HW2 Data Report for Math/CS 471

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09/16/2017

Abstract

This is the HW2 report. This report is made all through LaTeX.

1 Question 1

We run the script and see the output. The iteration is ten times.

2 Question 2

Please see the newtonS.f90.Template file. The do-while loop is used for approximating the absolute error.

3 Question 3

In newtonS.f90.Template file, I add linear and quadratic convergence right after the x output. So for every iteration there should be a total of 6 outputs.

4 Question 4

4.1 Convergence

4.2 Convergence discussion

f'(x) is ('1.d0', '2.d0*x', 'cos(x)-2.d0*x*sin(x*x)'). For the first one, when x comes closer to its root, there should not be very big change happening since f'(x) is already a constant. However, when it comes to the second one, it is difficult to determine convergence since x still exists in the equation. The third is better since they are $\cos()$ and $\sin()$, which is more possible to give a final rate of convergence I think when it comes to quadratic convergence. The data also proves this quadratic convergence when it reaches fifth and sixth iteration in case three.

4.3 Modified Newton's method discussion

I think I will implement Modified Newton's method on case two since the multiplicity m of the root is known in advance (I know what f'(x) is as soon as I have a x). $-m \times (f'(x)/f(x)) = x_{n+1} - x_n$. So I will just modify newtonS.f90.Template file.

5 Question 5

>> is actally more useful.

6 Question 6

One good method writing double slashes at the end of each output data row has not come up though.

7 Tabulated Data

Explanation: Table 1: the first character is the case ('x', 'x*x', 'sin(x)+cos(x*x)'). The second number is the iteration number. The third one is x value, the forth one is dx. Table 2: Explanation: the first character is the case ('x', 'x*x', 'sin(x)+cos(x*x)'). The second numbe is the iteration number. The third one is linear convergence and the forth one is quadratic convergence.

7.1 Table1without convergence

```
x 01 0.000000000000000E+00 0.5000000000000E+00
 x*x 01 -0.2500000000000000E+00 0.250000000000000E+00
x*x 02 -0.12500000000000000E+00 0.125000000000000E+00
x*x 03 -0.6250000000000000E-01 0.62500000000000E-01
x*x 04 -0.3125000000000000E-01 0.312500000000000E-01
x*x 05 -0.1562500000000000E-01 0.156250000000000E-01
x*x 06 -0.7812500000000000E-02 0.781250000000000E-02
       -0.3906250000000000E-02 0.390625000000000E-02
       -0.1953125000000000E-02 0.195312500000000E-02
x*x 09 -0.9765625000000000E-03 0.9765625000000000E-03
x*x 10 -0.4882812500000000E-03 0.4882812500000000E-03
     11 -0.2441406250000000E-03 0.2441406250000000E-03
x*x 12 -0.1220703125000000E-03 0.1220703125000000E-03
x*x 13 -0.6103515625000000E-04 0.6103515625000000E-04
        -0.3051757812500000E-04 0.3051757812500000E-04
        -0.1525878906250000E-04 0.1525878906250000E-04
    16 -0.7629394531250000E-05 0.7629394531250000E-05
    17 -0.3814697265625000E-05 0.3814697265625000E-05
     18 -0.1907348632812500E-05 0.1907348632812500E-05
x*x 19 -0.9536743164062500E-06 0.9536743164062500E-06
x*x 20 -0.4768371582031250E-06 0.4768371582031250E-06
x*x 21 -0.2384185791015625E-06 0.2384185791015625E-06
       -0.1192092895507812E-06 0.1192092895507812E-06
x*x 23 -0.5960464477539062E-07 0.5960464477539062E-07
x*x 24 -0.2980232238769531E-07 0.2980232238769531E-07
```

```
x*x 25 -0.1490116119384766E-07 0.1490116119384766E-07
       x*x 26 -0.7450580596923828E-08 0.7450580596923828E-08
       x*x 27 -0.3725290298461914E-08 0.3725290298461914E-08
       x*x 28 -0.1862645149230957E-08 0.1862645149230957E-08
           29 -0.9313225746154785E-09 0.9313225746154785E-09
           30 -0.4656612873077393E-09 0.4656612873077393E-09
           31 -0.2328306436538696E-09 0.2328306436538696E-09
           32 \ -0.1164153218269348 \\ E-09 \ 0.1164153218269348 \\ E-09
           33 -0.5820766091346741E-10 0.5820766091346741E-10
       x*x 34 -0.2910383045673370E-10 0.2910383045673370E-10
          35 -0.1455191522836685E-10 0.1455191522836685E-10
       x*x 36 -0.7275957614183426E-11 0.7275957614183426E-11
           37 -0.3637978807091713E-11 0.3637978807091713E-11
       x*x 38 -0.1818989403545856E-11 0.1818989403545856E-11
       x*x 39 -0.9094947017729282E-12 0.9094947017729282E-12
           40 -0.4547473508864641E-12 0.4547473508864641E-12
       x*x 41 -0.2273736754432321E-12 0.2273736754432321E-12
       x*x 42 -0.1136868377216160E-12 0.1136868377216160E-12
       x*x 43 -0.5684341886080801E-13 0.5684341886080801E-13
       x*x 44 -0.2842170943040401E-13 0.2842170943040401E-13
       x*x 45 -0.1421085471520200E-13 0.1421085471520200E-13
       x*x 46 -0.7105427357601002E-14 0.7105427357601002E-14
       x*x 47 -0.3552713678800501E-14 0.3552713678800501E-14
       x*x 48 -0.1776356839400250E-14 0.1776356839400250E-14
       x*x 49 -0.8881784197001252E-15 0.8881784197001252E-15
\sin(x) + \cos(x^*x) 01 -0.9351046647281536E+00 -0.4351046647281536E+00
\sin(x) + \cos(x^*x) 02 -0.8546415960180649E+00 0.8046306871008869E-01
\sin(x) + \cos(x^*x) 03 -0.8493901358009870E+00 0.5251460217077924E-02
\sin(x) + \cos(x^*x) 04 -0.8493688627401134E+00 0.2127306087358230E-04
\sin(x) + \cos(x^*x) 05 -0.8493688623926731E+00 0.3474402480610000E-09
\sin(x) + \cos(x^*x) 06 -0.8493688623926731E+00 -0.000000000000000E+00
```

