HW2 Data Report for Math/CS 471

Xiaomeng Li

09/16/2017

Abstract

This is the HW2 report. This report is made all through LaTeX.

1 Question 1

We run the script and see the output. The iteration is ten times.

2 Question 2

Please see the newtonS.f90.Template file. The do-while loop is used for approximating the absolute error.

3 Question 3

In newtonS.f90.Template file, I add linear and quadratic convergence right after the x output. So for every iteration there should be a total of 6 outputs.

4 Question 4

4.1 Convergence

Rate of convergence for $\sin(x)+\cos(x^*x)$ is 0.1633240749027686E-04 and 0.7677507053309917E+00. The results are read from the file when the iteration reaches maximum.

4.2 Convergence discussion

f'(x) is ('1.d0', '2.d0*x', 'cos(x)-2.d0*x*sin(x*x)'). For the first one, when x comes closer to its root, there should not be very big change happening since f'(x) is already a constant. However, when it comes to the second one, it is difficult to determine convergence since x still exists in the equation. The third is better since they are $\cos()$ and $\sin()$, which is more possible to give a final rate of convergence I think when it comes to quadratic convergence. The data also proves this quadratic convergence when it reaches fifth and sixth iteration in case three.

4.3 Modified Newton's method discussion

I think I will implement Modified Newton's method on case two since the multiplicity m of the root is known in advance (I know what f'(x) is as soon as I have a x). $-m \times (f'(x)/f(x)) = x_{n+1} - x_n$. So I will just modify newtonS.f90.Template file.

5 Question 5

>> is actally more useful.

6 Question 6

One good method writing double slashes at the end of each output data row has not come up though.

7 Tabulated Data

Explanation: the first character is the case ('x', 'x*x', ' $\sin(x) + \cos(x*x)$ '). The second number is the iteration number. The third one is x value, the

forth one is dx, the fifth one is linear convergence and the sixth one is quadratic convergence.

7.1 Convergence Results Table

```
x 01 0.000000000000000E+00 0.50000000000000E+00
    0.1000000000000000E+01 0.100000000000000E+01
0.50000000000000000E+00 0.5000000000000000E+00
x*x 01 -0.250000000000000E+00 0.25000000000000E+00
    0.1000000000000000E+01 0.100000000000000E+01
x*x 02 -0.1250000000000000E+00 0.12500000000000E+00
    0.25000000000000000E+00 0.2500000000000000E+00
x*x 03 -0.625000000000000E-01 0.62500000000000E-01
    0.5000000000000000E+00 0.200000000000000E+01
x*x 04 -0.312500000000000E-01 0.31250000000000E-01
    0.5000000000000000E + 00 \quad 0.400000000000000E + 01
x*x 05 -0.1562500000000000E-01 0.156250000000000E-01
    0.50000000000000000E+00 0.800000000000000E+01
x*x 06 -0.781250000000000E-02 0.78125000000000E-02
    0.5000000000000000E+00 0.160000000000000E+02
x*x 07 -0.3906250000000000E-02 0.390625000000000E-02
    0.5000000000000000E+00 0.320000000000000E+02
x*x 08 -0.1953125000000000E-02 0.195312500000000E-02
    0.5000000000000000E+00 0.640000000000000E+02
x*x 09 -0.9765625000000000E-03 0.976562500000000E-03
    0.5000000000000000E+00 0.128000000000000E+03
x*x 10 -0.4882812500000000E-03 0.4882812500000000E-03
    0.5000000000000000E+00 0.256000000000000E+03
x*x 11 -0.2441406250000000E-03 0.2441406250000000E-03
    0.5000000000000000E+00 0.512000000000000E+03
x*x 12 -0.1220703125000000E-03 0.1220703125000000E-03
    0.5000000000000000E+00 0.10240000000000E+04
x*x 13 -0.6103515625000000E-04 0.6103515625000000E-04
    0.50000000000000000E+00 0.204800000000000E+04
x*x 14 -0.3051757812500000E-04 0.3051757812500000E-04
    0.5000000000000000E+00 0.409600000000000E+04
x*x 15 -0.1525878906250000E-04 0.1525878906250000E-04
```

