HW3 Data Report for Math/CS 471

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Abstract

This is the HW3 report. This report is made all through LaTeX. This report's data was produced by Fortran 90 and the plots were produced by matlab. Finally, the author used Perl to run Latex, write discussion, and incorporate the four plots.

1 Problem Description

In this assignment, an integral is given with two different efficients (k=pi and k=pi*pi). The author will use two different computing methods to approximate the value of intergral I. The language for computing is Fortran. The knowledge will cover Trapezoidal rule as well as Gauss Quadrature. Others like Makefile and MATLAB plotting will also be used. The author wants to find out when the error: abs(theoretical value - approximate value) will be smaller than 10^{-10} and how many iterarions (n) will be used for this convergence. It is worth noting that for Trapezoidal rule both cases coverge at last and they all stop before my iteration upper limit (n=100000 times). However, they didn't converge as small as 10^{-10} in Gauss Quadrature. The best error is about 10^{-8}. The theoretical values for both k = pi and k = pi* pi are from Trapezoidal rule when the author set the equidistant grids be 10^8, which should be able to give a good approximation close to theoretical value. All of them all start with n = 2 and XL = -1 and XR = 1.

2 Method 1: Trapezoidal rule

2.1 introduction

Trapezoidal rule approximates the integrals using equiditant grids. The order for this method is 2 in Trapezoidal rule. The author uses Fortran to write a program for composite trapezoidal and try to find the value which is more closer to the theoretical value of this integral. The N I used is 10^8 which should be accurate enough for giving a good theoretical value. Then the author will use different numbers of equidistant grids (n) to find different approximate integral values and plot their absolute value of error: abs(theoretical value - approximate value) corresponding to n's values. Both k=pi and k=pi*pi will be in the same picture.

2.2 Figures1

2.2.1 Question 1

Question: Plot the error against n using a logarithmic scale for both axes.
Ans: Fig1 is the figure for the Trapezoidal rule plot. I plot all the n whoses error is bigger than 10^{-10}. My n starts from 2 and XL = -1 and XR = 1. Both x axis plot and y axis plot are logarithmic scale.

2.2.2 Question 2

Question: Read off the order of the method from the slopes and theory discussion.
Ans: Yes, actually based on the plots and the shape of the curve, I find the slope with k=pi is much larger than that with k=pi*pi. Trapezoidal rule with k =

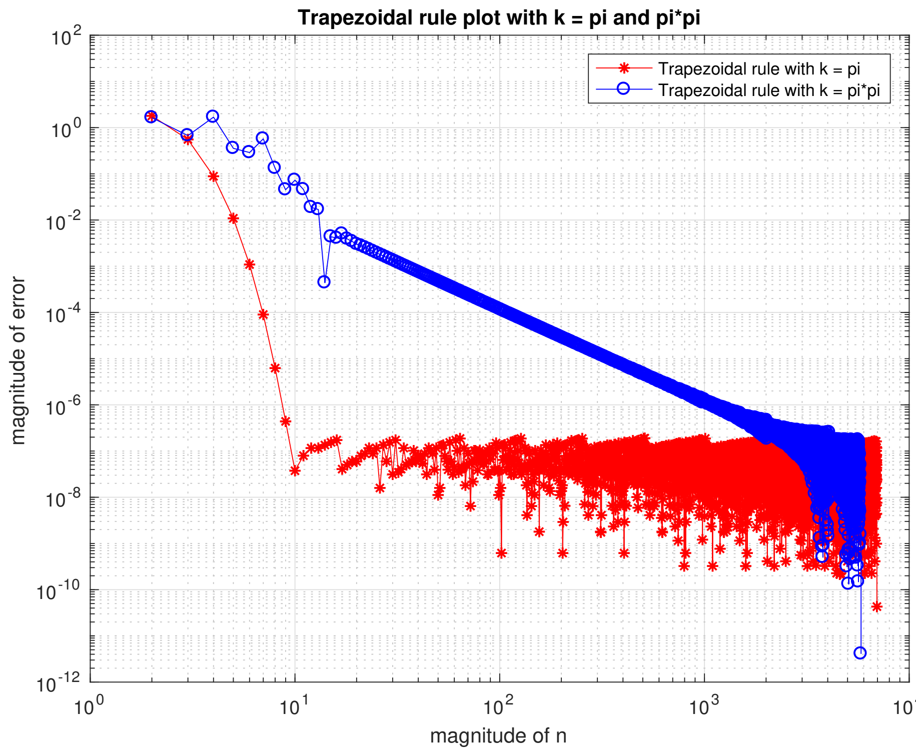

Figure 1: Trapezoidal rule plot with k = pi and pi*pi.

pi converges faster. From the perspective of theory, remember when Professor derived the Trapezoidal rule, there is a Trap Error which includes a term: $f(x)^{"}$. (just in case, it is f(x) double prime) This term is required to be bonded with XL and XR. When the coefficient k is much lager (pi*pi ¿ pi) the results got from $f(x)^{"}$ would be much larger because when you differentiate it twice, the coefficients go outside. At this moment the curve will become flat, therefore, it comes to theoretical value slower (Big Trap Error slowers the curve and makes it flat).

