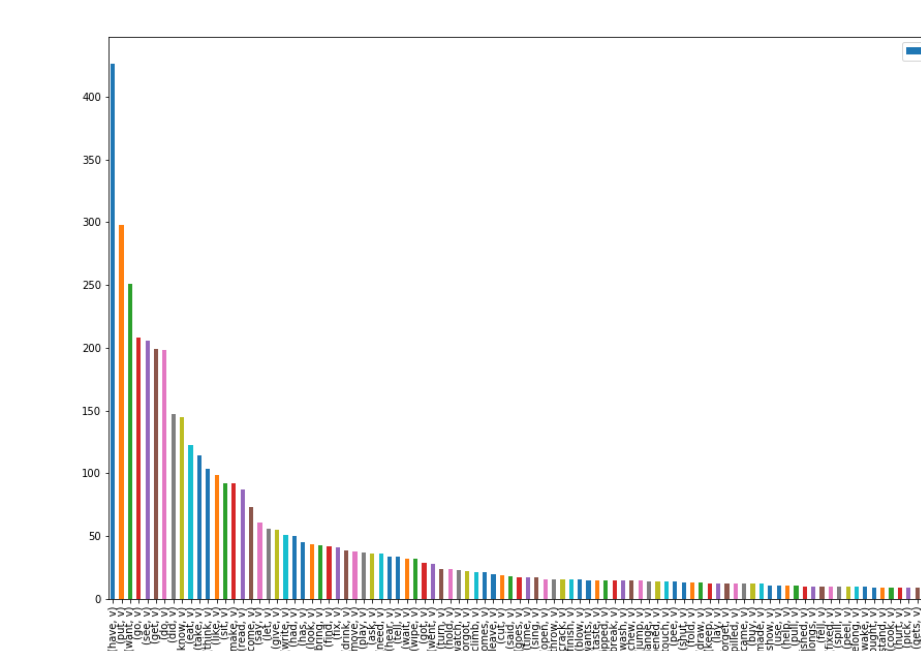


Fig. 4. Eve's mother's distribution  
(log = -0.68)

#### 2. Better represent $N$

Instead of using children's own utterances ( $U_c$ ) to represent children's effective vocabulary, we propose that  $N$  can be estimated through parents' input ( $U_p$ ) and children's production ( $U_c$ ), both of which can be extracted from corpora. Since children do not absorb parents' input completely, and since their productions do not represent their entire linguistic knowledge, we introduce  $\lambda$  to represent comprehension cost (%) and  $\delta$  to represent production loss (%). Since  $\lambda$  and  $\delta$  represent the comprehension cost and production loss, they should range between 0% - 100%. To estimate the minimum value of  $\lambda$ , we can use the proportions of words that are in the parents' utterances that are found in children's utterances. Similarly, the maximum value of  $\delta$  should be equal to the maximum of  $\lambda$  if we make the bold assumption that children understands everything the parents said. In addition, in order to compensate for loss of data due to undersampling, we introduce  $X_c$  and  $X_p$  for the missing data. The estimated  $N$  is shown in below:

$$(1) N = (U_p + X_p) \cdot \lambda = (U_c + X_c) / \delta$$

### CONCLUSION

According to TP, when  $\theta \geq e$ , acquisition can take place. Using this method, instead of simply comparing  $\theta$  and  $e$ , we evaluate the plausibility of  $\theta \geq e$ , namely the possible values for  $X_p$  and  $X_c$ . The number of verbs ( $U$ ) and irregular past tense verbs ( $e$ ) are generated automatically using the NLTK Python package.

First, we calculated the minimum value for  $N$  using the formula  $\theta = N/H(N)$ , by making  $\theta=e$ , using the estimate of the harmonic number for parents' and children's word distributions.

Estimating N using Parent's input			Estimating N using Children's output		
	Adam	Eve		Adam	Eve
e	29	24	e	16	16
log	-0.72	-0.68	log	-0.64	-0.69
Estimated N	~500	~450	Estimated N	~320	~240

Then we insert the minimal value for  $N$  to the formula in (2) and generate an estimated value for  $X_p$  and  $X_c$ . TP is confirmed if  $X_p < U_p$  and  $X_c < U_c$ .

Estimating X for parents' input			Estimating X for Children's output		
	Adam	Eve		Adam	Eve
U	354	229	U	294	147
N	~500	~450	N	~320	~240
Estimated X	146	221	Estimated X	26	93
X<U?	Yes	Yes	X<U?	Yes	Yes

The findings of this study show first that the distribution of verb production of is not Zipfian for either parents or children (shown in Figure 1-4) and, second, that the new method produces a plausible  $X$ . TP's predictions are confirmed for corpus data.

### REFERENCE

- Selected Reference:**  
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[2] Yang, C. (2005). On productivity. *Linguistic variation yearbook*, 5(1), 265-302.  
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[4] Schuler, K. D., Yang, C., & Newport, E. L. (2016). Testing the Tolerance Principle: Children form productive rules when it is more computationally efficient to do so. In *CogSci*.

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