- $0.5000000000000000E + 00 \quad 0.819200000000000E + 04$
- x*x 16 -0.7629394531250000E-05 0.7629394531250000E-05 0.50000000000000E+00 0.163840000000000E+05
- x*x 17 -0.3814697265625000E-05 0.3814697265625000E-05 0.50000000000000E+00 0.327680000000000E+05
- x*x 19 -0.9536743164062500E-06 0.9536743164062500E-06 0.50000000000000E+00 0.131072000000000E+06

- x*x 22 -0.1192092895507812E-06 0.1192092895507812E-06 0.500000000000000E+00 0.1048576000000000E+07
- x*x 23 -0.5960464477539062E-07 0.5960464477539062E-07 0.500000000000000E+00 0.2097152000000000E+07

- x*x 34 -0.2910383045673370E-10 0.2910383045673370E-10

```
0.50000000000000000E+00 0.4294967296000000E+10
      x*x 35 -0.1455191522836685E-10 0.1455191522836685E-10
          0.50000000000000000E+00 0.8589934592000000E+10
      x*x 36 -0.7275957614183426E-11 0.7275957614183426E-11
          0.5000000000000000E+00 0.1717986918400000E+11
      x*x 37 -0.3637978807091713E-11 0.3637978807091713E-11
          0.50000000000000000E+00 0.3435973836800000E+11
      x*x 38 -0.1818989403545856E-11 0.1818989403545856E-11
          0.50000000000000000E+00 0.6871947673600000E+11
      x*x 39 -0.9094947017729282E-12 0.9094947017729282E-12
          0.5000000000000000E+00 0.1374389534720000E+12
      x*x 40 -0.4547473508864641E-12 0.4547473508864641E-12
          0.50000000000000000E+00 0.2748779069440000E+12
      x*x 41 -0.2273736754432321E-12 0.2273736754432321E-12
          0.50000000000000000E+00 0.5497558138880000E+12
      x*x 42 -0.1136868377216160E-12 0.1136868377216160E-12
          0.50000000000000000E+00 0.1099511627776000E+13
      x*x 43 -0.5684341886080801E-13 0.5684341886080801E-13
          0.5000000000000000E+00 0.2199023255552000E+13
      x*x 44 -0.2842170943040401E-13 0.2842170943040401E-13
          0.50000000000000000E+00 0.4398046511104000E+13
      x*x 45 -0.1421085471520200E-13 0.1421085471520200E-13
          0.5000000000000000E+00 0.8796093022208000E+13
      x*x 46 -0.7105427357601002E-14 0.7105427357601002E-14
          0.50000000000000000E+00 0.1759218604441600E+14
      x*x 47 -0.3552713678800501E-14 0.3552713678800501E-14
          0.50000000000000000E+00 0.3518437208883200E+14
      x*x 48 -0.1776356839400250E-14 0.1776356839400250E-14
          0.50000000000000000E+00 0.7036874417766400E+14
      x*x 49 -0.8881784197001252E-15 0.8881784197001252E-15
          0.50000000000000000E+00 0.1407374883553280E+15
\sin(x) + \cos(x^*x) 01 -0.9351046647281536E+00 -0.4351046647281536E+00
          0.10000000000000000E+01 0.100000000000000E+01
\sin(x) + \cos(x^*x) 02 -0.8546415960180649E+00 0.8046306871008869E-01
          0.4351046647281536E+00 0.4351046647281536E+00
\sin(x) + \cos(x^*x) 03 -0.8493901358009870E+00 0.5251460217077924E-02
          0.1849280764672121E+00 0.4250197514723320E+00
\sin(x) + \cos(x^*x) 04 -0.8493688627401134E+00 0.2127306087358230E-04
```

 $\begin{array}{c} 0.6526547273506428\text{E-O1} & 0.8111233362254442\text{E+O0} \\ \sin(\text{x}) + \cos(\text{x*x}) & 05 & -0.8493688623926731\text{E+O0} & 0.3474402480610000\text{E-O9} \\ 0.4050884895680481\text{E-O2} & 0.7713825732711236\text{E+O0} \\ \sin(\text{x}) + \cos(\text{x*x}) & 06 & -0.8493688623926731\text{E+O0} & -0.000000000000000\text{E+O0} \\ \underline{0.1633240749027686\text{E-O4}} & 0.7677507053309917\text{E+O0} \end{array}$