Geometric Part*

Xiao Ming Xiu[1]

October 19, 2017

***Abstract*— Given 3 2-D points' coordinates in two different Reference Systems, Barcode System (BS) and Camera System (CS), output two things: $\vec{t}$ and θ, defined as the CS origin's coordinates in BS and the rotated angle of CS relative to BS.**

I. Problem Setting

A. Given

Notation:

$$BS---S$$

$$CS---S'$$

Three Points' coordinates in both systems are known:

$$P1:(x_1,y_1),(x'_1,y'_1)$$

$$P2:(x_2,y_2),(x'_2,y'_2)$$

$$P3:(x_3,y_3),(x'_3,y'_3)$$

B. Want

$$\vec{t}=(t_x,t_y)$$

$$\theta$$

where,
$\vec{t}$ is defined by S' origin in S.
θ is defined by

$$\begin{aligned}(\varepsilon'_x,\varepsilon'_y)&=(\varepsilon_x,\varepsilon_y)*R\\&=(\varepsilon_x,\varepsilon_y)*\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & cos\theta\end{bmatrix}\end{aligned} \tag{1}$$

($\varepsilon_x,\varepsilon_y$ denotes the basis vector in S. $\varepsilon'_x,\varepsilon'_y$ denotes the basis vector in S'.)

C. Relationship Equation

Denote $\vec{P}$ (column vector)as vector coordinate representation in S and $\vec{P}'$ in S'. We have

$$\vec{P}=\vec{t}+R*\vec{P}' \tag{2}$$

Write in explicit form:

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}t_x\\t_y\end{bmatrix}+\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & cos\theta\end{bmatrix}*\begin{bmatrix}x'\\y'\end{bmatrix} \tag{3}$$

Eq. (3) is necessary and sufficient.
You have 3 unknown variables, t_x,t_y,θ. Given 1 pair coordinates $\vec{P}$ and $\vec{P}'$, you will get 2 independent equations. At least, 2 points are needed to solve all the 3 variables, though 1 equation is over determined.

II. Question: what is minimal set of input sufficient to solve?

Theorem 1. *Given 6 Rits of $x_1,y_1,x'_1,y'_1,x_2,x'_2$ is sufficient.*

3 independent equations have already been prepared up from Eq.(3). Only left with a Two-fold ambiguity.

Remark 1. The set of 1.5 points, not 2, is the minimum input.

Remark 2. Do there exist the following case: the input given format cannot exactly fit the sufficient and necessary solution of Problem, at least have some input as overdetermined condition and called unavoidable waste input?

III. Solution

Eliminate θ first from Eq. (3) by Left Multiply (based on $R^T*R=I$) : (Eq. (3) represents 2 equations.)

$$(x-t_x)^2+(y-t_y)^2={x'}^2+{y'}^2 \tag{4}$$

Rely on Point P1 and P2. Substitute Eq.(4) as:

$$(x_1-t_x)^2+(y_1-t_y)^2={x'_1}^2+{y'_1}^2 \tag{5}$$

$$(x_2-t_x)^2+(y_2-t_y)^2={x'_2}^2+{y'_2}^2 \tag{6}$$

Use Eq.(5)- Eq.(6), we get

$$t_y=kt_x+b \tag{7}$$

where

$$k=-\frac{x_1-x_2}{y_1-y_2} \tag{8}$$

$$\begin{aligned}b=&\frac{1}{2}*\{y_1+y_2\\&-\frac{1}{y_1-y_2}[{x'_1}^2+{y'_1}^2-{x'_2}^2-{y'_2}^2-x_1^2+x_2^2]\}\end{aligned} \tag{9}$$

Substitute t_y in Eq. (5)

$$At_x^2+Bt_x+C=0 \tag{10}$$

Where,

$$\begin{aligned}A&=1+k^2\\B&=-2[x_1+k(y_1-b)]\\C&=x_1^2+(y_1-b)^2-x_1'^2-y_1'^2\end{aligned} \tag{11}$$

*This work was supported by Standard-Robots Ltd.,co.

[1]Xiao Ming Xiu is with Visual SLAM Algorithm Group, Standard-Robots Ltd.,co. Shenzhen, China. `stevenxiu@aliyun.com`

Fig. 1. 0cm Case No Lean

Fig. 2. 2cm Case No Lean

Solve this quadratic equation

$$\Delta = B^2 - 4AC$$

$$t_x = \frac{-B \pm \sqrt{\Delta}}{2A} \tag{12}$$

The corresponding t_y can be derived from Eq. (7).

Here is a two-fold ambiguity. Only one of the roots are correct. Check them by the third point P3:

$$(x_3 - t_x)^2 + (y_3 - t_y)^2 = {x'_3}^2 + {y'_3}^2 \tag{13}$$

Define

$$\delta_{check} = |(x_3 - t_x)^2 + (y_3 - t_y)^2 - ({x'_3}^2 + {y'_3}^2)| ? = ?0 \tag{14}$$

The right root will satisfy Eq. (13). In practice we can set the threshold of δ_{check}.

IV. Experiments of Oct.17

In the evening of Oct.17, a 2cm translation experiment was conducted.

A. 2cm case data

Notice you need to transform the S' pixel unit to cm.

TABLE I
2cm1017.jpg's P1,P2,P3

2cmNoLean	BS (cm)	CS(pixel)
P1	(0,0)	(267,455)
P2	$(3*cos(30°), 3*sin(30°))$	(271,256)
P3	$(-3*cos(30°), 3*sin(30°))$	(466,460)

Output:

$$t_x = 7.94855$$

$$t_y = -0.215983$$

$$\theta = 120°$$

B. 0cm case data

Output:

$$t_x = 8.82632$$

$$t_y = -2.04685$$

$$\theta = 118.005°$$

TABLE II
0cm1017.jpg's P1,P2,P3

0cmNoLean	BS (cm)	CS(pixel)
P1	(0,0)	(399,458)
P2	$(3*cos(30°), 3*sin(30°))$	(406,257)
P3	$(-3*cos(30°), 3*sin(30°))$	(599,463)

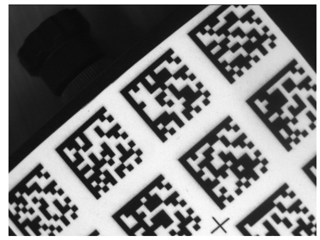

Fig. 3. 0cm Case No Lean

Fig. 4. 2cm Case No Lean

C. Translation Calculation

$$\begin{aligned} Translation &= \sqrt{(7.94855 - 8.82632)^2 + (-0.215983 + 2.04685)^2} \\ &= 2.03cm \end{aligned} \tag{15}$$

The real translation is just 2 cm. So the precision has attained to 0.3mm.

V. Experiments of Oct.19

In the evening of Oct.19, just use the data collected in Sep., to check the algorithm again in Lean case.