2.2.3 Question 3

Question: Why k = pi converges faster and what is special with k=pi?
Ans: The Euler–Maclaurin formula according to WiKi is used for detailed error analysis in numerical quadrature, which explains the superior performance of the trapezoidal rule on smooth periodic functions. In the equation $S - I = \sum_{k=1}^{p} Bk/k! * (f(XR)^{k-1} - f(XL)^{k-1}) + R$. S is the sum and I is the Integral value. Bk is the kth Bernoulli number and R is an error term. Smaller cofficient

k=pi leads to smaller (S-I) value when the differentiation happens, which makes the approimation converge faster.

2.3 Trapezoidal rule results discussion

Using Trapezoidal rule the author got the Fig.1 data plot. Luckily, both of them finally convege. Besides they all make the error smaller than 10^{-10} before n comes to 10^5. The case where coefficient k = pi converges faster than that where k = pi*pi.

3 Method 2: Gauss Quadrature

3.1 introduction

In this part, Gauss Quadrature method is used to approximate the results of this integral. The author uses Fortran to call a subroutine and tries to find the value which is more close to the theoretical value of this integral. The N I used is 10^8 which should be accurate enough for giving a good theoretical value. The theoretical value is from Trapezoidal rule with n = 10^8. Then the author will use different numbers of n (number of Gauss points) to find different approximate integral values and plot their absolute value of error: abs(theoretical value - approximate value) corresponding to n's values. Both k=pi and k=pi*pi will be in the same picture.

3.2 Figures2

3.2.1 Question 1

Question: Plot the error against n using a logarithmic scale for both axes.
Ans: Fig2 is the figure for the Gauss Quadrature plot. I plot all the first 5000 n's error. Almost all of them have the error larger than 10^{-8}. My n starts from 2 and XL = -1 and XR = 1. Both x axis plot and y axis plot are logarithmic scale.

3.2.2 Question 2

Question: Try some different C and a to get a good C^{-an} plot fitting the curves.
Ans: For the case k = pi, the author chooses C = 1000 and a = 0.1.
For the case k = pi*pi, the author chooses C = 70 and a = 0.09.

3.3 Gauss Quadrature results discussion

Using Gauss Quadrature, the author has the result data plot in Fig 2. It is clear that with coefficient k = pi, the data converges faster. The author uses five thousand iterations for both k = pi and k =pi*pi. It looks that neither of them could make the error smaller than 10^{-10}.

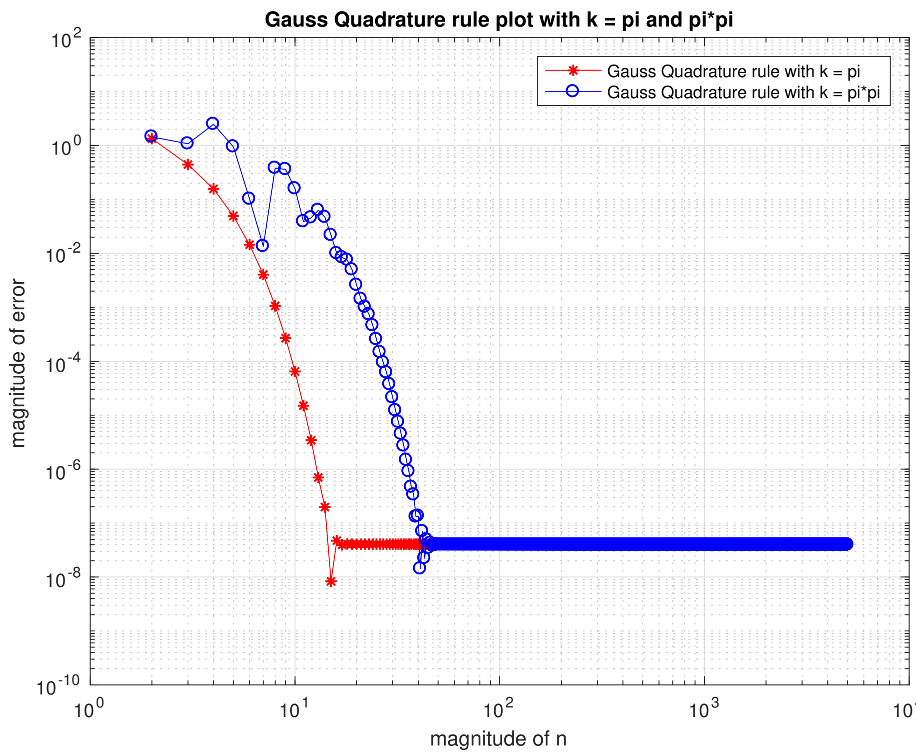

Figure 2: Gauss Quadrature rule plot with k = pi and pi*pi.