7.2 Table2with convergence

```
x^*x 11 0.5000000000000000E+00 0.51200000000000E+03
    x*x 12 0.500000000000000E+00 0.10240000000000E+04
        14 \ 0.50000000000000000E+00 \ 0.409600000000000E+04
        15 0.500000000000000E+00 0.81920000000000E+04
        16 0.500000000000000E+00 0.16384000000000E+05
        0.50000000000000000E+00 0.6553600000000000E+05
          0.50000000000000000E+00 0.131072000000000E+06
           0.50000000000000000E+00 0.262144000000000E+06
        21
           0.50000000000000000E+00 0.524288000000000E+06
          0.50000000000000000E+00 0.1048576000000000E+07
        23 0.500000000000000E+00 0.209715200000000E+07
        24 0.500000000000000E+00 0.419430400000000E+07
          0.50000000000000000E+00 0.8388608000000000E+07
    x^*x
          0.50000000000000000E+00 0.1677721600000000E+08
          0.500000000000000000E+00 0.3355443200000000E+08
    x^*x
           0.50000000000000000E+00 0.6710886400000000E+08
           0.50000000000000000E+00 0.1342177280000000E+09
        0.50000000000000000E+00 0.5368709120000000E+09
        0.50000000000000000E+00 0.2147483648000000E+10
        0.50000000000000000E+00 0.8589934592000000E+10
           0.50000000000000000E+00 0.1717986918400000E+11
        36
           0.50000000000000000E+00 0.3435973836800000E+11
          0.500000000000000000E+00 0.6871947673600000E+11
          0.50000000000000000E+00 0.2748779069440000E+12
        41 \quad 0.50000000000000000E + 00 \quad 0.5497558138880000E + 12
       42 0.500000000000000E+00 0.1099511627776000E+13
          0.50000000000000000E+00 0.2199023255552000E+13
        44
           0.50000000000000000E+00 0.4398046511104000E+13
          46 0.500000000000000E+00 0.1759218604441600E+14
        47 0.500000000000000E+00 0.3518437208883200E+14
        48 0.500000000000000E+00 0.7036874417766400E+14
    x*x 49 0.500000000000000E+00 0.1407374883553280E+15
\sin(x) + \cos(x^*x) 01 0.10000000000000E+01 0.100000000000E+01
\sin(x) + \cos(x^*x) 02 0.4351046647281536E+00 0.4351046647281536E+00
\sin(x) + \cos(x^*x) 03 0.1849280764672121E+00 0.4250197514723320E+00
\sin(x) + \cos(x^*x) 04 0.6526547273506428E-01 0.8111233362254442E+00
\sin(x) + \cos(x^*x) 05 0.4050884895680481E-02 0.7713825732711236E+00
\sin(x) + \cos(x^*x) 06 0.1633240749027686E-04 0.7677507053309917E+00
```

 x^*x 10 0.5000000000000000E+00 0.25600000000000E+03