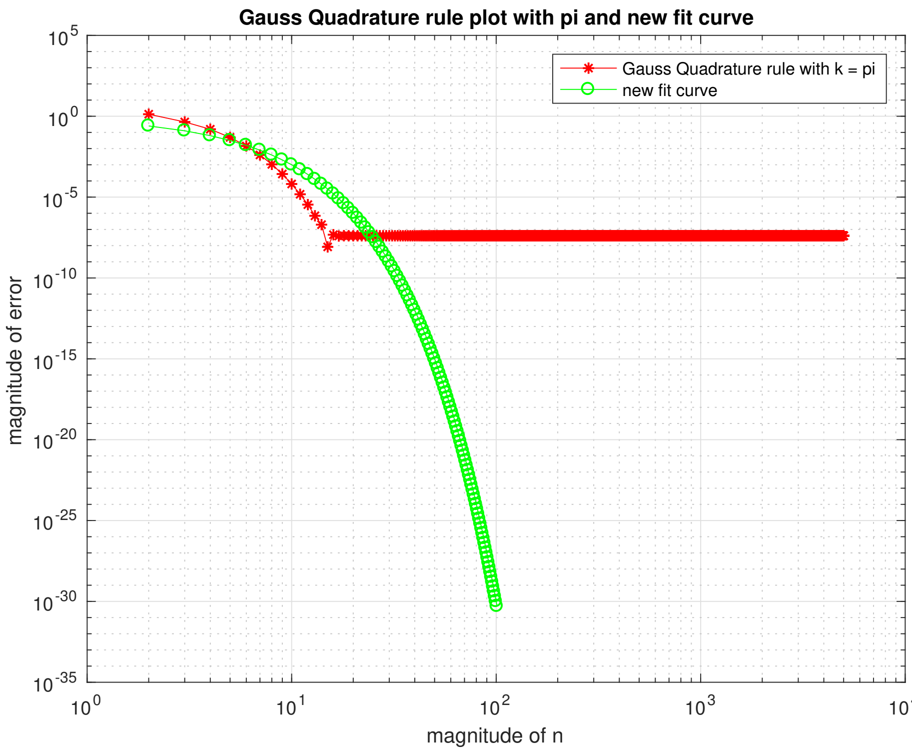

Figure 3: Gauss Quadrature rule plot with k = pi and new curve (C = 1000 and a = 0.1).

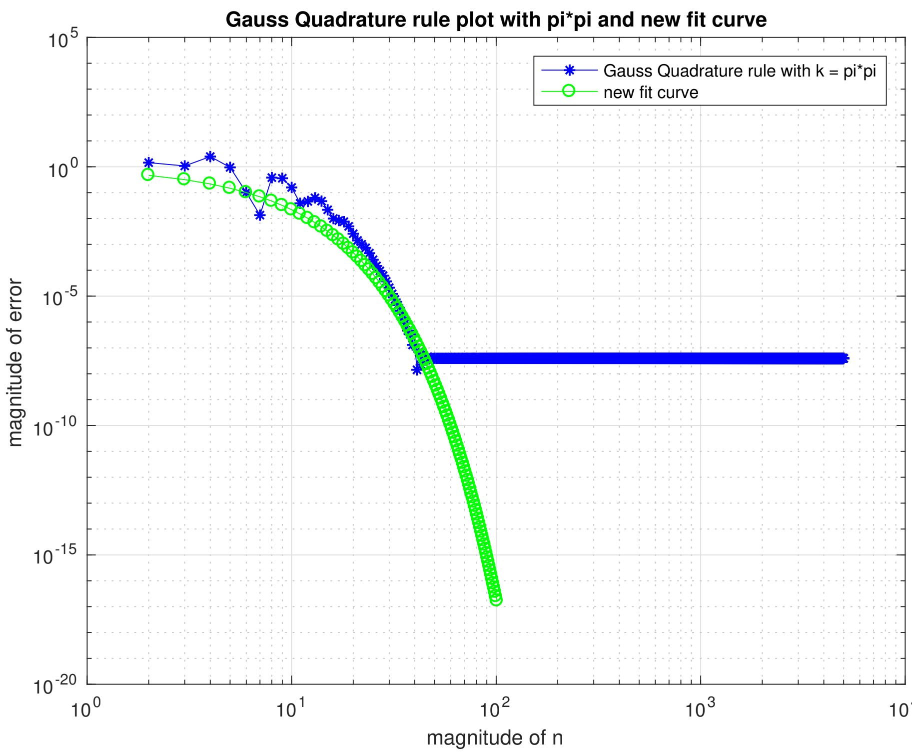

Figure 4: Gauss Quadrature rule plot with k = pi*pi and new curve (C = 70 and a = 0.09).